A Performance Guaranteed Distributed Multicast Algorithm for Long-Lived Directional Communications in WANETs

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Abstract. We consider the lifetime optimization problem for multicasting in wireless ad hoc networks, in which each node is equipped with a directional antenna and has limited energy supplies. In this paper, we propose a new distributed algorithm, whose performance in terms of providing long-lived multicast tree is guaranteed by our theoretical analysis. We prove that its approximation ratio is bounded by a finite number. In particular, the derived upper bound in a closed form shows that the algorithm can achieve global optimal in some cases. The real performance of this new proposed algorithm is also evaluated using simulation studies and the experimental results show that it outperforms other distributed algorithms.

Keywords: Wireless Ad Hoc Network, Multicast, Directional Antenna, Energy Efficiency, Approximation Algorithm.

1 Introduction

Energy conservation is of paramount importance for the wide deployment of wireless ad hoc networks (WANETs) in the forms of mobile ad hoc networks (MANETs) and wireless sensor networks (WSNs) due to their potentially extensive civil and military applications. Multicasting plays an important role in typical WANETs where bandwidth is scarce and hosts have limited battery power. In addition, many routing protocols for MANETs need a broadcast / multicast as a communication primitive to update their states and maintain the routes between nodes. Multicast is also widely used in WSNs to disseminate information, *e.g.* environmental changes, to other nodes in the network. Therefore, it is essential to develop efficient multicast protocols that are optimized for energy consumption. There are two energy-aware metrics and their corresponding problem formulations that have been most widely studied: (1) to minimize the energy consumption and (2) to maximize the network operating lifetime.

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Both problems have received equal attentions, *e.g.* the work [1-6] for the first problem and [7-15] for the second. In this paper we have focused on the second problem.

The network operating lifetime is typically defined as the duration of the network operation time until the battery depletion of the first node in the network. Some work has considered maximizing the network lifetime in a network for broadcast session, *e.g.* [7-10], or for multicast session, *e.g.* [10-14]. Some optimal solutions [12-14] with polynomial time complexity show that such optimization problem belongs to P. Over the last few years, energy efficient communication in wireless ad hoc networks with directional antennas has received more and more attention. This is because directional communications can save transmission power by concentrating RF energy where it is needed [17, 18]. The same optimization problem using directional antennas has been studied in [15-20] and has been proven to be a NP-hard problem [20]. The exact solution for such difficult problem is presented in [19] based a MILP (mixed integer linear programming) formulation.

The most desirable work [16] proposed two distributed algorithms DMMT-OA/DMMT-DA (Distributed Min-Max Tree algorithm for Omnidirectional / Directional Antennas) to provide long-lived multicasting in WANETs with directional antennas. Simulation results have also shown that these two distributed multicast algorithms for directional communications outperform other centralized multicast algorithms, *e.g.* in [15, 17, 18]. The advance of this work inspires us to further investigate the distributed solutions for this optimization problem. A careful observation on the DMMT-DA algorithm leads to a new distributed algorithm with improved performance. The proposed algorithm uses a node-centric point of view, in stead of the traditional link-centric manner [15, 16], such that it can avoid some cases that are far from optimal. We then use a graph theoretic approach to analysis its theoretical performance in terms of approximation ratio. The derived bound, in a closed analytical expression, of this approximation ratio shows that our proposed algorithm is a constant-factor approximation algorithm. In order to evaluate the real performance of our proposed algorithm, we use simulation as well to compare against a set of distributed algorithms and find that it outperforms other proposals.

2 System Model and Problem Formulation

We model our wireless ad hoc network as a simple directed graph G with a finite node set N ($|N| = n$) and an arc set A corresponding to the unidirectional wireless communication links. Each node is equipped with a directional antenna, which concentrates RF transmission power to where it is needed. We assume a widely used propagation model [1] for adaptive antennas [15-18], in which the antenna at each node v can switch its orientation to any desired direction with transmission power uniformly distributed across its adjustable beamwidth θ_v between $\theta_{\min}$ and 2π. The transmission power p_{vu} to support a link (v, u) separated by a distance r_{vu} ($r_{vu} > 1$) is therefore proportional to r_{vu}^{α} and θ_v with unit signal detection threshold, where the propagation loss exponent α typically takes on a value between 2 and 4. We further assume that any node $v \in N$ can choose its transmission power, strictly within some minimum and maximum levels $p_{\min}$ and $p_{\max}$, respectively, which are positive constant numbers. The transmission power p_{vu} thus can be expressed as follows.

$$p_{vu} = p(r_{vu}, \theta_v) \tag{1}$$

$$p(r, \theta) \equiv \max(p_{\min}, \theta \cdot r^{\alpha} / 2\pi) \leq p_{\max} \tag{2}$$

Let $\varepsilon = \{\varepsilon_v > 0 \mid v \in N\}$ be the energy supply associated with each node in G. The residual lifetime τ_{vu} of an arc $(v, u) \in A(T_s)$ is therefore ε_v / p_{vu}.

We consider a source-initiated multicast with multicast members $M = \{s\} \cup D$ ($|M| = m$), where s is the source node and D are destination nodes. All the nodes involved in the multicast form a multicast tree rooted at the node s, *i.e.* a rooted tree T_s, with a tree node set $N(T_s)$ and a tree arc set $A(T_s)$. We define a rooted tree as a directed acyclic graph with a source node with no incoming arcs, and each other node v has exactly one incoming arc. A node with no out-going arcs is called a leaf node, and all other nodes are internal nodes (also called relay nodes). We use $\Lambda_v^+(T_s)$ and $\lambda_v^+(T_s)$ to denote the child node set and the out-degree (*i.e.* the number of child nodes) of node v in the tree T_s, respectively.

Let Ω_M be the family of all rooted multicast trees spanning nodes in M. The maximum-lifetime multicast problem can thus be expressed as

$$\max_{T_s \in \Omega_M} \min_{(v,u) \in T_s} (\tau_{vu}) = 1 / \min_{T_s \in \Omega_M} \max_{(v,u) \in T_s} (1/\tau_{vu}) . \tag{3}$$

Note that if we assign the tree arc weight function w_{vu} as the reciprocal of the lifetime of the arc (v, u), *i.e.*

$$w_{vu} = 1/\tau_{vu} = p(r_{vu}, \theta_v) / \varepsilon_v , \tag{4}$$

our optimization problem is equivalent to the *min-max tree problem*, which is to determine a directed tree T_s including all the multicast members (*i.e.*, $M \in N(T_s)$) such that the maximum arc weight is minimized. The corresponding optimal solution is just the reciprocal of the lifetime of the maximum-lifetime multicast tree.

Given a multicast tree T_s, we use $\delta_o(T_s)$ and $\delta_d(T_s)$ to denote the maximum arc weight of the same tree in a network instance $G(N, A)$ with omni-directional antennas and directional antennas, respectively, *i.e.*

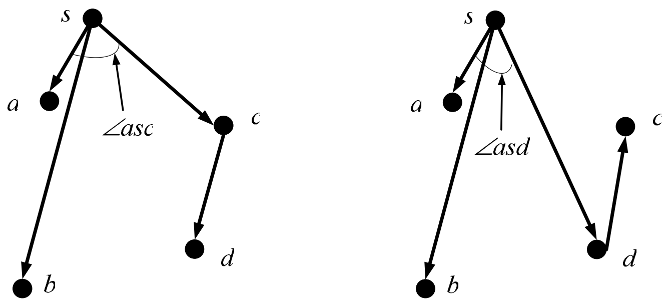
$$\delta_o(T_s) \equiv \max_{(v,u) \in A(T_s)} (p(r_{vu}, 2\pi) / \varepsilon_v) , \tag{5}$$

$$\delta_d(T_s) \equiv \max_{(v,u) \in A(T_s)} (p(r_{vu}, \theta_v) / \varepsilon_v) . \tag{6}$$

The arc with the above weights (5) and (6) is called the *omni-directional* and *directional* bottleneck arc, respectively. Note that the beamwidth θ_v at node v in (6) should be set as the smallest possible angle in the range between $\theta_{\min}$ and 2π to provide the beam-coverage for *all* nodes in $\Lambda_v^+(T_s)$. It has been proven in the recent literature that the Problem (3) belongs to P [12-14] and NP-hard [20] for networks with omni-directional antennas and directional antennas, respectively.

3 A New Distributed Algorithm

As mentioned earlier, the DMMT-DA algorithm [16] is one of the best solutions and especially beneficial to WANETs because of its distributed scheme. It runs in rounds to create a tree and each round includes as many nodes as possible on a minimum arc-weight (defined in Equation 4) basis until all nodes are in the tree. However, the following observation leads to the design of new heuristic algorithm that can improve the performance of DMMT-DA further.



(a) $T_1 = \{(s, a), (s, b), (s, c), (c, d)\}$ (b) $T_2 = \{(s, a), (s, b), (s, d), (d, c)\}$

Fig. 1. An example to show how the performance of DMMT-DA can be improved

A 5-node network instance is given in Fig. 1, in which source node s and all destination nodes a, b, c and d have the same energy level ε. Note that the Euclidean distance between each pair of nodes is exactly indicated in Fig. 1. We consider an intermediate solution obtained from the DMMT-DA algorithm with tree arcs (s, a) and (s, b). In the following iteration, we assume that arc (s, c) will be included into the tree by DMMT-DA because it has the minimum weight, *i.e.* $w_{sc} < w_{sd}$, or equivalently $p(r_{sc}, \angle asc) < p(r_{sd}, \angle asd)$, in which the symbol $\angle xyz$ denotes the angle between the two rays of yx and yz. Finally, the multicast tree T_1 is achieved by DMMT-DA as shown in Fig. 1a with $\delta_d(T_1) = p(r_{sb}, \angle asc) / \varepsilon$. Now we consider an alternative arc (s, d) to be included into the tree in the same iteration and the final tree should be T_2 as shown in Fig. 1b with $\delta_d(T_2) = p(r_{sb}, \angle asd) / \varepsilon$. It is obvious that T_2 is a better solution, *i.e.* $\delta_d(T_1) > \delta_d(T_2)$, because $\angle asc > \angle asc$. In other words, the solutions found by DMMT-DA based on the arc-weight may sometimes be far deviated from the optimum.

The above example motivates us to apply a node-centric approach, *i.e.* to use a node-weight instead of an arc-weight defined in (4) as the criteria, to increment a multicast tree such that the performance of DMMT-DA would be improved. In this section, we propose a new algorithm, DMMT-NC (Distributed Min-Max Tree algorithm with Node-Centric approach), for the min-max tree problem. The multicast tree is constructed in a distributed and incremental manner. Initially, the multicast tree T_s only contains the source node. It then iteratively performs a *Search-and-Grow* procedure until the tree contains all the nodes in M. The final multicast tree T_s is therefore obtained by pruning all transmissions that are not needed to reach the nodes in M.

We use T_s^i to denote an intermediate tree constructed by the DMMT-NC algorithm after the i-th node is added into the tree. As implied by the name of the algorithm, each node v maintains a node weigh w_v^i ($0 \le i \le n$-1) at each step a tree is incremented, which is defined as follows.

$$w_v^i \equiv \min_{u \in N - N(T_s^i)} (w_{vu}^i) \tag{7}$$

$$w_{vu}^i \equiv p(r_{vu}^i, \varphi_{vu}^i) / \varepsilon_v . u \in N - N(T_s^i) \tag{8}$$

$$r_{vu}^i \equiv \max_{x \in \{u\} \cup \Lambda_v^+(T_s^i)} (r_{vx}) \tag{9}$$

$$\varphi_{vu}^i \equiv \min \left\{ \theta_v \middle| \ \theta_v \text{ covers each node in } \{u\} \cup \Lambda_v^+(T_s^i) \right\} \tag{10}$$

Note that the variable r_{vu}^i denotes the longest Euclidean distance between node v and any node x already included in the tree T_s^i and a node u outside the tree. Similarly, variable φ_{vu}^i denotes the minimum beamwidth required by node v to cover all its child nodes already in the tree T_s^i as well as an additional node u outside the tree. In this way, the tree incremental operation by including the candidate node u, satisfying the condition

$$w_{vu}^i = w_v^i , \tag{11}$$

would lead to the lifetime of the resulting intermediate tree to be maximized over all possible choices of any node could be included into the tree. This approach is based on a node's point of view, which is different from other proposed algorithms.

In the following, we give a formal description of the DMMT-NC algorithm. The formulations shall help us understand the subsequent theoretical analysis that the proposed heuristic algorithm has an approximation ratio bounded by a constant number. The description of the DMMT-NC algorithm in pseudo code is given in Fig. 2.

The DMMT-NC Algorithm

```
(1)  Initialize i = 0, N(T_s^i) = {s} and A(T_s^i) = φ;
(2)  Repeat
         // Search Phase
(3)      δ ≡ min{w_v^i | v ∈ N(T_s^i)};
         // Grow Phase
(4)      while (∃v ∈ N(T_s^i), w_v^i ≤ δ ∧ ∃u ∈ N − N(T_s^i), w_vu^i = w_v^i)
(5)          i = i + 1;
(6)          N(T_s^i) = N(T_s^(i−1)) ∪ {u}, A(T_s^i) = A(T_s^(i−1)) ∪ {(v,u)};
(7)          Update w_v^i for each v ∈ N(T_s^i) using (7 – 10);
(8)  until (M ⊆ N(T_s));
(9)  Obtain the final multicast tree T_s by pruning T_s^i.
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Fig. 2. The DMMT-NC Algorithm

4 Theoretical Performance Analysis

In this section, we study the theoretical performance of the proposed algorithm in terms of approximation ratio[1]. We use δ_o^* and δ_d^* to denote the optimal solutions for the min-max tree problem under *omni-directional* and *directional* scenarios, respectively, *i.e.*

$$\delta_o^* = \min_{T_s \in \Omega_M} \delta_o(T_s) \,, \tag{12}$$

$$\delta_d^* = \min_{T_s \in \Omega_M} \delta_d(T_s) \,. \tag{13}$$

Given a multicast tree T_s obtained by the DMMT-NC algorithm, its approximation ratio ρ can be expressed as

$$\rho = \delta_d(T_s) / \delta_d^* \,. \tag{14}$$

In the following, we first provide several fundamental results that shall be used to derive the upper bound of the approximation ratio for the heuristic algorithm DMMT-NC. Let C_X denote the cut connecting a node partition X and $N–X$, in which the first node set X must include the source node s and the second node set $N - X$ must include at least one destination node, *i.e.*

$$C_X \equiv \{(v, u) \mid v \in X \wedge u \in N - X \wedge s \in X \wedge D \not\subset X\}. \tag{15}$$

We use $\psi(C_X)$ to denote the minimum weight of the cut links under omni-directional scenarios, *i.e.*

$$\psi(C_X) = \min_{(v,u)\in C_X} (p(r_{vu}, 2\pi) / \varepsilon_v) \,. \tag{16}$$

Theorem 1. If $G(N, A)$ is connected then for any cut C_X, then

$$\delta^* \geq \delta_0 \cdot \theta_{\min} / 2\pi \,. \tag{17}$$

Proof: Note that there is at least one destination node z ($z \in D$) belonging to $N - X$, *i.e.*, $z \in N - X$, because $D \not\subset X$. Let T_s^* be a min-max tree of network G with omni-directional antenna. There must exist an arc $(x, y) \in A(T_s^*)$ connecting X and $N - X$ (*i.e.*, $(x, y) \in C_X$) in order to satisfy that there must exist a directed path from s to the destination node z along the links in the tree T_s^*. Therefore, we can obtain (17) as follows. $\delta_o^* = \delta_0(T_s^*) = \max_{(v,u)\in A(T_s^*)} p(r_{vu}, 2\pi)/\varepsilon_v \geq p(r_{xy}, 2\pi)/\varepsilon_x \geq \min_{(v,u)\in C_X} p(r_{vu}, 2\pi)/\varepsilon_v = \psi(C_X)$. □

It is a straightforward exercise to obtain the following conclusion if a function $K_{vu}(\theta_1, \theta_2)$ is defined as

$$K_{vu}(\theta_1, \theta_2) \equiv p(r_{vu}, \theta_1) / p(r_{vu}, \theta_2) \,. \tag{18}$$

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[1] An algorithm for a problem has an approximation ratio of $\rho(n)$ if, for any input of size n, the expected cost c of the solution produced by the algorithm is within a factor of $\rho(n)$ of the cost c^* of an optimal solution: $\max\{c / c^*, c^* / c\} \leq \rho(n)$.

Lemma 1. For any $(v, u) \in A$, $K_{vu}(\theta_1, \theta_2)$ satisfies

$$\begin{cases} \max(\theta_1 / \theta_2, p_{\min} / p_{\max}) \le K_{vu}(\theta_1, \theta_2) \le 1 & \theta_1 \le \theta_2 \\ 1 \le K_{vu}(\theta_1, \theta_2) \le \min(\theta_1 / \theta_2, p_{\max} / p_{\min}) & \theta_1 \ge \theta_2 \end{cases}. \tag{19}$$

Theorem 2. The optimal solutions δ_o^* and δ_d^* satisfy

$$\delta_d^* \ge \max(\theta_{\min} / 2\pi, p_{\min} / p_{\max}) \cdot \delta_o^* . \tag{20}$$

Proof: Considering $\theta_v \ge \theta_{\min}$ for any given multicast tree T_s and using Lemma 1, we then have the following derivations.

$$\begin{aligned} \delta_d^* &= \min_{T_s \in \Omega_M} \max_{(v,u)\in T_s} (p(r_{vu}, \theta_v) / \varepsilon_v) \\ &= \min_{T_s \in \Omega_M} \max_{(v,u)\in T_s} (K_{vu}(\theta_v, 2\pi) \cdot p(r_{vu}, 2\pi) / \varepsilon_v) \\ &\ge \min_{T_s \in \Omega_M} \max_{(v,u)\in T_s} (\max(\theta_v / 2\pi, p_{\min} / p_{\max}) \cdot p(r_{vu}, 2\pi) / \varepsilon_v) \\ &\ge \min_{T_s \in \Omega_M} \max_{(v,u)\in T_s} (\max(\theta_{\min} / 2\pi, p_{\min} / p_{\max}) \cdot p(r_{vu}, 2\pi) / \varepsilon_v) \\ &= \max(\theta_{\min} / 2\pi, p_{\min} / p_{\max}) \cdot \min_{T_s \in \Omega_M} \max_{(v,u)\in T_s} (p(r_{vu}, 2\pi) / \varepsilon_v) \\ &= \max(\theta_{\min} / 2\pi, p_{\min} / p_{\max}) \cdot \delta_o^* \end{aligned}$$

□

We now turn our attention to the most interesting and difficult task on deriving the approximation ratio of the DMMT-NC algorithm. Suppose that T_s is the final multicast tree obtained from the algorithm described in Fig. 2 and the *directional* bottleneck arc (v, u) of T_s is the i-th arc added into the tree, *i.e.* the intermediate tree is T_s^{i+1} after arc (v, u) is included. Let φ_v be the beamwidth applied by the node v in T_s. The solution $\delta_d(T_s)$ can thus be expressed as follows.

$$\begin{aligned} \delta_d(T_s) &= p(r_{vu}, \varphi_v) / \varepsilon_v \\ &= K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot p(r_{vu}, \varphi_{vu}^i) / \varepsilon_v \\ &\le K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot p(r_{vu}^i, \varphi_{vu}^i) / \varepsilon_v \\ &= K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot w_{vu}^i \\ &= K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot w_v^i \end{aligned}. \tag{21}$$

We further assume that arc (v', u') is the first one added into the tree in the same round of the *Search-and-Grow* iteration (described in Fig. 2) as arc (v, u) is included and the resulting intermediate tree is T_s^{j+1} ($j \le i$). Based on the description in Fig. 2, we have $w_v^i \le w_{v'}^j$. Now we define a cut C_X, where $X \equiv N(T_s^j)$, and let arc (x, y) be the one in C_X such that

$$\psi(C_X) = p(r_{xy}, 2\pi) / \varepsilon_x . \tag{22}$$

Recall that just before the intermediate tree T_s^{j+1} is formed, arc (v', u'), instead of (x, y), is chosen to be included, which implies $w_{v'}^j \le w_x^j$. By summarizing the above analysis, we have the derivation as follows.

$$w_v^i \le w_{v'}^j \le w_x^j \le w_{xy}^j = p(r_{xy}^j, \varphi_{xy}^j) / \varepsilon_x \tag{23}$$

We consider the following two cases.

Case 1: $r_{xy}^j = r_{xy}$

The above equation can be rewritten as

$$w_v^i \le p(r_{xy}, \varphi_{xy}^j) / \varepsilon_x = K_{xy}(\varphi_{xy}^j, 2\pi) \cdot p(r_{xy}, 2\pi) / \varepsilon_x . \tag{24}$$

Case 2: $r_{xy}^j = r_{xz_1}$ as shown in Fig. 3.

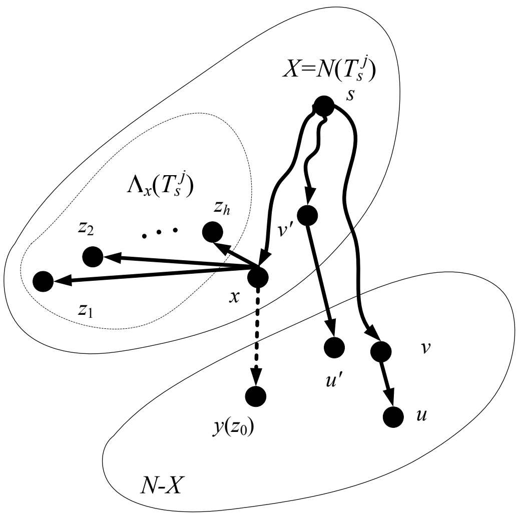


Fig. 3. Illustration used to derive the approximation ratio of DMMT-NC

This means arc (x, z_1) is already in the tree T_s^j and we assume the resulting tree is $T_s^{k_1}$ ($k_1 < j$) just after it is included. Considering arc (x, z_1), instead of (x, y), is chosen to be included at that moment, we have $w_{xz_1}^{k_1} \le w_{xy}^{k_1}$ or equivalently

$$p(r_{xz_1}^{k_1}, \varphi_{xz_1}^{k_1}) \le p(r_{xy}^{k_1}, \varphi_{xy}^{k_1}) . \tag{25}$$

Furthermore, the condition $r_{xy}^j = r_{xz_1}$ also implies

$$r_{xz_1}^{k_1} = r_{xz_1} . \tag{26}$$

Now equation (23) under case 2 can be rewritten as follows by combining (25) and (26).

$$\begin{aligned} w_v^i &\le p(r_{xz_1}, \varphi_{xy}^j) / \varepsilon_x \\ &= K_{xz_1}(\varphi_{xy}^j, \varphi_{xz_1}^{k_1}) \cdot p(r_{xz_1}, \varphi_{xz_1}^{k_1}) / \varepsilon_x \\ &= K_{xz_1}(\varphi_{xy}^j, \varphi_{xz_1}^{k_1}) \cdot p(r_{xz_1}^{k_1}, \varphi_{xz_1}^{k_1}) / \varepsilon_x \\ &\le K_{xz_1}(\varphi_{xy}^j, \varphi_{xz_1}^{k_1}) \cdot p(r_{xy}^{k_1}, \varphi_{xy}^{k_1}) / \varepsilon_x \end{aligned} \tag{27}$$

Comparing (27) and (23), we can conclude that the above equation can be further derived similarly under two cases of 1) $r_{xy}^{k_1} = r_{xy}$ or 2) $r_{xy}^{k_1} = r_{xz_2}$ as shown in Fig. 3 until Case 1) is met.

Generally, we assume that the Case 1) is met at the h-round of the above derivation iteration, *i.e.*

$$\begin{cases} r_{xy}^{k_l} = r_{xz_{l+1}} & 0 \le l \le h-1 \\ r_{xy}^{k_l} = r_{xy} & l = h \end{cases}, \tag{28}$$

and the following equation will be eventually achieved.

$$\begin{aligned} w_v^i &\le \prod_{l=1}^{h} K_{xz_l}(\varphi_{xz_{l-1}}^{k_{l-1}}, \varphi_{xz_l}^{k_l}) \cdot p(r_{xy}^{k_h}, \varphi_{xy}^{k_h}) / \varepsilon_x \\ &= \prod_{l=1}^{h} K_{xz_l}(\varphi_{xz_{l-1}}^{k_{l-1}}, \varphi_{xz_l}^{k_l}) \cdot p(r_{xy}, \varphi_{xy}^{k_h}) / \varepsilon_x \\ &= \prod_{l=1}^{h} K_{xz_l}(\varphi_{xz_{l-1}}^{k_{l-1}}, \varphi_{xz_l}^{k_l}) \cdot K_{xy}(\varphi_{xy}^{k_h}, 2\pi) \cdot p(r_{xy}, 2\pi) / \varepsilon_x \\ &= H \cdot p(r_{xy}, 2\pi) / \varepsilon_x \end{aligned} \tag{29}$$

Note that item H in (29) is defined as

$$H \equiv K_{xy}(\varphi_{xy}^{k_h}, 2\pi) \cdot \prod_{l=1}^{h} K_{xz_l}(\varphi_{xz_{l-1}}^{k_{l-1}}, \varphi_{xz_l}^{k_l}) \tag{30}$$

and the boundary conditions of (29) are given below.

$$k_0 = j, z_0 = y, 0 \le h \le \lambda_x^+(T_s^j) \tag{31}$$

Finally, combining (21), (29), (22), (17) and (20) sequentially, we obtain

$$\begin{aligned} \delta_d(T_s) &\le K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot w_v^i \\ &\le K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot H \cdot p(r_{xy}, 2\pi) / \varepsilon_x \\ &= K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot H \cdot \psi(C_X) \\ &\le K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot H \cdot \delta_o^* \\ &\le K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot H \cdot \delta_d^* / \max(\theta_{\min} / 2\pi, p_{\min} / p_{\max}). \end{aligned} \tag{32}$$

The above analysis now allows us to obtain the following conclusion.

Theorem 3. The DMMT-NC algorithm is a constant-factor approximation algorithm with an approximation ratio ρ bounded by

$$\rho \le \mu_\rho \equiv H \cdot K_{vu}(\varphi_v, \varphi_{vu}^i) \cdot \min(2\pi / \theta_{\min}, p_{\max} / p_{\min}) . \tag{33}$$

It is a straightforward exercise based on (19) to verify that μ_ρ is bounded by a constant number. In particular, we can conclude $H \le 1$ since

$$\varphi_{xz_{l-1}}^{k_{l-1}} \le \varphi_{xz_l}^{k_l} \le 2\pi, \quad 1 \le l \le h . \tag{34}$$

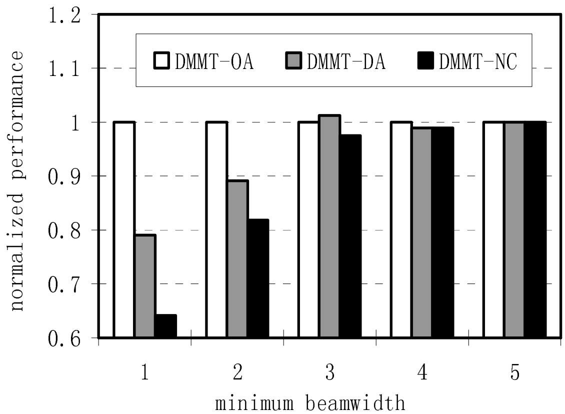
On the other hand, it is not sure $K_{vu}(\varphi_v, \varphi_{vu}^i) \le 1$ because the relation of φ_v and φ_{vu}^i is not deterministic in general due to the post-pruning operation.

5 Experimental Performance Evaluation

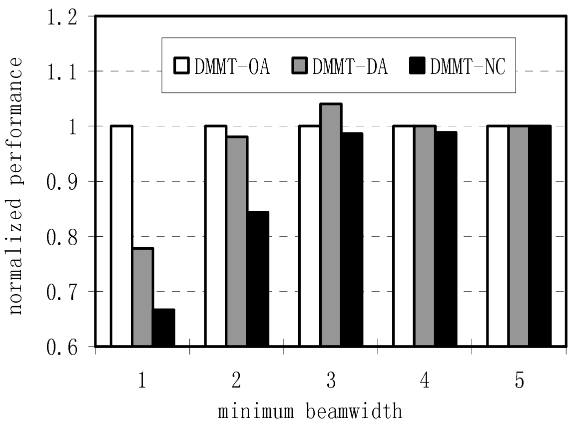
We have performed a simulation study for evaluating a set of distributed algorithms DMMT-OA [16], DMMT-DA [16] and the new proposed DMMT-NC. Their solutions

Table 1. Parameter values for simulation

Parameters	Values
n	100
m	50 and 100
θ_{min}	15°, 30°, 60°, 90°, and 360°
p_{max}	10
p_{min}	0.1
$(E(\varepsilon), D(\varepsilon))$	(500, 200)
α	2



(a) $m = 50$



(b) $m = 100$

Fig. 4. Normalized performance as a function of the minumum beamwidths 15°, 30°, 60°, 90°, and 360° (corsponding to the numbers 1 – 5 on the x-axle) under various multicast sizes

are denoted as δ_1, δ_2 and δ_3, respectively. We use the metric δ_i/δ_1 ($i = 1, 2, 3$) to evaluate their relative performance, which allows us to facilitate the comparison of different algorithms over a wide range of network examples. In each network example, a number of nodes are randomly generated within a square region 10×10. The values of parameters used in simulation are given in Table 1. We randomly generated 100 different network examples, and we present here the average over those examples for all cases.

Fig. 4 depicts graphically the normalized performances over different connected network topologies. The x-axis represents the minimum beamwidths 15°, 30°, 60°, 90° and 360° (corresponding to the numbers 1 - 5 on the x-axle) and the y-axis presents the mean of δ_i/δ_1 for all three distributed algorithms. Referring to the multicast size $m = 50$ in Fig. 4a, we observe that the new proposed distributed algorithm DMMT-NC improves the other two algorithms significantly when the minimum beamwidth is small. In particular, such improvement is over 30% and 15% compared to DMMT-OA and DMMT-DA, respectively. On the other hand, once the minimum beamwidth increases (greater than 90°), all algorithms converge to the same performance (optimal solutions [16]). A similar observation can be made for the broadcasting scenarios $m = 100$ as shown in Fig. 4b.

6 Conclusion

We have presented a new distributed long-lived multicast algorithm for directional communications in wireless ad hoc networks. Our proofs show that it is a constant-factor approximation algorithm. Our efforts are also validated via the simulation study, in which the experimental results show that our new algorithm has better performance than other distributed algorithms.

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