Truthful Spectrum Auction Design for Secondary Networks

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Outline

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Introduction

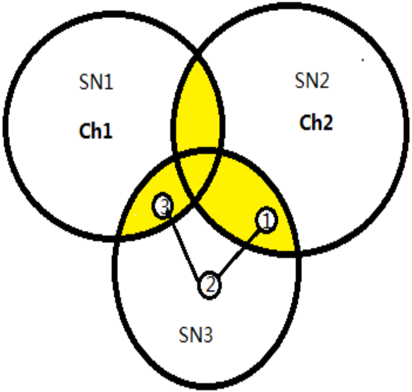
Problem

How to achieve truthful spectrum auction when users have multihop routing demands?

Solution

model non-licensed users as secondary networks and allocate channels to SNs in a coordinated fashion that maximizes social welfare of the system using a key technique that decompose a linear program (LP) solution for assignment into a set of integer program (IP) solutions.

Introduction



Key challenges

Crux

- how to ensure the auction is truthful in multihop network
- how to maximizes social welfare of the system

truthfulness

- no incentive to lie
- the probability of bid b_i is non-decreasing
- there exists a minimum bid b_i^*

maximizing social welfare

- interference
- reuse of channels
- equivalent to the graph coloring problem, and is NP-hard

Crux

- The path to bid for is naturally best made by the auctioneer, i.e., the PN.
- A bid from an SN includes just a price it wishes to pay, with two nodes it wishes to connect using a path.
- Not only that they transmit along multihop paths, but each path can be assigned with distinct channels at different links.

Key results and contributions

- First design a simple heuristic auction for spectrum allocation to SNs, which guarantees both truthfulness and interference-free channel allocation
- Next design a randomized auction, which is truthful in expectation, and is provably approximate optimal in social welfare

Truthful Auction Design

- We denote by $p(i)$ and b_i the payment and bid of agent i, and ω_i as nonnegative valuation of each agent i. Then the utility of i is a function of all the bids: $u_i(b_i, b_{-i}) = \omega_i - p(i)$
- We assume that each agent i is selfish and rational
- An auction is truthful if for any agent i with any $b_i \neq \omega_i$,any b_{-i}, we have

$$u_i(\omega_i, b_{-i}) \geq u_i(b_i, b_{-i}) \tag{1}$$

- A randomized auction is truthful in expectation if (1) holds in expectation.

Truthful Auction Design

Theorem 1. Let $P_i(b_i)$ be the probability of agent i with bid b_i winning an auction. An auction is truthful if and only if the followings hold for a fixed b_{-i}

- $P_i(b_i)$ is monotonically non-decreasing in b_i;
- Agent i bidding b_i is charged $b_i P_i(b_i) - \int P_i(b)db$

- That is, there must be an crucial bid b_i^*, such that the agent i will win if he bids at least b_i^*.

System Model

- SNs as agents and the PN as the auctioneer
- We use l_{uv}^{i} to denote the link from node u_i to node v_i belonging to SN i, and f_{uv}^{i} to denote the amount of flow on link l_{uv}^{i}.
- Before the auction starts, each SN i submits to the auctioneer a compound bid, defined as $B_i = (G_i(\xi_i; \nu_i); s_i; d_i; b_i)$ (G_i is connectivity graph). Then the conflict graph can be centrally obtained by the auctioneer.

System Model

- $x(c, l_{uv}^i)$ is a binary var: whether channel c is allocated to link l_{uv}^i
- Two links l_{uv}^i and l_{pq}^j interfere if a node in u,v is within the interference range of a node in p,q, and cannot be assigned the same channel if $i \neq j$.
- We denote by $R_T(u_i)$ and $R_I(u_i)$ the transmission range and interference range of node u_i, $\Delta = \frac{R_I(u_i)}{R_T(u_i)}$, $R_I(u_i) \geq R_T(u_i)$

Constraints

- Channel Interference Constraints:

$$x(c, l_{uv}^{i}) + x(c, l_{pq}^{j}) \leq 1 \quad (2)$$

- Capacity Constraints:

$$\sum f_{uv}^{i} \leq \sum x(c, l_{uv}^{i}) \leq 1 \quad (3)$$

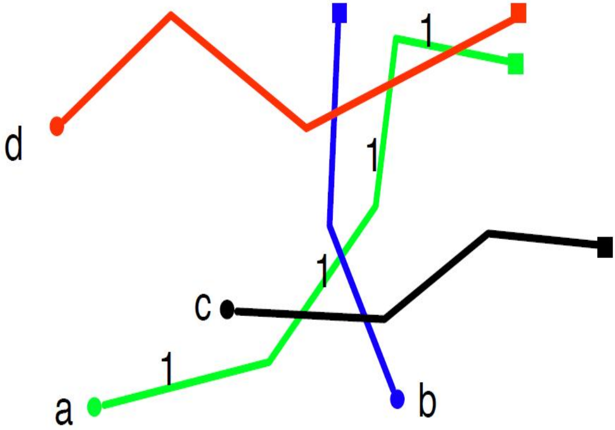
Algorithm1:greedy style allocation

- Compute the shortest path for each agent as its end-to-end path
- Virtual bid of SN i is($I_s(i)$ is the set of SNs that interfere with i along the path,j means the number j hop and total m):

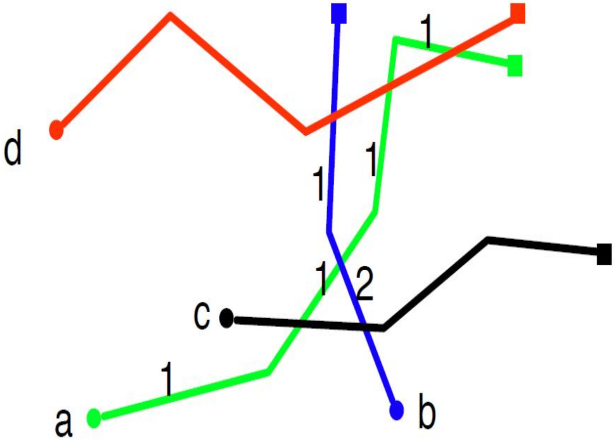
$$\phi(i) = \sum \frac{b_i}{m \cdot I_s(ij)} \quad (4)$$

- Assign minimum indexed available channels along the paths to each link

Example



Example



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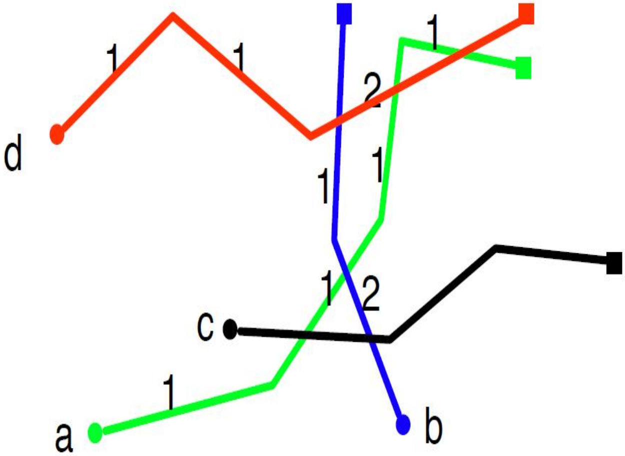
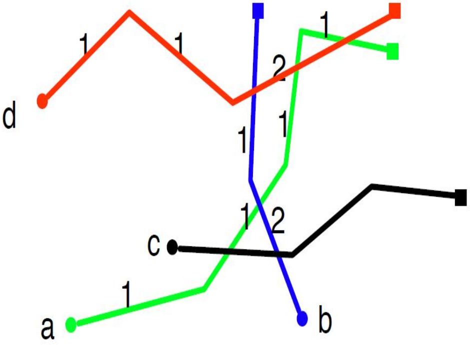


Algorithm2:Payment Calculation

- In every hop, calculate the average bid $S(i) = \frac{1}{n} \cdot \sum \frac{b_j}{m}$
- Agent i’ s payment can be computed as follows:

$$p(i) = \sum S(i) \tag{5}$$

Example



Truthful

Theorem 2. The auction in Algorithms 1 and 2 is truthful and individually rational.

- $r(b_i) = 0$ when agent i doesn’ t receive a channel and $r(b_i) = 1$ when agent i receives a channel

Let w_i and w_i^* be agent i’ s bid when being truthful and not truthful

$\omega_i^* < \omega_i$

- $r(\omega_i^*) = 0$,$r(\omega_i) = 0$ no incentive to lie
- $r(\omega_i^*) = 1$,$r(\omega_i) = 0$ impossible
- $r(\omega_i^*) = 0$,$r(\omega_i) = 1$ rational, no incentive to lie
- $r(\omega_i^*) = 1$,$r(\omega_i) = 1$ the critical bidder does not change

$\omega_i^* > \omega_i$

- $r(\omega_i^*) = 0$,$r(\omega_i) = 0$ no incentive to lie
- $r(\omega_i^*) = 1$,$r(\omega_i) = 0$ negative utility $p(i) > \omega(i)$
- $r(\omega_i^*) = 0$,$r(\omega_i) = 1$ impossible
- $r(\omega_i^*) = 1$,$r(\omega_i) = 1$ the critical bidder does not change

Future work

- Improve the performance guarantee of the randomized auction, by proving a tighter bound on social welfare approximation


Thanks