

# Sensing Processes Participation Game of Smartphones in Participatory Sensing Systems

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**Abstract**—Participatory sensing has gained more and more attention in recent years, since it provides a promising approach which enables large-scale data collection and analysis with a crowd of public workers. In a participatory sensing system, the platform recruits workers to participate in multiple sensing processes. However, sensing processes are usually constrained by limited budgets, while the demands of workers are highly heterogeneous due to various factors. It is then crucial for workers to participate in the sensing processes effectively while their costs are covered by the payments. In this paper, we propose a general sensing processes participation game framework with heterogeneous workers and sensing processes to address this issue. We show that it is NP-hard to find a sensing processes participation solution which maximizes the number of satisfied workers. Inspired by the finite improvement property of the game, we design and implement a sensing processes participation algorithm which is guaranteed to reach a pure Nash equilibrium in polynomial time, by allowing workers to change their strategy profiles asynchronously due to satisfaction incentives. We also demonstrate that the performance of the algorithm is close to optimal when workers and sensing processes are not very heterogeneous, by bounding the price of anarchy. Simulation results show that our algorithm is effective and efficient.

**Index Terms**—Participatory sensing, smartphones, participation game, congestion game, sensing data.

## I. INTRODUCTION

Over the past few years, the usage of smartphones is expanded incredibly all over the world. Significant improvements on 4G cellular networks and hardware manufacturing make the smartphone more attractive and accessible. It is estimated that a total of 1.5 billion smartphones will be shipped worldwide in 2017, according to the International Data Corporation (IDC) Worldwide Quarterly Mobile Phone Tracker [1]. The popularity of smartphones has greatly changed our lifestyles in many aspects including business, health care, social networks, environment monitoring, safety, and transportation [2].

Nowadays multiple cheap but powerful sensors (i.e., GPS, microphone, camera, digital compass, accelerometer and gyroscope) have been embedded in the smartphone to enhance user experience, while the computational performance is boosted by powerful processors. All of these enhancements have made smartphone a powerful handheld device for large-scale data sensing and processing. It is feasible to build participatory sensing systems to collect environment and social sensing data with personal smartphones. Previous studies have proposed numerous systems and applications to utilize the potential of participatory sensing, including Zee [3] and FreeLoc [4] for indoor localization, GigaSight [5] for video collection, PMP

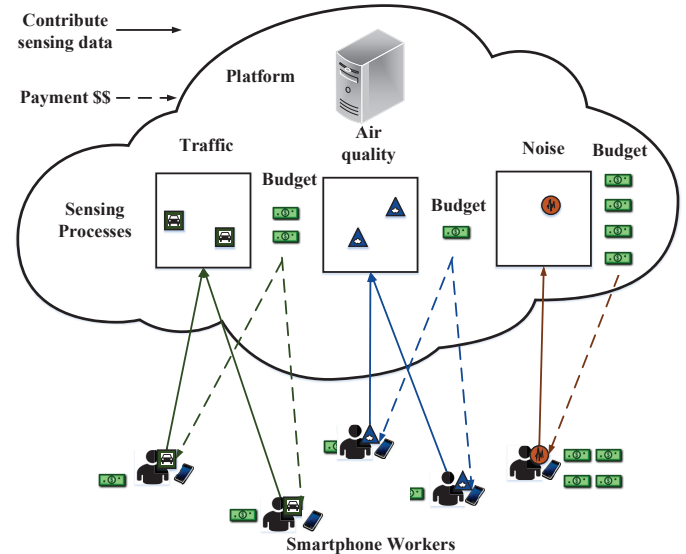
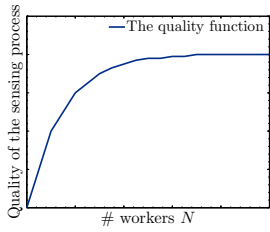


Figure 1. A participatory sensing system.

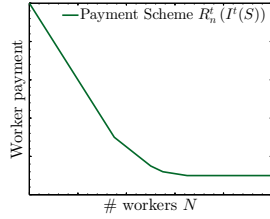
[6] for privacy protection, a crowd counting solution [7] based on audio tones, a bus arrival time prediction system [8] and an autonomous place naming system [9].

We introduce the structure of a participatory sensing system in Fig. 1. A participatory sensing *platform* possesses some continuous *sensing processes*, each of them focus on the collection of a kind of sensing data (i.e., traffic flow, air quality or noise level). A sensing process consists of a series of tasks, and the task is a specified part of the sensing process (i.e., observing the traffic flow at a crossroad for 10 minutes, measuring the noise level of a bus station for 5 minutes). The platform publicizes the descriptions and requirements of the sensing processes to a large group of smartphone *workers*. A worker who is interested joins one of the sensing processes and accomplishes one task to contribute the sensing process. After the collection of the sensing data, the platform computes and delivers the payment to each worker.

Considering that participating in the sensing process naturally incurs a *cost* for the worker, it is essential for the worker to receive a payment, which is usually in the form of a monetary reward. However, the budgets of the sensing processes is usually severely constrained, while the quality of the sensing process depends crucially on the number of participating workers. Since sensing data collected by different users may be redundant, the issue of marginal effect arises as



**Figure 2.** Illustration of the marginal effect of the sensing process [10].



**Figure 3.** Illustration of the payment scheme.

the number of participating workers grows larger [10]. The more workers participating in the same sensing process, the less value of each submitted task. It naturally implies that the payment scheme of the sensing process is not a fixed value, but decreases as the number of workers participating the same sensing process increases (see Fig. 2 and Fig. 3 for illustrations).

Moreover, workers may have different goals and costs due to various factors even when joining the same sensing process. The factors include the schedule of worker activities, the energy cost of data sensing, processing and transmission, the geographical distance travelled for the task, the worker’s skill level in required fields, even the discomfort of the worker caused by accomplishing the task [11]. Workers therefore have satisfaction states when participating in the sensing processes. If the payment is not sufficient enough to compensate the resource consumption, the potential privacy breach and its own interest, a worker will refuse to participate. It is then important to make sure the participatory sensing system satisfies as many workers as possible.

Several studies [12] [13] on the incentive mechanism design have been proposed for participatory sensing systems. Yang et al. [13] propose a platform-centric model and a user-centric model for participatory sensing systems, and two incentive mechanisms for each model. In [12], a reverse auction based incentive mechanism called RADP is proposed to minimize the incentive cost. Auction based incentive mechanisms are conducted by the platform in a centralized manner, and none of them consider the issues of marginal effect and data redundancy. Requiring users to report costs to the platform is also impractical due to the issue of privacy.

In this paper, we propose a general sensing processes participation game framework for participatory sensing systems with heterogenous workers and sensing processes. The idea of the sensing processes participation game is based on the congestion games [14] in game theory. There are multiple *players* and *resources* in congestion games, where a player selects a resource to use. The utility of the player is not only related to the resource it selects, but also the number of the players who are sharing the same resource. By thinking the players as smartphone workers, and the resources as sensing processes, the sensing processes participation game can be modeled as a congestion game. With *the finite improvement property* of the sensing processes participation game, We have designed and implemented a distributed sensing processes participation

algorithm to find *the best response strategy* for everyone. The workers are allowed to compute their best response strategies and update their strategy profile asynchronously. The system will eventually reach a pure Nash equilibrium in polynomial time. We have performed the simulation and results show that the performance of our sensing processes participation algorithm is guaranteed in most general cases when workers have similar costs and goals.

The main contributions of our work are listed as follows.

- We propose a general sensing processes participation game framework for participatory sensing systems with heterogenous workers and sensing processes, and the privacy information of the worker is preserved and protected. We obtain several significant analytic results of our game framework.
- We prove that it is NP-hard to find a social optimum solution of a sensing processes participation game, which is the motivation of our work.
- We prove that every sensing processes participation game possesses the finite improvement property, which enables us to design a fast sensing processes participation algorithm. The algorithm allows the workers to self-organize into a *satisfied* pure Nash equilibrium easily. Every worker can find its best response strategy with the algorithm. We also prove that the performance of the algorithm is close to the social optimal solution when the workers are not too heterogenous.
- We conduct extensive simulations to evaluate the performance of the sensing processes participation algorithm. Results show that our algorithm is effective and efficient.

The rest of the paper is organized as follows. Related works are discussed in Section II. We introduce the sensing processes participation game model in Section III, then propose and study its property in Section IV. The sensing processes participation algorithm is proposed in Section V. We will evaluate the performance of the algorithm in Section VI. Finally the paper is concluded Section VII.

## II. RELATED WORK

Participatory sensing systems and applications are proposed in many studies to collect sensing data from smartphone users. Rai et al. [3] propose a participatory sensing system called Zee to collect site-specific calibration data of radio frequency fingerprinting for indoor localization. GigaSight is a scalable Internet system proposed in [5], which is capable of continuous collecting videos from remote devices.

To estimate the number of mobile devices in an area, [7] propose a crowd counting solution based on audio tones with the help of microphones and speaker phones. Zhou et al. [8] propose a bus arrival time prediction system with multiple sensing resources, including cell tower signals, movement statuses and audio recordings. An autonomous place naming system is proposed in [9] with the crowdsourcing data and the information extracted from social networks.

There are several research studies on incentive mechanisms for mobile phone sensing [11], [13], [15], [16], [17]. In [11],

a reverse auction incentive mechanism is proposed for user participation level determination and payment allocation, while preserving the specific participation cost of the users. Yang et al. [13] propose two incentive mechanisms, one based on Stackelberg game for a platform-centric model and another based on auction for a user-centric model. The knowledge of user participation cost is required in the first mechanism, which is not practical due to the issue of privacy.

Luo et al. [15] propose two incentive schemes to maximize the fairness and the social welfare of information service crowdsourcing scenarios, where the user gets the service as a payment. A reputation mechanism has been proposed in [16] for improving the performance of pricing schemes in crowdsourcing applications, while Liu et al. [17] propose an efficient network management framework to tackle a series of challenges including motivating the users to participate.

All of the above incentive mechanisms are centralized approaches toward the issue of incentive. As the number of users grows larger, the users are becoming more and more heterogenous, which means the platform has to gather massive amount of user information to perform the centralized optimization. Moreover, sensing data collected from workers have marginal effect, since the participating sensing system suffers from problems of data redundancy [10]. Therefore the payment for a smartphone worker shall decrease as the number of the workers participating the same sensing process increases. None of the above mechanisms address this issue.

There are several congestion game based algorithms proposed in [18] and [19], in the context of spectrum sharing. Due to the fact that the channels of the spectrum are uncontrolled, an unlicensed user is selfish and interested in utilizing its own transmission bandwidth when the licensed user is dormant. Southwell et al. [18] propose a spectrum mobility game to utilize the social welfare of the unused channels, while in [19], the issue of *spectrum reuse* that different users can share the same channel without introducing interference when far enough is discussed. These approaches are not readily to be applied in the context of participatory sensing, since they considered specific properties pertain to the spectrum sharing problems.

### III. SENSING PROCESSES PARTICIPATION GAME

In this section, we formally define the sensing processes participation game in participatory sensing. We will first introduce the system model of participatory sensing, then we propose the sensing processes participation game. We also propose the key game concepts and a simplified form of sensing processes participation game for the convenience of discussion.

#### A. Preliminaries

We introduce the participatory sensing system shown in Fig. 1. The system consists of a participatory sensing *platform* and many smartphone *workers*. The whole process of participatory sensing is as follows. The platform first formalizes and publicizes *sensing processes*. A sensing process is a series of

tasks aimed at collecting the same kind of sensing data (i.e. traffic status, air quality and noise level). Each worker joins and contributes to a sensing process by accomplishing one of the tasks belonging to the sensing process. Assume there are a set of  $\mathcal{N} = \{1, 2, 3, \dots, N\}$  workers are interested in participating the sensing processes, where  $N \geq 2$ . Participating the sensing process naturally incurs a *cost* for the worker, therefore a *payment* is demanded as a compensation for its effort in accomplishing the task. Each sensing process has a limited budget  $B$ , which is shared by all the workers participating the same sensing process. Since the issue of data redundancy and marginal effect arise when the number of players joining the same sensing process increases [10], the payment of the worker is not only related to itself, but also the number of workers participating in the same sensing process. Worker participation is the most important element for providing an adequate level of service quality [12]. A worker joins one sensing process and submit it to the platform. The workers then conduct the sensing task belonging to the sensing processes and send the data to the platform. The platform collects the sensing data and sends the payment to the workers.

We make an important assumption that each worker has a *goal* on the payoff. When the payoff of the worker is higher than its goal, the worker is then satisfied and will not change to other sensing processes. This is reasonable since that most of the workers are not insatiably greedy. Former studies on incentive mechanisms also assume that the worker valuation of sensing data is finite and can be reached [12], [13]. On the contrary, when the payoff of the worker is lower than its goal, the worker would rather refuse to participate. It is then important to make sure workers are satisfied when participating in the sensing processes. The inspiration of our work comes from [20] and [21]. In [20], the authors propose a game where players are willing to satisfy their demands, and the goal of the game is to find the satisfaction equilibria where all the players are satisfied. [21] extends the idea of satisfaction players in Qos satisfaction games in the context of spectrum sharing.

The focus of this paper is to present the sensing processes participation game framework to model the satisfaction state of the workers, which we call a sensing process participation problem. Other issues such as design and implementation of a participating system, privacy [22], [23], energy saving [17] and application development [24] are out of the scope of this paper. Our sensing processes participation game is capable of catching different details of participating systems, which will be discussed in Subsection III-B.

#### B. Game Model

A **sensing processes participation game** is defined by a tuple  $(\mathcal{N}, \mathcal{T}, (S_n)_{n \in \mathcal{N}}, (P_n^t)_{n \in \mathcal{N}, t \in \mathcal{T}}, (G_n^t)_{n \in \mathcal{N}, t \in \mathcal{T}})$  where:

- $\mathcal{N} = \{1, 2, 3, \dots, N\}$  is the set of **participatory workers**, also referred as **players**.
- $\mathcal{T} = \{1, 2, 3, \dots, T\}$  is the set of available **sensing processes**. A worker can only join one sensing process

at a time. Since workers are allowed to be **dormant**, we introduce element 0 as a virtual sensing process. The payoff of a worker who joins the sensing process 0 (i.e., not participating in any sensing processes) is always 0. When the sensing processes cannot satisfy a worker due to limited budget, it is beneficial for the worker to be dormant by joining the sensing process 0. The **strategy set** of sensing processes with virtual sensing process 0 therefore is  $\mathcal{T} = \{0, 1, 2, 3, \dots, T\}$ .

- $S = \{s_1, s_2, s_3, \dots, s_N\} \in \mathcal{T}^N$  is the **strategy profile** of the sensing processes participation game. The **strategy** of a worker  $n$  is  $s_n \in \mathcal{T}^N$ . We use the strategy profile  $S$  to represent the state of the whole system.
- $P_n^t(I^t(S))$  is a non-negative and non-increasing function which characterizes the payoff of a worker  $n$  with the strategy of joining a sensing process  $t$ . Specifically, we have  $P_n^t(I^t(S)) = R_n^t(I^t(S)) - C_n^t$ , with an integer  $I^t(S) = |\{n \in \mathcal{N} : s_n = t\}|$  as the **congestion level** of the sensing process  $t$  (i.e., the number of workers who join the sensing process  $t$ ). The congestion level of virtual sensing process 0 is always 0, since no dormant worker will affect the others' utility. We present the details of the parameters in  $P_n^t(\cdot)$  as follows.
  - $R_n^t(I^t(S))$  is non-negative and non-increasing function which characterizes the payment of a player. For a worker  $n$  joining a sensing process  $t$ , the payment  $R_n^t$  is determined, given the congestion level  $I^t(S)$ . In general, as the number of workers  $I^t(S)$  participating the same sensing process  $t$  increases, the payment to each worker  $R_n^t(I^t(S))$  decreases. We leave the specific definition of function  $R_n^t(I^t(S))$  to the platform to cover most of the details of the sensing processes participation. For example, a sensing process wants to simply share its budget  $B_t$  to all the participating workers, then we have  $R_n^t(I^t(S)) = \frac{B_t}{I^t(S)}$ .
  - $C_n^t$  is the cost of a worker  $n$  for participating a sensing process  $t$ . We allow workers to specify their own cost functions, i.e., different workers may have the different  $C_n^t$  even on the same sensing process  $t$ . In this way, we are able to model the cost of the worker due to participating sensing processes. Important factors that affect the cost include the schedule of participatory sensing, the energy cost of the sensing process, the geographical distance travelled, even the discomfort of the worker caused by accomplishing the task. Workers can feel free to take all of the factors into consideration and have their own ideal costs for each sensing process.
- $G_n^t \geq 0$  is the goal of the payoff of the worker  $n$ . Workers may have different preferences for the same sensing process due to different reasons such as the interest level of the sensing process and the worker's skill level in required fields. It is reasonable to allow different workers to have different goals of payoff even for the

same sensing process. We will further use  $G_n^t$  to verify whether the worker  $n$  is satisfied when participating.

We define the **utility**  $U_n(S)$  of a worker  $n$  in strategy profile  $S$  is

$$U_n(S) = \begin{cases} 1, & \text{if } s_n \neq 0 \text{ and } P_n^t(I^t(S)) \geq G_n^t, \\ 0, & \text{if } s_n = 0, \\ -1, & \text{if } s_n \neq 0 \text{ and } P_n^t(I^t(S)) < G_n^t. \end{cases} \quad (1)$$

Accordingly, the satisfaction state of the worker is divided into three categories.

- A **satisfied worker** is a worker who joins a sensing process  $s_n \neq 0$  and receives the payment  $R_n^t(I^t(S))$ , which is not smaller than the worker goal  $G_n^t$ . The satisfied worker will not change its strategy, and the utility of the worker is 1.
- A **dormant worker** is a worker who is not participating in any sensing processes. Since the worker will not receive any payment and spend any cost, the payoff of a dormant worker is always 0. Being dormant is not a good strategy when more than one sensing process which will make the worker satisfied is available, but definitely better than unsatisfied. The utility of a dormant worker is therefore 0.
- An **unsatisfied worker** is a worker who joins a sensing sensing process  $s_n \neq 0$  but receives the payment  $R_n^t(I^t(S))$ , which is smaller than the worker goal  $G_n^t$ . The worker is not satisfied and will refuse to participate in the sensing process, and the utility of the worker is -1.

It is clear that an unsatisfied worker can join the proper sensing process to be dormant or satisfied, and therefore increases its utility. We assume that all the workers are *rational*, that is, all the workers want to increase its utility when possible. It is suggested that if we allow workers to change their strategy freely with enough time, The final strategy profile will not contain any unsatisfied workers, which satisfies the property of **individual rationality**. We also say a sensing process  $t$  is **satisfying** if given the strategy profile of other workers, changing a worker  $n$ 's strategy to  $t$  will make the worker satisfied.

### C. Key Game Concepts

We introduce the key game concepts of sensing processes participation game as follows.

**Definition 1 (Social Welfare).** *The social welfare  $\sum_{n=1}^N U_n(S)$  of a strategy profile  $S$  is the sum of all the players' utilities.*

**Definition 2 (Social Optimum).** *A strategy profile  $S$  is **Social Optimum** when it maximizes the social welfare.*

**Definition 3 (Better Response Update).** *The event where a player changes its strategy from  $p$  to  $q$  is a **better response update** if and only if  $U_n(q, S_{-n}) > U_n(p, S_{-n})$ , where the argument of the function is written as  $S = (s_n, S_{-n})$  with  $S_{-n} = (s_1, s_2, s_3, \dots, s_{n-1}, s_{n+1}, \dots, s_N)$  representing the strategy profile of all the players except player  $n$ .*

**Definition 4 (Pure Nash Equilibrium).** A strategy profile  $S$  is a **Pure Nash Equilibrium** if no players under  $S$  can perform a better response update, i.e.,  $U_n(s_n, S_{-n}) > U_n(q, S_{-n})$  for any  $q \in \tilde{\mathcal{T}}$  and  $n \in \mathcal{N}$ .

**Definition 5 (Finite Improvement Property).** A game has the **Finite Improvement Property** if any asynchronous better response update process (i.e., no more than one player is allowed to perform better response update at any given time) terminates at a pure Nash equilibrium within a finite number of updates.

#### D. Transformation to an Equivalent Number of Workers Threshold Form

Considering both the goal and the cost is specified by workers, we introduce an equivalent form of the sensing processes participation game for the convenience of discussion. The key idea of transformation is to relate the payment of a worker to its satisfaction level, therefore reduce the size of parameters.

In the sensing processes participation game model, the goal  $G_n^t$  and cost  $C_n^t$  of a worker  $n$  are determined, given its strategy profile of the sensing process  $t$ . We could use the demand  $D_n^t = G_n^t + C_n^t$  of the worker  $n$  participating in sensing process  $t$  and the payment  $R_n^t(I^t(S))$  to determine whether the worker is satisfied. Since the payment function  $R_n^t(I^t(S))$  is non-negative and non-increasing with the congestion level  $I^t(S)$ , there must exist a critical congestion threshold value  $V_n^t$  corresponding to the demand  $D_n^t$ , that is, if and only if  $I^t(S) > V_n^t$ , there exists  $R_n^t(I^t(S)) > D_n^t$ . We formally define the threshold  $V_n^t$  of a sensing process  $t$  with respect to a worker  $n$  to be an integer as follows, given the pair  $(P_n^t, G_n^t)$ .

- if  $R_n^t(I^t(S)) > D_n^t$  for each  $I^t \in \mathcal{N}$ , then  $V_n^t = 0$  (i.e., even the player is the only player participating in the sensing process, it will not be satisfied).
- if  $R_n^t(I^t(S)) < D_n^t$  for each  $I^t \in \mathcal{N}$ , then  $V_n^t = N + 1$  (i.e., the player is always satisfied even all the players are participating the same sensing process).
- Otherwise  $V_n^t$  is equal to the maximum integer  $I^t \in \mathcal{N}$  such that  $R_n^t(I^t(S)) > D_n^t$ .

The definition of  $V_n^t$  guarantees that

$$R_n^t(I^t(S)) > D_n^t \Leftrightarrow I^t(S) < V_n^t. \quad (2)$$

We then formally present a sensing processes participation game  $g = (\mathcal{N}, \mathcal{T}, (S_n)_{n \in \mathcal{N}}, (P_n^t)_{n \in \mathcal{N}, t \in \mathcal{T}}, (G_n^t)_{n \in \mathcal{N}, t \in \mathcal{T}})$  in the number of workers threshold form  $g' = (\mathcal{N}, \mathcal{T}, (S_n)_{n \in \mathcal{N}}, (V_n^t)_{n \in \mathcal{N}, t \in \mathcal{T}})$ . The utility of a worker  $n$  is accordingly changed to

$$U_n(S) = \begin{cases} 1, & \text{if } s_n \text{ and } I^t(S) < V_n^t, \\ 0, & \text{if } s_n = 0, \\ -1, & \text{if } s_n \neq 0 \text{ and } I^t(S) > V_n^t. \end{cases} \quad (3)$$

Since the transformation guarantees that the utility  $U_n(S)$  of player  $n$  in  $g$  is the same as that of player  $n$  in  $g'$  for any strategy profile  $S$  and player  $n$ , the original game  $g$  is thus

equivalent to the game  $g'$ . We will use the number of workers threshold form to analyze the sensing processes participation game for the rest of the paper.

## IV. THE PROPERTY OF THE SENSING PROCESSES PARTICIPATION GAME

In this section, we present the properties of the sensing processes participation game, including the computational complexity of finding the social optimum, the existence of pure Nash equilibrium and the finite improvement property.

### A. Computational Complexity of Finding the Social Optimum

Considering that the workers and sensing processes are heterogenous with different goals, costs and budgets, finding the social optimum of a sensing processes participation game is challenging. We show the computational complexity of finding the social optimum in Theorem 1.

**Theorem 1.** *The problem of finding a social optimum of a sensing processes participation game is NP-hard.*

*Proof:* For the convenience of discussion, we call the problem of finding a social optimum of a sensing processes participation game as the sensing processes participation problem. We first introduce the 3-dimensional matching decision problem.

**Definition 6 (3-dimensional matching).** *LET  $\mathcal{X}, \mathcal{Y}$ , and  $\mathcal{Z}$  be three disjoint sets, and let  $\mathcal{T}$  be a subset of  $\mathcal{X} \times \mathcal{Y} \times \mathcal{Z}$ . That is,  $\mathcal{T} \subseteq \{(x, y, z) : x \in \mathcal{X}, y \in \mathcal{Y}, z \in \mathcal{Z}\}$ . If  $\mathcal{M} \subseteq \mathcal{T}$ , and for distinct triples  $(x_1, y_1, z_1) \in \mathcal{M}$  and  $(x_2, y_2, z_2) \in \mathcal{M}$ , we have  $x_1 \neq x_2$ ,  $y_1 \neq y_2$  and  $z_1 \neq z_2$ , then  $\mathcal{M}$  is a 3-dimensional matching.*

We refer to an element  $(x, y, z) \in \mathcal{T}$  as an *edge*. The 3-dimensional matching decision problem is as follows. Suppose that the set sizes satisfy  $|\mathcal{X}| = |\mathcal{Y}| = |\mathcal{Z}| = I$ . Given an input  $\mathcal{T}$  and  $|\mathcal{T}| \geq I$ , decide whether there exists a 3-dimensional matching  $\mathcal{M} \subseteq \mathcal{T}$  with the maximum size  $|\mathcal{M}| = I$ . The 3-dimensional matching decision problem is a well-known NP-complete problem in Karp's 21 NP-complete problems [25]. We then prove that the sensing processes participation problem is NP-hard, by showing that if an oracle can solve the sensing processes participation problem, then the 3-dimensional matching decision problem can be solved in polynomial time.

Given an instance of 3-dimensional matching  $((\mathcal{X}, \mathcal{Y}, \mathcal{Z}), \mathcal{T})$  with  $|\mathcal{X}| = |\mathcal{Y}| = |\mathcal{Z}| = I$  and  $|\mathcal{T}| = J \geq I$ , we can construct an instance of sensing processes participation problem as follows. Let set  $\theta = X \cup Y \cup Z$ . Each element  $n \in \theta$  is regarded as worker  $n$ . We introduce the additional worker set  $\vartheta$  to contain the  $J - I$  workers, and the total number of the workers in  $\theta$  and  $\vartheta$  is  $3I + J - I = 2I + J$ . An edge  $(x, y, z)$  is a sensing process, and the set of sensing process is  $\mathcal{T}$ , with the number of sensing process  $J$ . We therefore define the congestion thresholds  $V_n^t$  of a worker  $n$  and a sensing process  $t$  as follows. For a worker  $n$  in set  $\theta$  participating in the sensing process  $t = (x, y, z)$ , if  $n$  is

an element of the edge  $t$  in  $\mathcal{T}$  (i.e., one of the following is true:  $t = x$ ,  $t = y$  or  $t = z$ ), we set  $V_n^t = 3$ . Otherwise we set  $V_n^t = 1$ . For a worker  $n$  in the set  $\vartheta$  participating in the sensing process  $t = (x, y, z)$ , we set  $V_n^t = 1$ . It is clear that 3 workers can participate in the same sensing process simultaneously and they are all satisfied, if and only if they form an edge in  $\mathcal{T}$ . Since a worker can only join one sensing process, given a sensing processes participation solution based on a set of sensing processes, each of which has 3 satisfied workers, which corresponds to a 3-dimensional matching in  $\mathcal{T}$ . In this case, we have the global optimal solution with  $2I + J$  workers and  $J$  sensing processes, if and only if there exists a 3-dimensional matching  $\mathcal{M} \subseteq \mathcal{T}$  with the maximum size  $|\mathcal{M}| = I$ . The strategy profile of the solution is that  $I$  sensing processes are having 3 satisfied workers and  $J - I$  sensing processes are having 1 satisfied worker.

If we have an oracle to find the global optimal solution of a sensing processes participation problem with  $2I + J$  workers and  $J$  sensing processes, we can decide that whether there exists a 3-dimensional matching  $\mathcal{M} \subseteq \mathcal{T}$  with the maximum size  $|\mathcal{M}| = I$  in a polynomial time  $\mathcal{O}(1)$ . It implies that the 3-dimensional matching decision problem is polynomially reducible to the sensing processes participation problem, therefore the sensing processes participation problem is NP-hard. ■

The proof of Theorem 1 is based upon showing that the 3-dimensional matching decision problem (which is well known to be NP-complete [25]) can be reduced to the problem of finding a social optimum of a sensing processes participation game with  $V_n^t \in \{1, 3\}$  for any worker  $n$  and sensing process  $t$ . Theorem 1 is the motivation for our study, since it suggests that solving the sensing processes participation problem is fundamentally difficult. Exploring solutions with a game based approach is therefore reasonable and makes sense, with the inspiration of congestion game based models proposed in [18] [19].

### B. Characterization of Pure Nash Equilibria

Since we allow workers to update their strategy profiles based on satisfaction incentives, it is important to understand that whether the final strategy profile is a pure Nash equilibrium. It is obvious that no worker is unsatisfied when the strategy profile is a pure Nash equilibrium. To make this clear, consider a strategy profile  $S$  where a worker is satisfied or dormant. The worker can always change its strategy profile to the virtual sensing process (i.e., choose to be dormant) to improve its utility. We show that every sensing processes participation game has the finite improvement property in Theorem 2, which naturally leads to the existence of pure Nash equilibrium.

**Theorem 2.** *Every  $N$ -player sensing processes participation game has the finite improvement property. Any asynchronous (i.e., only one better response update at a time) better response update process with no more than  $3N^2 + 4N$  steps is guaranteed to reach a pure Nash equilibrium, regardless of the*

*initial strategy profile of the game. If for any worker  $n$  in the game, the initial strategy profile  $s_n = 0$ , then a pure Nash equilibrium is guaranteed to be reached in  $2N^2 + 2N$  steps.*

*Proof:* We define the function  $\Phi$  which maps each strategy profile  $S$  to a real integer. We have  $\Phi(S) = 2 \sum_{n \in \mathcal{N}: S_n \neq 0} V_n^{S_n} - \sum_{n \in \mathcal{N}: S_n \neq 0} I^{S_n}(S)$ , where  $V_n^{S_n}$  is the critical threshold of player  $n$  when it joins the sensing process  $S_n$ , and  $I^{S_n}(S)$  is the congestion level of the sensing process  $S_n$ .

Suppose that player  $n'$  performs a better response update by changing its strategy from  $p \in \tilde{\mathcal{T}} = \{0, 1, 2, 3, \dots, T\}$  to  $q \in \tilde{\mathcal{T}} = \{0, 1, 2, 3, \dots, T\}$ , therefore the strategy profile is changed from  $S_p = S(p, S_{-n'}) = (s_1, s_2, s_3, \dots, s_{n'-1}, p, s_{n'+1}, \dots, s_N)$  to  $S_q = S(q, S_{-n'}) = (s_1, s_2, s_3, \dots, s_{n'-1}, q, s_{n'+1}, \dots, s_N)$ .

For the next three possible cases, we will show that  $\Phi(S_q) > \Phi(S_p) + 1$ :

- 1)  $p \neq 0, q = 0$  (i.e., the player becomes dormant).
- 2)  $p = 0, q \neq 0$  (i.e., the player changes its strategy from dormant to satisfying sensing processes).
- 3)  $p \neq 0, q \neq 0$  (i.e., the player changes its strategy from one sensing process to another).

In case 1), where  $p \neq 0, q = 0$ , we have  $\Phi(S_q) = \Phi(S_p) - 2V_{n'}^p - (-2I^p(S_p) + 1)$ . The update where the player  $n'$  change its strategy profile from sensing process  $p$  to 0 decreases the sum of thresholds by  $2V_{n'}^p$ , and decreases the sum of congestion levels by  $2I^p(S_p) - 1$ . It is clear that the congestion level of player  $n'$  is decreased from  $I^p(S_p)$  to 0, and there are  $I^p(S_p) - 1$  other players are participating the same sensing process  $p$ . Each player experiences a decrement of 1 and the sum of congestion levels of these players decreases by  $I^p(S_p) - 1$ . Since the case is a better response update, we have  $U_{n'}(S_p) = -1$  and  $U_{n'}(S_q) = 0$ . It follows that  $V_{n'}^p < I^p(S_p)$ . Since both  $V_{n'}^p$  and  $I^p(S_p)$  are integers, it implies that  $V_{n'}^p \leq I^p(S_p) - 1$ . It follows that  $\Phi(S_q) - \Phi(S_p) = 2I^p(S_p) - 2V_{n'}^p - 1 \geq 1$ .

In case 2), where  $p = 0, q \neq 0$ , we have  $\Phi(S_q) = \Phi(S_p) + 2V_{n'}^q - (2I^q(S_p) + 1)$ . The update where the player  $n'$  changes from sensing process 0 to  $q$  increases the sum of thresholds by  $2V_{n'}^q$ , and increases the sum of congestion levels by  $2I^q(S_p) + 1$ . It is clear that the congestion level of player  $n'$  is increased from 0 to  $I^q(S_p) + 1$ , since there are  $I^q(S_p)$  other players are participating in the same sensing process  $q$ . Each player experiences an increment of 1 and the sum of congestion levels of these players increases by  $I^q(S_p)$ . Also, since the case is a better response update, we have  $U_{n'}(S_p) = 0$  and  $U_{n'}(S_q) = 1$ . It follows that  $V_{n'}^q > I^q(S_p)$ . It follows that  $\Phi(S_q) - \Phi(S_p) = 2V_{n'}^q - 2I^q(S_p) - 1 \geq 1$ .

In case 3), where  $p \neq 0, q \neq 0$ , we have  $U_{n'}(S_p) = -1$  and  $U_{n'}(S_q) = 1$ . This case of the better response update is the combination of the above two cases of better update responses in a certain order, the first case (i.e., the player changes its strategy from sensing process  $p$  to 0), followed by the second case (i.e., the player changes its strategy from sensing process 0 to  $q$ ). Performing the above two cases in the order is equal

to the case that the player changes its strategy from sensing process  $p$  to  $q$ , and in each case the value of  $\Phi$  is increased by 1. It follows that  $\Phi(S_q) - \Phi(S_p) > 2 > 1$ .

Without loss of generality, we can suppose that  $-1 < V_n^t < N + 1, \forall n \in \mathcal{N}, \forall t \in \mathcal{T}$ , since thresholds greater than  $N + 1$  induces the same kind of behavior as thresholds equal to  $N + 1$  (i.e., the players possessing the thresholds can never be satisfied) and thresholds less than  $-1$  induces the same kind of behavior as thresholds equal to  $-1$  (i.e., the players possessing the thresholds are always satisfied). For any strategy profile  $S$ , we have  $-N < \sum_{n \in \mathcal{N}: S_n \neq 0} V_n^{S_n} < N(N + 1)$ . Also, we have  $0 < I_n^t < N, \forall n \in \mathcal{N}, \forall t \in \mathcal{T}$ , it follows that  $0 < \sum_{n \in \mathcal{N}: S_n \neq 0} I^{S_n}(S) < N^2$ . From these inequalities, we have  $-(N^2 + 2N) < \Phi(S) < 2N^2 + 2N$ .

When the participatory system starts to evolve into a better state (i.e., allowing workers to update their strategies), the value of  $\Phi(S)$  can not be less than  $-(N^2 + 2N)$ . With every better response update, the value of  $\Phi(S)$  increases by 1. Suppose that we have performed  $k$  better response updates from strategy profile  $S_p$ , and the current strategy profile is  $S_q$ . We must have  $k - (N^2 + 2N) < \Phi(S_q) = \Phi(S_p) + k < 2N^2 + 2N$ . It follows that  $k \leq 3N^2 + 4N$ . Especially, when the system starts from the strategy profile that all the players are dormant, the value of  $\Phi(S)$  is 0, it follows similarly that  $k \leq 2N^2 + 2N$ .

We have shown that when we start the sensing processes participation process and allow players to update their strategies, the number of the better update process is no more than  $3N^2 + 4N$ . This implies that the system *must* reach a strategy profile  $r$  from which no better update response is available. Such strategy profile  $r$  must be a pure Nash equilibrium by definition. ■

Theorem 2 is important since that it implies that allowing workers to update their strategy profiles will finally reach a stable state effectively. The feature of the finite improvement property enables us to design a fast sensing processes participation algorithm in Section V. Note that although Theorem 2 implies that pure Equilibrium exists and can be found in polynomial time, there are conditions where multiple pure Nash equilibria exist. Under these conditions, it is not guaranteed that the most beneficial pure Nash equilibrium will be reached. For a sensing processes participation game, reaching the state of certain pure Nash equilibrium is difficult since players perform better response updates randomly.

### C. Price of Anarchy

Although Theorem 1 shows that finding a social optimum of a sensing processes participation game is difficult, finding a pure Nash equilibrium of the game is proved to be relatively easy, shown by Theorem 2. It naturally raises the question that how the social welfare of pure Nash equilibrium compare to that of a social optimum.

We discuss this issue with the concept of the price of anarchy [26]. The definition of the price of anarchy in sensing processes participation game is

$$PoA = \frac{\max \left\{ \sum_{n=1}^N U_n(S) : S \in \tilde{\mathcal{T}}^{\mathcal{N}} \right\}}{\min \left\{ \sum_{n=1}^N U_n(S) : S \in S_{NE} \right\}}, \quad (4)$$

where  $S_{NE}$  is the set of all the pure Nash equilibria,  $\tilde{\mathcal{T}}^{\mathcal{N}}$  is the set of all the strategy profile of our game. The value of the price of anarchy is the maximum of the social welfare of all the strategy profiles, divided by the minimum welfare of a pure Nash equilibrium.

**Theorem 3.** Consider a  $N$ -player sensing processes participation game  $(\mathcal{N}, \mathcal{T}, (S_n)_{n \in \mathcal{N}}, (V_n^t)_{n \in \mathcal{N}, t \in \mathcal{T}})$ . Assume that  $V_n^t \geq 1$  for each player  $n$  and each sensing process  $t$ . The price of anarchy of the game satisfies

$$PoA \leq \min \left\{ N, \frac{\max \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}}{\min \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}} \right\}. \quad (5)$$

*Proof:* We introduce a lemma first before proving the result of the price of anarchy. Let  $W(S)$  denote the number of satisfied workers in a strategy profile  $S$ .

**Lemma 1.** Suppose that  $S_p$  is a pure Nash equilibrium of the sensing processes participation game, and  $S_q$  is a social optimum of the sensing processes participation game. The following statements are true:

- 1) There are no unsatisfied workers in  $S_p$ .
- 2) We have  $\sum_{n=1}^N U_n(S_p) = W(S_p) = \sum_{t=1}^T I^t(S_p)$ .
- 3) There are no unsatisfied workers in  $S_q$ .
- 4) We have  $\sum_{n=1}^N U_n(S_q) = W(S_q) = \sum_{t=1}^T I^t(S_q)$ .

*Proof:*  $S_p$  is a pure Nash equilibrium if and only if no one can perform a better response update in  $S_p$ , implied in Subsection III-C. Assume that there is a worker who is unsatisfied in  $S_p$ , the worker can improve its utility by performing a better response update. This contradicts the assumption that  $S_p$  is a pure Nash equilibrium. This proves Statement 1).

Statement 2) implies that for any worker  $n$  under  $S_p$ , we have  $U_n(S_p) \in \{0, 1\}$ . A worker is satisfied if and only if  $U_n(S_p) = 1$ . This implies that the number of satisfied worker  $W(S_p)$  equals the social welfare  $\sum_{n=1}^N U_n(S_p) =$  of the strategy profile  $S_p$ . Moreover, every non-dormant worker is satisfied in  $S_p$ , and  $\sum_{t=1}^T I^t(S_p)$  is the number of all the non-dormant workers in  $S_p$ , we must have  $W(S_p) = \sum_{t=1}^T I^t(S_p)$ . This proves Statement 2).

To prove Statement 3), note that by making a worker to change its strategy from unsatisfied to dormant, the social welfare of a strategy profile with an unsatisfied worker can be increased. The strategy profile therefore is not a social optimum.

The proof of Statement 4) is similar to the proof of Statement 2), and is hence omitted. ■

Then we use Lemma 1 to prove Theorem 3.

Let  $S_p$  be a pure Nash equilibrium of the sensing processes participation game which minimizes the social welfare among all the pure Nash equilibria (note that the existence of a pure Nash equilibrium is guaranteed by Theorem 2), and  $S_q$  is a

social optimum of the sensing processes participation game. It is clear that all the four statements in Lemma hold in this scenario. With Equation 4, Statement 2) and Statement 4) of Lemma 1, the definition of the price of anarchy is

$$PoA = \frac{\sum_{n=1}^N U_n(S_q)}{\sum_{n=1}^N U_n(S_p)} = \frac{W(S_q)}{W(S_p)}. \quad (6)$$

We will then prove Statement (7)-(10) one by one.

$$W(S_q) \in \{1, 2, \dots, N\}. \quad (7)$$

$$W(S_q) \leq T \max \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}. \quad (8)$$

$$W(S_p) \in \{1, 2, \dots, N\}. \quad (9)$$

If  $W(S_p) < N$ , then  $W(S_p) \geq T \min \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}$ . (10)

We first consider the strategy profile  $S_z$  where the worker  $n = 1$  joins the sensing process  $t = 1$ , and all the other workers are dormant. Note that according to the assumption  $V_n^t \geq 1$  for any worker  $n$  and any sensing process  $t$ , we have  $I^1(S_z) \leq V_1^1$ , then the worker  $n = 1$  must be satisfied. The social welfare of  $S_z$  is then  $\sum_{n=1}^N U_n(S_z) = 1$ . Since  $S_q$  is the social optimum, its social welfare must not be less than that of  $S_z$ , then we have  $\sum_{n=1}^N U_n(S_q) \geq \sum_{n=1}^N U_n(S_z) = 1$ . Combing the result here with the Statement 3) of Lemma 1 implies that  $W(S_q) \geq 1$ . It is clear that  $W(S_q)$  is not bigger than the number of workers  $N$ , hence we have proved Statement (7).

Let  $t' \in \{1, 2, \dots, T\}$  denote the sensing process that have the most participating workers in  $S_q$ , and we have  $I^{t'}(S_q) = \max \{I^t(S_q) : t \in \mathcal{T}\}$ . Statement 4) of Lemma 1 implies that

$$W(S_q) = \sum_{t=1}^T I^t(S_q) \leq \sum_{t=1}^T I^{t'}(S_q) = T I^{t'}(S_q). \quad (11)$$

Since Statement (7) gives  $W(S_q) \geq 1$ , and combining this with Inequality (11) gives  $T I^{t'}(S_q) \geq 1$ . By the definition of  $I^{t'}(S_q)$  we also have  $1 \leq I^{t'}(S_q)$ , since  $I^{t'}(S_q)$  is an integer. It follows that there must be some worker  $n'$  participating in the sensing process  $t'$  under  $S_q$ . Statement (1) of Lemma 1 gives that  $n'$  is satisfied when participating the sensing process  $t'$  under  $S_q$ . We must have

$$I^{t'}(S_q) \leq V_{n'}^{t'} \leq \max \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}. \quad (12)$$

By combining Inequality (11) and (12), we have  $W(S_q) \leq T I^{t'}(S_q) \leq T \max \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}$ . Hence we have proved Statement (8).

We then prove that  $W(S_p) \geq 1$  by contradiction. If  $W(S_p) \geq 1$  were false, Since  $W(S_p)$  is an integer, we would have  $W(S_p) = 0$ , which means there are no worker participating in any sensing processes. However, the worker  $n = 1$  could do a better response update by participating in

the sensing process  $t = 1$ , Since  $V_n^t \geq 1$ . This contradicts our assumption that  $S_p$  is a pure Nash equilibrium, hence we must have  $W(S_p) \geq 1$ . It is also clear that  $W(S_p)$  is not greater than  $N$ , hence we have proved Statement (9).

We suppose that  $W(S_p) < N$  to prove Statement (10). It is clear that there are some workers which are not satisfied under  $S_p$ . Since Statement 1) of Lemma 1 implies that every worker which is not satisfied under  $S_p$  is dormant, it follows that there must be some worker  $n^*$  that is dormant in  $S_p$ . Since  $S_p$  is a pure Nash equilibrium, the player  $n^*$  cannot perform a better response by changing its strategy to sensing process  $t$ . For each sensing process  $t \in \{1, 2, \dots, T\}$ , we must have

$$I^t(S_p) \geq V_{n^*}^t \geq \min \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}. \quad (13)$$

Combining Statement 4) of Lemma 2 with Inequality (13), we have  $W(S_p) = \sum_{t=1}^T I^t(S_p) \geq \sum_{t=1}^T \min \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\} = T \min \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}$ , hence Statement (10) is proved.

Now we can prove Theorem 3. Statement (6) gives that  $PoA = \frac{W(S_q)}{W(S_p)}$ . With Statement (7) gives  $W(S_q) \leq N$  and Statement (9) gives  $W(S_p) \geq 1$ , we have

$$PoA \leq N. \quad (14)$$

We then consider two cases of PoA. In the first case, Suppose that  $W(S_q) = W(S_p)$ , and we have  $PoA = 1$ . Since  $1 \leq \frac{\max \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}}{\min \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}}$ , Theorem 3 holds in this case. In the second case, where  $W(S_q) \neq W(S_p)$ , we must have  $W(S_q) \geq W(S_p)$  since  $S_q$  is a social optimum. Combining this with Statement (7) and Statement (9), we have  $W(S_p) < N$ . It follows from Statement (10) that we must have  $W(S_p) \geq T \min \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}$ . Since  $PoA = \frac{W(S_q)}{W(S_p)}$ , we have

$$\frac{W(S_q)}{PoA} \geq T \min \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}. \quad (15)$$

Combining this with Inequality (8), we have

$$PoA \leq \frac{T \max \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}}{T \min \{V_n^t : n \in \mathcal{N}, t \in \mathcal{T}\}}. \quad (16)$$

We cancel the  $T$ s from Inequality (16) and combine it with the Inequality (14), therefore we have Inequality (5) in Theorem 3. ■

We make the assumption in Theorem 3 that when the player is the only player of the participating sensing process, it cannot be unsatisfied, therefore the PoA makes sense by avoiding ‘‘Division by Zero’’. Theorem 3 implies that the performance of all the Nash equilibrium is close to the global optimal, when the maximum threshold of a worker  $n$  to a sensing process  $t$  is close to the minimum of that. It means that the NP-hard problem of finding a social optimal (Theorem 1) can be approximated by any better response updates process (Theorem 2), when the demands of the workers are not too diverse.



## V. SENSING PROCESSES PARTICIPATION ALGORITHM

### A. Overview

In this section, we propose a fast sensing processes participation (SPP) algorithm to find a pure Nash equilibrium of sensing processes participation games, where no worker receives less payment than its goal. The basic idea of the algorithm is to exploit the finite improve property and allow the workers to update their strategies asynchronously (i.e., one by one at a time). As described in Subsection III-B, we allow workers to be dormant by joining the added virtual sensing process 0. The utility of a dormant worker is always 0.

### B. Design of sensing processes participation Algorithm

We describe the process of sensing processes participation algorithm in the following two phases.

- 1) **Phase 1: Sensing Process Announcement.** the platform announces the set of sensing processes  $\mathcal{T}$  and the associated payment functions  $(R_n^t(I^t(S)))_{t \in \mathcal{T}}$ . All the  $N$  workers who are willing to participate register at the platform, and the initial strategy profile of worker  $s_n$  is generated as 0 (i.e., dormant).
- 2) **Phase 2: Strategy Update Process.** the strategy update process is based on the principle of better response updates. This phase consists of three steps.
  - **Step 1: Select the Update Worker.** the platform publicizes all the number of workers participating the same sensing processes  $(R_n^t(I^t(S)))_{t \in \mathcal{T}}$ . Then it randomly selects a worker  $n'$ , which is allowed to update its strategy profile.
  - **Step 2: Best Response Generation.** the selected worker  $n'$  computes its set of best response  $\mathcal{B}_{n'}(S)$  (i.e., changing the present strategy  $s_n$  to the elements of the better response set is a better response update). The definition of  $\mathcal{B}_{n'}(S)$  is

$$\mathcal{B}_{n'}(S) = \left\{ t^* : t^* = \arg \max_{t \in \mathcal{T}} U_{n'}(s_{n'}, S_{-n'}) \right. \\ \left. \text{and } U_{n'}(t^*, S_{-n'}) > U_{n'}(S) \right\}$$

If  $\mathcal{B}_{n'}(S) \neq \emptyset$ , the worker  $n'$  will randomly join a satisfying sensing process  $t^* \in \mathcal{B}_{n'}(S)$  and submit its update to the platform. Else the worker reports that the update is not available.

- **Step 3: Update the Strategy Profile.** If the platform receives the update from worker  $n'$ , it updates the strategy profile  $S$  and the congestion levels  $(I^t(S))_{t \in \mathcal{T}}$ . If the update is not available, then go back to step 1.

The privacy of the data specified by the worker such as goal  $G_n^t$  and cost  $C_n^t$  is preserved, Since all the data the worker submits to the platform is its strategy  $s_n$ . The algorithm will reach a pure Nash equilibrium in less than  $2N^2 + 2N$  steps (Theorem 2). The algorithm is implemented as a sensing processes participation solution in Algorithm 1.

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### Algorithm 1: Sensing processes participation algorithm

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**Input** The set of workers  $\mathcal{N}$ ,  
The set of sensing processes  $\mathcal{T}$ ,  
The payoff functions  $(R_n^t(I^t(S)))_{t \in \mathcal{T}}$ .  
**Initialization:** For each worker  $n$ , the initial strategy profile  $s_n = 0$ .  
**Output** The strategy profile  $S$ .  
1: **while** for any worker  $n$  and the strategy profile  $S$ ,  $\mathcal{B}_n(S) \neq \emptyset$   
**do**  
2: The platform publicizes the congestion levels  $(I^t(S))_{t \in \mathcal{T}}$   
to all the workers.  
3: The platform randomly selects a worker  $n'$ .  
4: The worker computes its set of best response  $\mathcal{B}_{n'}(S)$ .  
5: **if**  $\mathcal{B}_{n'}(S) \neq \emptyset$  the worker **then**  
6: randomly joins a sensing process  $t^* \in \mathcal{B}_{n'}(S)$ .  
7: reports the update  $t^*$  to the platform.  
8: **else**  
9: reports the update is not available and ask the platform to  
randomly select a worker again.  
10: **end if**  
11: The platform receives the update  $t^*$  and  
12: updates the strategy profile  $S$  and the congestion  
levels  $(I^t(S))_{t \in \mathcal{T}}$ .  
13: **end while**  
14: **return** The strategy profile  $S$

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## VI. SIMULATIONS

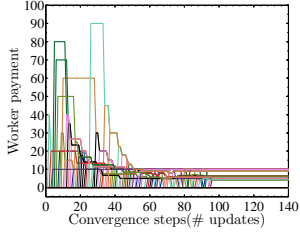
We evaluate the performance of the sensing processes participation algorithm with two simulations. To compare with the sensing processes participation algorithm, we have also implemented a centralized baseline which simply assumes that the demands of all the workers are equal. The platform then initializes the strategy profile of each worker according to the proportional distribution of the budget of a single sensing process in the sum of all the budgets. Each worker checks if the expected payment is higher than its own demand. If not, then the worker is unsatisfied and will not participate in the sensing progress.

We check the dynamics of worker payments and the performance loss of the algorithm compared to the social optimum in the first simulation. In the second simulation, we consider the performance metrics including the social welfare and the running time. The impact parameters include the number of workers, the number of tasks and the range of worker demands.

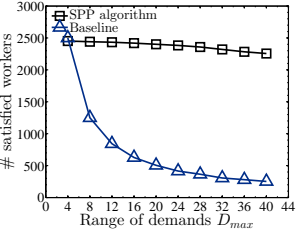
### A. Simulation Setup

We first implement a simulation of participatory sensing system with  $N = 100$  workers and  $T = 9$  sensing processes. The worker can be dormant by joining the virtual sensing process  $t = 0$ , and the payoff of the worker is always 0. The payment function  $R_n^t(I^t(S))$  of a worker  $n$  participating in sensing process  $t$  is given as  $R_n^t(I^t(S)) = \frac{B_t}{I^t(S)}$ . We assume that workers only have two types of demands, a low demand  $D = 4$  and a high demand  $D = 9$ , regardless of the sensing process joined. The sensing process  $t$  has the budget  $B_t = 50t$ , while the virtual sensing process  $t = 0$  has the budget  $B_0 = 0$ . The fraction of the workers with a high demand is varied from 0% to 100%.

In the second simulation, we consider a participatory sensing system containing  $N$  workers and  $T$  sensing processes with the budgets  $B_t$ . The number of the workers is varied



**Figure 4.** Dynamics of the worker payment with  $N = 100$  workers and  $T = 9$  sensing processes.



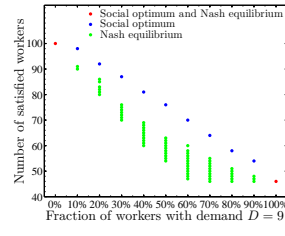
**Figure 8.** Impact of the range of demand  $D_{max}$  on social welfare  $U(S)$  while  $N = 2500$  and  $T = 24$ .

from 1000 to 5500, and the number of the sensing processes  $T$  is varied from 9 to 49. The virtual sensing process  $t = 0$  and the payoff function  $R_n^t(I^t(S))$  of a worker  $n$  participating in sensing process  $t$  is the same as that in the first simulation. We assume that the demand of each worker  $D_n^t$  (i.e., the sum of the goal and the cost of the worker) is uniformly distributed over  $[1, D_{max}]$  with  $D_{max}$  varied from 4 to 40. The budget of the sensing process  $t$  is  $B_t = 40t$ . Each measurement is averaged over 50 instances.

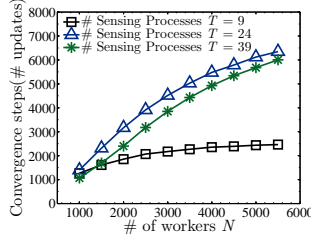
### B. Dynamic Convergence Status and Price of Anarchy

**The dynamic payments of the participating workers.** We first evaluate the finite improvement property in the better response update process. Fig. 4 shows the dynamics of  $N = 100$  workers' payments with  $T = 9$  sensing processes, which demonstrates that the system will eventually reach a pure Nash equilibrium with the sensing processes participation algorithm. Note that there are multiple workers painted with the same color in Fig. 4.

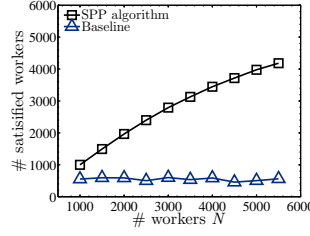
**The price of anarchy.** The issue of the price of anarchy has been discussed in Subsection IV-C. Now we use the social optimum as a benchmark to evaluate the performance of our algorithm. We vary the fraction of workers with a demand of  $D = 9$  from 0% to 100%. As the fraction of high demand workers becomes larger, the system is more congested and less workers can be satisfied. The results of the performance of our algorithm and social optima are shown in Fig. 5. Note that there are multiple pure Nash equilibria exist in one scenario of the sensing processes participation game, while the algorithm randomly selects one. Our algorithm achieves at least 82.9% and 67.2% performance of the social optima, when the best and the worst pure Nash equilibrium is reached. What's more, when the demands of the workers are heterogenous (i.e., the



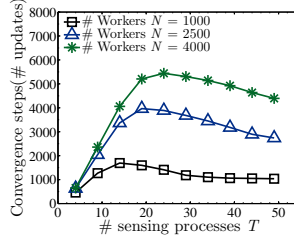
**Figure 5.** Numbers of satisfied workers at Nash equilibria and social optima with  $N = 100$  workers and  $T = 9$  sensing processes.



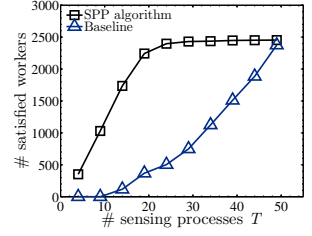
**Figure 9.** Impact of the number of workers  $N$  on convergence steps while  $D_{max} = 20$ .



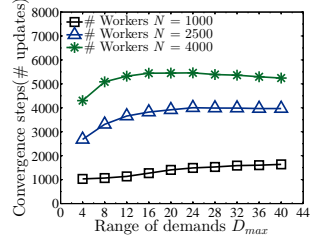
**Figure 6.** Impact of the number of workers  $N$  on social welfare  $U(S)$  while  $T = 24$  and  $D_{max} = 20$ .



**Figure 10.** Impact of the number of sensing processes  $T$  on convergence steps while  $D_{max} = 20$ .



**Figure 7.** Impact of the number of sensing processes  $T$  on social welfare  $U(S)$  while  $N = 2500$  and  $D_{max} = 20$ .



**Figure 11.** Impact of the range of demand  $D_{max}$  on convergence steps while  $T = 24$ .

fraction of workers with  $D = 9$  is 0% or 100%), The social welfare of our algorithm is exactly the same compared to the social optima. It implies that our algorithm is especially effective when the workers are highly heterogenous.

### C. The Social Welfare of SPP Algorithm

We study the social welfare achieved by our algorithm compared to that by the baseline, shown in Fig. 6, Fig. 7 and Fig. 8.

We find that our algorithm performs well as the number of workers grows in Fig. 6. When there are  $N = 5500$  workers, 76% of the workers are satisfied with the sensing processes participation algorithm while there are only 10.2% of the workers satisfied with the baseline. The proportion of satisfied workers decreases as the number of workers increases.

The sensing processes participation algorithm utilizes the sensing processes effectively when the number of sensing processes is severely constrained, shown in Fig. 7. The congestion level of the system decreases when the number of sensing processes increases, reducing the difficulty of finding a satisfying sensing process. It is clear that the performance of the baseline approximates that of our algorithm only when there are sufficient sensing processes available. This is uncommon in participatory sensing systems.

When the range of worker demands increases, finding a satisfying sensing process is growing to be difficult. However, the fraction of satisfied workers of the pure Nash equilibria by our algorithm is at least 90%, while the performance of the baseline is greatly affected as the range of demand increases. Workers who have relatively high demands are naturally less competitive than those with small demands, and the algorithm forces them to join a satisfying sensing process with less payment.

## D. The Number of Convergence Steps of SPP Algorithm

We then investigate the computational efficiency of the algorithm. Since our algorithm is based on the better response update process, we will use the number of convergence steps to represent the running time of our algorithm.

The impact of the number of workers on the number of the convergence steps with  $T = 10, 25, 40$  sensing processes is shown in Fig. 9. Our algorithm scales well as the number of workers grows, which is important for an NP-hard sensing processes participation problem.

The impact of the number of sensing processes can be divided in two cases. Increasing the number of sensing processes will also increase the number of satisfied workers when there are only few sensing processes available to numerous workers. In this case, the number of the convergence steps is increased. In another case, when the number of workers is close to social optimal, The worker's search space of better response updates is enlarged, which eventually reduces the number of convergence steps. Fig. 10 shows our expected result with  $N = 1000, 2500, 4000$  workers.

At last, we study the impact of the range of worker demands on the convergence time. Fig. 11 shows the result with  $N = 1000, 2500, 4000$  workers. More better response updates are needed when the range of demand increases, since workers with high demands need to update themselves more frequently to find a satisfying sensing process. However, the number of better response updates is slightly reduced when the range of worker demands is at a certain level. It is suggested that the number of workers with high demands on sensing processes is increased, and they will hardly have a chance to update their better responses.

## VII. CONCLUSION

This paper presents a sensing processes participation game model for the sensing processes participation problem in participatory sensing, motivated by the observation that finding a global optimal solution of the problem is NP-hard. Our game model is based on congestion games in game theory. The properties of the sensing processes participation game including the convergence dynamics and the price of anarchy are studied, which enables us to design a fast sensing processes participation algorithm to find a pure Nash equilibrium in polynomial time. When the demands of all the workers are not too diverse, the social welfare of Nash equilibrium approximates the social optimum. We have conducted simulations to show the efficiency of our algorithm.

The future work can be carried along the following directions. First, since the price of anarchy implies that the social welfare of a pure Nash equilibrium can be close to the social optimum, it is necessary to explore the cases that workers or sensing processes are homogenous. Second, we shall examine the conditions that multiple workers are allowed to update their strategy profiles at the same time, to reduce the convergence time of the sensing processes participation algorithm.

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