Lab01-Proof

Exercises for Algorithms by Xiaofeng Gao, 2016 Spring Semester

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1. Prove that for any integer $n > 2$, there is a prime p satisfying $n < p < n!$. (Hint: consider a prime factor p of $n! - 1$ and use proof by contradiction)

2. Use minimal counterexample principle to prove that: for every integer $n > 17$, there exist integers $i_n \geq 0$ and $j_n \geq 0$, such that $n = i_n \times 4 + j_n \times 7$.

3. Suppose $a_0 = 1$, $a_1 = 2$, $a_2 = 3$, $a_k = a_{k-1} + a_{k-2} + a_{k-3}$ for $k \geq 3$. Use strong principle of mathematical induction to prove that $a_n \leq 2^n$ for all integers $n \geq 0$.

4. Prove, by Mathematical Induction, that

$$(n+1)^2 + (n+2)^2 + (n+3)^2 + \cdots + (2n)^2 = \frac{n(2n+1)(7n+1)}{6}$$

   is true for all natural numbers n.