Lab07-Graph Algorithms II

Exercises for Algorithms by Xiaofeng Gao, 2016 Spring Semester

Name:_____ Student ID:_____ Email: _____

1. Given a 2d grid map of 1s (land) and 0s (water), design a BFS-style algorithm to count the number of islands. An island is surrounded by water and is formed by connecting adjacent lands horizontally or vertically. You may assume all four edges of the grid are all surrounded by water. e.g. If the input grid is like the right one, the algorithm should return 3.

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11000
11000
00100
00011
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2. Consider a directed graph in which the only negative edges are those that leave s; all other edges are positive. Can Dijkstra's algorithm, started as s, fail on such a graph? Prove your answer.

3. A *bottleneck spanning tree* T of an undirected graph G is a spanning tree of G whose largest edge weight is minimum over all spanning trees of G. We say that the value of the bottleneck spanning tree is the weight of the maximum-weight edge in T.

   (a) Argue that a minimum spanning tree is a bottleneck spanning tree.

   (b) Give a linear-time algorithm that given a graph G and an integer b, determines whether the value of the bottleneck spanning tree is at most b.

4. In an undirected graph $G = (V, E)$, a vertex v is an articulation point if v is in every path from w to x (w, x are vertices different from v). Subsequently, we call a connected graph biconnected if and only if it has no articulation points. The maximal biconnected subgraph in a connected graph G is defined as G's *Biconnected Component*.

   (a) Prove that in a DFS tree, a vertex (not root) v is an articulation point if and only if :

   - v is not a leaf.
   - Some subtree of v has no back edge incident with a proper ancestor of v.

   (b) Design an efficient algorithm to find all the *Biconnected Components* in an undirected graph using the above characteristics. (hint: it is a DFS-style algorithm, and could be easily modified to find strong connected components in a directed graph.)