Lab08-PathAndFlow

Exercises for Algorithms by Xiaofeng Gao, 2016 Spring Semester

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1. During the procedure of Bellman-Ford algorithm in a directed Graph $G = (V, E)$, we use $successor[v]$ to maintain the first vertex (after v itself) on its path to the destination t. Now let G_p denote the directed "pointer graph" whose vertices are V, and whose edges are $(v, successor[v])$. Prove that if G_p contains a cycle C, then this cycle must have negative cost.

2. In the Floyd-Warshall algorithm, $c_{ij}^{(k)}$ is defined as the weight of a shortest path from i to j with intermediate vertices belonging to the set $\{1, 2, \cdots, k\}$, which is recursively defined as

$$d_{ij}^{(k)} = \begin{cases} w_{ij} & k = 0, \\ \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}) & k \geq 1. \end{cases} \tag{1}$$

   Specifically, we define $\pi_{ij}^{(k)}$ as the predecessor of vertex j on a shortest path from vertex i with all intermediate vertices in the set $\{1, 2, \cdots, k\}$. Please give its recursive definition in a similar way.

3. Johnson's algorithm needs a label $h(v)$ for each $v \in V$ to do the graph reweighting job by $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$. Naturally, the new set of edge weights $\hat{w}$ must satisfy an important property: For all edges (u, v), the new weight $\hat{w}(u, v)$ is nonnegative. Think about how we can ensure this property with certain $h(v)$.

4. Given a digraph $G = (V, E)$ and two nodes s and t.

   (a) Design an efficient algorithm to find the max number of *edge-disjoint* $s \to t$ path, and prove its correctness. (Two paths are *edge-disjoint* if they have no edge in common.)

   (b) Prove that the max number of edge-disjoint $s \to t$ paths is equal to the min number of edges whose removal disconnects t from s.