Lab10-Reduction

Exercises for Algorithms by Xiaofeng Gao, 2016 Spring Semester

Name:________ Student ID:________ Email: _____

1. What is the "certificate" and "certifier" for the following problems?

   (a) *CLIQUE*: Given an undirected graph, is there a subset S of k nodes such that there is an edge connecting every pair of nodes in S?

   (b) *3-COLORE*: Given a planar map, can it be colored with 3 colors? (Each edge should connect two nodes with different colors.)

   (c) *GRAPH ISOMORPHISM*: Given two graphs G and H, an isomorphism of them is a bijection between the vertex sets of G and H

   $$f : V(G) \to V(H)$$

   such that any two vertices u and v of G are adjacent in G iff $f(u)$ and $f(v)$ are adjacent in H. The graph isomorphism problem is to determine whether two finite graphs are isomorphic.

2. A *dominating set* for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one vertex in D. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for G. The *Dominating Set* (DS) problem concerns finding a minimum $\gamma(G)$ for a given graph G.

   Prove that: Independent Set $\equiv_p$ Dominating Set.

3. Given an integer $m \times m$ matrix A and an integer m-vector b, the *0-1 Integer Programming* problem asks whether there exists an integer n-vector x with elements in the set $\{0, 1\}$ such that $Ax \leq b$.

   Prove that: *0-1 Integer Programming* problem is NP-complete. (Hint: Reduction from *3-SAT*.)

4. A *Hamiltonian Cycle* in a graph is a cycle that visits every vertex exactly once. This is very different from an *Eulerian Cycle*, which is actually a closed walk that traverses every edge exactly once. Eulerian Cycles are easy to find and construct in linear time using a variant of depth-first search.

   Prove that: Finding a *Hamiltonian Cycle* in a graph is NP-Complete. (Hint: Using *Vertex Cover* for reduction will be most appreciated, while other reductions will get half the score.)