## Lab05-Numbering Programs

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

\* Please upload your assignment to FTP or submit a paper version on the next class \* If there is any problem, please contact: nongeek.zv@gmail.com \* Name:\_\_\_\_\_ StudentId: \_\_\_\_\_ Email: \_\_\_\_\_

- 1. Show that there is a total computable function k such that for each n,
  - (a) k(n) is an index of the function  $\lfloor \sqrt[n]{x} \rfloor$ .
  - (b)  $W_{k(n)}^{(m)} = \{(y_1, \dots, y_m) : y_1 + y_2 + \dots + y_m = n\} \ (m \ge 1).$
  - (c)  $E_{k(n)} = W_n$ .
- 2. (a) Find  $P_{1028}$ . Distinguish what are  $\phi_{1028}(x)$  and  $\phi_{1028}^{(n)}(x_1, \dots, x_n)$  and their corresponding  $W_{1028}(x), E_{1028}(x)$  and  $W_{1028}^{(n)}(x), E_{1028}^{(n)}(x)$ ;
  - (b) Let P be the program J(1,2,4), Z(1), S(1). Calculate  $\gamma(P)$ .
- 3. (a) (Cantor) Show that the set of all functions from  $\mathbb{N}$  to  $\mathbb{N}$  is not denumerable.
  - (b) Show that the set of all non-computable total functions from  $\mathbb{N}$  to  $\mathbb{N}$  is not denumerable.
- 4. Alternative Selection of  $\pi$

The  $\pi$  function where  $\pi(x, y) = 2^x(2y + 1) - 1$  can enumerate linearly all pairs of natural numbers  $(x, y) \in \mathbb{N} \times \mathbb{N}$ . However, it does not generate a trace in the first quadrant of the plane. Correspondingly, instead of applying this  $\pi$  function, we can define an alternative bijection  $\pi'$ , such that  $\pi' : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$  and it grows horizontally and vertically according to the right figure. Thus we have:

 $\pi'(0,0) = 0, \ \pi'(0,1) = 1, \ \pi'(1,0) = 2, \\ \pi'(1,1) = 3, \ \pi'(0,2) = 4, \ \pi'(1,2) = 5, \\ \pi'(2,0) = 6, \ \pi'(2,1) = 7, \ \pi'(2,2) = 8, \ \text{etc.}$ 



Now please develop a mathematical formula for  $\pi'$ , (like the notation of original  $\pi$ ), and prove the correctness of your design.