Lab08-Recursively Enumerable Set

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

$*$ Please upload your assignment to FTP or submit a paper version on the next class

$*$ If there is any problem, please contact: steinsgate@sjtu.edu.cn

$*$ Name:________ StudentId: ________ Email: ________

1. Let A, B be subsets of $\mathbb{N}$. Define sets $A \oplus B$ and $A \otimes B$ by

$$A \oplus B = \{2x \mid x \in A\} \cup \{2x+1 \mid x \in B\},$$
$$A \otimes B = \{\pi(x,y) \mid x \in A \text{ and } y \in B\},$$

where π is the pairing function $\pi(x,y) = 2^x(2y+1) - 1$. Prove that

(a) $A \oplus B$ is recursive iff A and B are both recursive.

(b) If $A, B \neq \varnothing$, then $A \otimes B$ is recursive iff A and B are both recursive.

2. Which of the following sets are recursive? Which are r.e.? Which have r.e. complement? Prove your judgements.

(a) $\{x \mid P_m(x) \downarrow \text{ in } t \text{ or fewer steps }\}$ (m, t are fixed).

(b) $\{x \mid x \text{ is a power of } 2\}$;

(c) $\{x \mid \phi_x \text{ is injective}\}$;

(d) $\{x \mid y \in E_x\}$ (y is fixed);

3. Prove following statements.

(a) Let $B \subseteq \mathbb{N}$ and $n > 1$; prove that B is r.e. then the predicate $M(x_1, \ldots, x_n)$ given by "$M(x_1, \ldots, x_n) \equiv 2^{x_1}3^{x_2}\ldots p_n^{x_n} \in B$" is partially decidable.

(b) Prove that $A \subseteq \mathbb{N}^n$ is r.e. iff $\{2^{x_1}3^{x_2}\ldots p_n^{x_n} \mid (x_1, \ldots, x_n) \in A\}$ is r.e..