Lab09-Recursively Enumerable Set(2)

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

$*$ Please upload your assignment to FTP or submit a paper version on the next class

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1. Suppose A is an r.e. set. Prove the following statements.

   (a) Show that the sets $\bigcup\limits_{x \in A} W_x$ and $\bigcup\limits_{x \in A} E_x$ are both r.e.

   (b) Show that $\bigcap\limits_{x \in A} W_x$ is not necessarily r.e. (*Hint*: $\forall t \in \mathbb{N}$ let $K_t = \{x : P_x(x) \downarrow \text{ in t steps}\}$. Show that for any t, K_t is recursive; moreover $K = \bigcup\limits_{t \in \mathbb{N}} K_t$ and $\overline{K} = \bigcup\limits_{t \in \mathbb{N}} \overline{K}_t$.)

2. Prove that $A \subseteq \mathbb{N}^n$ is r.e. iff $A = \varnothing$ or there is a total computable function $f : \mathbb{N} \to \mathbb{N}^n$ such that $A = Ran(\boldsymbol{f})$. (A *computable function* $\boldsymbol{f}$ from $\mathbb{N}$ to $\mathbb{N}^n$ is an n-tuple $\boldsymbol{f} = (f_1, \ldots, f_n)$ where each f_i is a unary computable function and $\boldsymbol{f}(x) = (f_1(x), \ldots, f_n(x))$.)

3. Suppose that f is a total computable function, A is a recursive set and B is an r.e.set. Show that $f^{-1}(A)$ is recursive and that $f(A)$, $f(B)$ and $f^{-1}(B)$ are r.e. but not necessarily recursive. What extra information about these sets can be obtained if f is a bijection?

4. A set D is the difference of r.e. sets (*d.r.e.*) iff $D = A - B$ where A, B are both *r.e.*.

   (a) Show that the set of all *d.r.e.* sets is closed under the formation of intersection.

   (b) Show that if $C_n = \{x \mid |W_x| = n\}$, then C_n is *d.r.e.* for all $n \geq 0$.