Lab10-Various Sets

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

∗ Please upload your assignment to FTP or submit a paper version on the next class
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1. Prove the following statements.
   (a) If B is r.e. and $A \cap B$ is productive, then A is productive.
   (b) If C is creative and A is an r.e. set such that $A \cap C = \varnothing$, then $C \cup A$ is creative.
2. Let $\mathscr{B}$ be a set of unary computable functions, and suppose that $g \in \mathscr{B}$ is such that for all finite $\theta \subseteq g$, $\theta \notin \mathscr{B}$. Prove that the set $\{x \mid \phi_x \in \mathscr{B}\}$ is productive.
3. If $A \oplus B = \{2x \mid x \in A\} \cup \{2x+1 \mid x \in B\}$, $A \otimes B = \{\pi(x,y) \mid x \in A \text{ and } y \in B\}$, prove the following statements.
   (a) Suppose B is r.e. If A is creative, then so are $A \oplus B$ and $A \otimes B$ (provided $B \neq \varnothing$).
   (b) If B is recursive, then the implications in (a) reverse.
   (c) If A, B are simple sets, prove that $A \otimes B$ is not simple but that $\overline{\overline{A} \otimes \overline{B}}$ is simple.