On the Multicast Lifetime of WANETs with Multibeam Antennas: Formulation, Algorithms, and Analysis

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Abstract—We explore the multicast lifetime capacity of energy-limited wireless ad hoc networks using directional multibeam antennas by formulating and solving the corresponding optimization problem. In such networks, each node is equipped with a practical smart antenna array that can be configured to support multiple beams with adjustable orientation and beamwidth. The special case of this optimization problem in networks with single beams have been extensively studied and shown to be NP-hard. In this paper, we provide a globally optimal solution to this problem by developing a general MILP formulation that can apply to various configurable antenna models, many of which are not supported by the existing formulations. In order to study the multicast lifetime capacity of large-scale networks, we also propose an efficient heuristic algorithm with guaranteed theoretical performance. In particular, we provide a sufficient condition to determine if its performance reaches optimum based on the analysis of its approximation ratio. These results are validated by experiments as well. The multicast lifetime capacity is then quantitatively studied by evaluating the proposed exact and heuristic algorithms using simulations. The experimental results also show that using two-beam antennas can exploit most lifetime capacity of the networks for multicast communications.

Index Terms—Wireless ad hoc networks, multicast, directional multibeam antenna, approximation algorithm

1 INTRODUCTION

N battery-powered wireless ad hoc networks (WANETs), energy supplied is likely to be a scarce resource and in some applications energy is entirely non-renewable. The network usability is limited by the battery energy in wireless devices. Energy conservation is of paramount importance for the wide deployment of wireless as hoc networks because the lifetime of batteries has not been improved as fast as processing speed of microprocessors. Multicasting plays an important role in typical multihop ad hoc networks where bandwidth is scarce and hosts have limited battery power. It is critical in applications where close collaboration of network hosts is required to carry out a given task. A multicast operation involves sending the piece of same information, called multicast packet, to all the members of the multicast group. The set of network nodes which may generate a multicast packet to be distributed to a multicast group are referred to as source nodes. Many routing protocols for wireless ad-hoc networks need a broadcast/

reprints@ieee.org, and reference the Digital Object Identifier below. Digital Object Identifier no. 10.1109/IC.2013.73 multicast as a communication primitive to update their states and maintain the routes between nodes. In addition, it is widely used in sensor networks to disseminate information, e.g., environmental changes, to other nodes in the network. Therefore, it is essential to develop efficient multicast protocols that are optimized to maximize the operating lifetime. Recent use of directional antennas in wireless communication has further enabled new approaches for energy saving in WANETs. This is because directional communications can save transmission power by concentrating RF energy where it is needed [1]. Some recently proposed exact algorithms, e.g., in [5]–[9], with polynomial time complexity show this optimization problem belonging to class P in networks with omnidirectional antennas. However, the same optimization problem in networks with directional antennas has been proven NP-hard [11]. Some exact and heuristic algorithms can be found in [10], [17] and [1], [9], [11], respectively.

The complexity of this energy-aware multicast problem may inhibit from providing optimal solutions for many network examples. As a result, most studies have focused on the logical problem of establishing energy-efficient structures for broadcast/multicast communications. Their approach is to assess the complexities one at a time and the study of underlying technologies, however, is not pursued at the same time. In other words, the multicast group is assumed to be assigned sufficient bandwidth and transceiver resources throughout the duration of the session. This paper shall further investigate this fundamental optimization problem along the same approach.

The lifetime of a multicast session is typically considered as the duration of the network operation time until the battery

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Fig. 1. Energy saving by directional K-beam antennas.

depletion of the first node in the network (e.g., [2]-[11]), although other definitions, like the time before a percentage of live nodes in the network, are possible. All existing solutions are based on a single-beam antenna model. However, the following observation shows that this antenna type may not flexible enough for energy efficient communications. Let us consider a broadcast scenario using the conventional single-beam antenna as shown in Fig. 1(a), in which a transmitting node needs to reach a set of downlink receiving nodes that are far separated in different directions. To exploit the single transmission property, a large beamwidth must be applied to cover all its desired receiving nodes. In this situation, the energy-saving feature of directional antennas has to be somewhat weakened. The extreme case of beam configuration would make directional antenna degenerate into omnidirectional antenna. This may result in the quick energy depletion of the transmitting nodes. On the other hand, the approach using single narrow-beam (to reach only one downlink node) but multiple transmissions (i.e., on the cost of throughput degradation) is usually impractical due to its lack of scalability in resource-constrained WANETs.

The above observation and consideration lead us to look at the multibeam antennas that can mitigate this inefficiency significantly. For example, the lifetime of the transmitting node using 2-beam antenna in Fig. 1(b) can be prolonged over 200% compared to the case in Fig. 1(a). Higher enhancement could be achieved using 3-beam antenna as shown in Fig. 1(c) because each beam applies the minimum beamwidth to cover only one node. The advances of the practical techniques on multiple beam antenna [12], [17], [20] and smart antenna array [13] make the above consideration meaningful and thus inspire us to study on this optimization problem further.

In order to explore the lifetime capacity of multicast in WANETs, we shall first develop an MILP (mixed integer linear programming) model for this optimization problem under a very general directional multibeam antenna model. To our best knowledge, this formulation is the first work that can accommodate so many various antenna configurations, including the multibeam antennas, that all existing work [10], [14], [15] for similar problems cannot. Many application scenarios can be solved based on this formulation using branch-and-cut or cutting planes techniques. Some numerical results using Integer Programming (IP) solver shall be presented for some network examples. The optimal solutions can be used to assess the performance of heuristic algorithms. Albeit valuable for theoretical reasons, practical usage of MILP is usually limited to a small input size, as is the usual case for many integer programming problems. Therefore, to deal with instances in real world applications in this paper, we shall also propose a practical heuristic algorithm for this

optimization problem. Our analysis shows that it is an approximation algorithm with an instance-dependent performance bound. An optimal solution determinant condition can be achieved from the close-form of its approximation ratio upperbound. Through a simulation study, we have evaluated the tradeoff between the costs, in terms of the maximum number of beams that the networks should support for each node, and the lifetime improvements by using the multibeam antenna technology. The experimental results show that using two-beam directional antennas can exploit most lifetime capacity of the networks for both multicast and broadcast communications.

The remainder of this paper is organized as follows. Section 2 describes our system model. Section 3 formulates this optimization problem as an MILP model. Section 4 presents a greedy algorithm for longest-lived multicast communications using directional multibeam antennas. Section 5 shows that the proposed algorithm has a bounded approximation ratio. These theoretical results help us extend the solutions by proposing an enhanced approximation algorithm in Section 6. Simulation studies for evaluating the real performance of the proposed algorithms and the tradeoff analysis between the equipment costs and performance improvements are given in Section 7. A sketch of some practical issues is discussed in Section 8. Finally, Section 9 summarizes our findings. For the convenience of the readers, the major notations used in this paper are listed in Table 1.

2 SYSTEM MODEL

Broadcasting is an inherent characteristic of wireless transmission because signal propagation occurs in all directions if networking units are equipped with omnidirectional antennas. In such a networking environment, a certain transmission power corresponds to an area of coverage, and a single transmission delivers a message to all nodes within the area. It has an obvious energy-saving benefit for broadcast and multicast applications. The directional antenna, on the other hand, permits energy savings by concentrating RF transmission energy to where it is needed. In particular, the smart antenna with multiple directional beams provide us an additional dimension to exploit the energy-saving features in WANETs.

Compared to the omnidirectional antenna, smart antenna would increase power consumption due to its more elaborated circuitry and DSP engines. As semiconductor process technologies continue to improve, the circuit power consumption of smart antenna transceivers in their transmit mode will continue decreasing. For example, the circuit power of the CMOS wireless transceivers at 2-5 GHz reported in the literature reduced over five times from 2005 to 2010 [30]. Meanwhile, the rest power consumption on amplifiers is primarily limited by the transmission power, which will not change over time. As a result, the transmission power will increasingly dominate the total power consumption and our paper will concentrate on the transmission power. To assess the complex trade-off one at a time, we assume in this paper that there is no mobility, even though the handling of mobility is one of the eventual goals of the approach outlined in this paper.

TABLE 1	
Notations	

χ^i_v	A binary variable which is equal to one if beam b_v^i is
	active, and zero otherwise
f^d_{vu}	A nonnegative continuous variable that represents the
	fictitious flow produced by the multicast initiator s
	going through arc (v, u) and terminated at node d
ω	The objective function which represents the reciprocal
	of multicast lifetime
ω_0	The optimal solution of ω when single beam with
	θ_{\min} is applied to all nodes
$\mu_{ ho}$	An upper-bound of approximation ratio of the MBLM
	algorithm
μ'_{o}	A tighter upper-bound of approximation ratio of the
, p	MBLM algorithm
B_v	$B_v = \left\{ b_{i_v} \mid i = 1, 2, \cdots, B_v \right\}$ is a set of beams that
-0	can be supported by each node v
$ heta^i$	The beamwidth of b^i
θ_{min}	The minimum beamwidth of each beam
c^{i}	A binary indicator which is equal to one if node u is
$^{\circ}vu$	located in beam b^i of node v and zero otherwise
r_{\cdots}	The distance between nodes v and u
r_{α}	The longest Euclidean distance between node <i>u</i> and
1.0	any of its child nodes in a given multicast tree
P	The energy supply associated with node v
K	The maximum number of active beams at each node
n^i	The transmission power of beam h^i if it is active
p_v	The transmission power as a function of transmission
$p(r, \sigma)$	range r and antenna beamwidth a
<i>n</i> .	The minimum transmission never for any hear
p_{\min}	The maximum transmission power for any beam
p_{\max}	The maximum manshiftshort power for any deam.
$\frac{4v}{t}$	The lifetime of pode <i>u</i>
v_{v}	The meanine of node v
(a, D)	A multicast sossion with a course node a and a act of
(s, D)	destination nodes D
C_{-}	$\frac{1}{2} \frac{1}{2} \frac{1}$
C(M, A)	An arcs crossing a node partition $(A, N - A)$
G(N, A)	A whereas network with a node set $N(N = n)$ and an are set A corresponding to logical wireless
	links that can be supported by beams
T	A multicast trac rectad at the rade a with a trace of
I_{S}	A municast tree rooted at the node s with a tree node set $N(T)$ and a tree are set $A(T)$
T	Set $IV(I_S)$ and a tree arc set $A(I_S)$
MBLM	A partially constructed multicast tree by the MBLM
$T^{x,y}$	algorithm The menticilly constructed multi-sections has the MDDM
MBLM	The partially constructed multicast tree by the MBLM
	algorithm before arc (x, y) is included
(x,y)	The added arc that just makes x the bottleneck node
(1)	of the tree constructed by the MBLM algorithm
(a,b)	The arc across $C_{N(T_{MBLM}^{x,y})}$ with the minimum
	$(p(r_{ab}, heta_{\min}) + q_a)/e_a$
$\Lambda^+(T)$	The child set of node v in the directed tree T

In our proposed directional multibeam antenna model, the beamwidth and orientation of each beam are freely adjustable as widely adopted in previous work [17]–[21]. The antenna arrays equipped for each node v can support a set of beams $B_v = \{b_v^i | i = 1, 2, ..., |B_v|\}$. Each beam $b_v^i \in B_v$ is defined as a set of neighboring nodes of v that must be covered by this beam and characterized by a beamwidth θ_v^i within θ_{\min} and 2π , where θ_{\min} is the minimum beamwidth determined by the physical characteristic of antennas. Furthermore, we assume that at most K beams can be active at a time, where K is a

small constant number, due to the constraint of antenna circuit complexity.

We use an example to illustrate the above model. Let α_{xy} ($0 \leq \alpha_{xy} < 2\pi$) be the angle measured counter-clockwise from the horizontal axis to the vector $\langle x, y \rangle$, from node x pointing to node y. Without loss of generality, we assume that $\alpha_1 < \alpha_2 < \cdots < \alpha_k$ are all such angles of vectors $\langle v, u_i \rangle$ ($i = 1, \ldots, k$) in an increasing order. The smallest beamwidth at v that covers neighboring node set $b_v \equiv \{u_1, \ldots, u_k\}$ is therefore

$$\theta_{v}(b_{v}) = \begin{cases} \theta_{\min} & k = 1, \\ \max\{\theta_{\min}, \min_{1} \leq i < k\{2\pi - \alpha_{i+1} + \alpha_{i}, \alpha_{k} - \alpha_{1}\}\} & k > 1. \end{cases}$$
(1)

We consider a widely used RF transmission model [1], [9]– [11], [17], [21] for directional antennas. For a given antenna beam that supports a link between two nodes separated by a distance r, the transmitted power required is proportional to r^{α} and the beamwidth θ , where the propagation loss exponent α typically takes on a value between 2 and 4. The transmission power for any beam is within the range p_{\min} and p_{\max} with $p_{\max} \ge p_{\min} > 0$. Without loss of generality, we further assume that all receivers have the same signal detection threshold, which is normalized to one, resulting in the RF transmission power p:

$$p(r, \theta) \equiv \max\left\{p_{\min}, \frac{r^{\alpha} \cdot \theta}{2\pi}\right\} \le p_{\max}.$$
 (2)

The wireless multibeam ad hoc network can thus be modeled as a simple directed graph G with a finite node set N(|N| = n) and an arc set A corresponding to the logical wireless links that can be supported by beams. A logical wireless link (v, u) exists, i.e., u is a neighbor of v, only when (v, u) can be supported by a beam at transmitter v. In multibeam networks, while a node can have many neighbors, the corresponding logical links may not be able to be supported at the same time due to the limits on the number of active beams. The RF power required on each arc is given by the function (2) which is dependent on the beam configuration of the node. A source-initiated multicast consists of a source node s and a set of destination nodes D. All the nodes $M = \{s\} \cup D$ involved in the multicast form a multicast tree rooted at the node *s*, i.e., a rooted tree T_s , with a tree node set $N(T_s)$ and a tree arc set $A(T_s)$. We define a rooted tree as a directed acyclic graph with a source node with no incoming arcs, and each other node has exactly one incoming arc. A node with no out-going arcs is called a leaf node, and all other nodes are internal.

Finally, we introduce several notations that will be used in the rest of the paper. We use $\Lambda_v^+(T)$ to denote the child node set of node v in the directed tree T. Given a multicast request (s, D), we use C_X to denote all arcs crossing a node partition (X, N - X) such that the first node set X must include at least the source node s and the second node set N - X must include at least one destination node in D, i.e.,

$$C_X \equiv \{(v, u) | v \in X \land u \in N - X\}.$$
(3)

Any C_X is referred as a cut of G with regards to a multicast request (s, D).



Fig. 2. Illustration for beam discretization.

3 OPTIMIZATION PROBLEM FORMULATION

In this section, we formulate the maximum-lifetime multicast problem under the general antenna model discussed in Section 2.

3.1 Beam Discretization

Although there are technically an infinite number of beam configurations for a directional antenna, it is sufficient to determine the solution of our optimization problem under the beam configurations such that any antenna beam can achieve minimum coverage for a subset of all neighboring nodes. In the following, we enumerate all possible beams at node v with neighboring nodes $\{u_i | i = 1, ..., k\}$. In general, we can classify all these beams into two sets: (c1) the beams that cover at least one neighboring node using θ_{\min} , and (c2) the beams that cover at least two neighboring nodes. Therefore, the set of beams B_v supported by v includes all beams belonging to $c1 \cup c2$ subject to achieving the smallest beamwidth that must satisfy (1).

The number of possible beams in c1 is at most k, i.e.,

$$|c1| \le k. \tag{4}$$

In order to evaluate the number of beams in c^2 , we use $B_v(u_i, u_j)$ ($i \neq j, i = 1, ..., k$ and j = 1, ..., k) to denote beams with boundaries that just cover u_i and u_j . Note that all possible beams in $B_v(u_i, u_j)$ are a series of concentric sectors with radiuses that can take at most k - 1 different values. The radius of each beam is determined by the farthest neighboring node in the sector. The beamwidth of each sector is the angle measured from $\langle v, u_i \rangle$ to $\langle v, u_j \rangle$ either clockwise, or counterclockwise, or both (when the farthest neighboring node is on the boundaries). Therefore, the number of beams in $B_v(u_i, u_j)$ is at most 2(k - 1). Considering that each beam in (c^2) must belong to $B_v(u_i, u_j)$ for some u_i , u_j , we have the following conclusion.

$$|c2| \le \sum_{i \ne j} |B_v(u_i, u_j)| \le {\binom{k}{2}} \cdot 2(k-1) = k(k-1)^2.$$
 (5)

Let $(u_i, u_j, u_k, +)$ or $(u_i, u_j, u_k, -)$ denote the beam in $B_v(u_i, u_j)$ defined by the boundaries that just cover u_i and u_j , the radius that reaches the farthest node u_k , and the orientation that is from $\langle v, u_i \rangle$ to $\langle v, u_j \rangle$ clockwise or counter-clockwise, respectively. We consider an example of beam discretization at node v with four neighboring nodes u_1, u_2, u_3

TABLE 2 The Corresponding Matrix $c_{vu_j}^i$ (i = 1, ..., 12 and j = 1, ..., 4) of the Example Given in Fig. 2

-	1	2	3	4	5	6	7	8	9	10	11	12
u_1	1	1	1	1	0	0	1	1	1	0	0	0
u_2	1	1	0	0	1	1	1	0	0	1	0	0
u_3	0	1	1	0	1	0	1	1	0	0	1	0
u_4	0	0	0	1	0	1	1	1	0	0	0	1

and u_4 depicted in Fig. 2. Given that the minimum beamwidth θ_{\min} can just cover single node, we list all valid beams of category c1 in Fig. 2(g) and category c2 in Fig. 2(a)–(f). Once the boundaries, radius and orientation (clockwise or counterclockwise) of a beam is determined, the set of nodes in such a beam is fixed. For example, nodes $\{u_1, u_2, u_3\}$ are inside beam $b_v^2 = (u_1, u_2, u_3, +)$ as shown in Fig. 2(a). A beam is considered as valid if the corresponding beamwidth satisfies (1), e.g., all beams in Fig. 2(a)–(g). A contradictory example is given in Fig. 2(h), where beam $(u_1, u_2, u_4, -)$ covering $\{u_1, u_2, u_4\}$ is invalid. This is because the beamwidth shown in Fig. 2(h) is not equal to the minimum one $\theta_v(u_1, u_2, u_4)$ given by (1). Actually, they can be covered by b_v^7 with a smaller beamwidth as illustrated in Fig. 2(e).

The beam sets c1 and c2 are disjoint only when the minimum beamwidth θ_{\min} covers a single node only. Otherwise, any beam in c1 covering multiple nodes will be enumerated again in c2. Therefore, the total number of valid beams to cover a non-empty subset of k neighboring nodes is at most |c1| + |c2| in the order of $O(k^3)$. Comparing to the exhaustive enumeration, which calculates a total $2^k - 1$ number of combinations, we find that most of these beams are identical and our method can significantly reduce the complexity of beam discretization procedure.

In order to describe the coverage of each beam configuration, a matrix with binary elements c_{vu}^i , $1 \le i \le |B_v|$ and $(v, u) \in A$, is recorded. By setting $c_{vu}^i = 1$, it represents node u in the coverage of beam b_v^i . Note that both the beam discretization and the corresponding coverage matrix can be obtained once the topology of the network is given. For example, Table 2 shows c_{vuj}^i (i = 1, ..., 12 and j = 1, ..., 4) of the example given in Fig. 2.

3.2 Problem Statement

In order to formulate the maximum lifetime multicast problem in multibeam WANETs, we define a set of optimization variables χ_{vv}^i , f_{vu}^i and ω as summarized in Table 1. The main idea is to extract a subgraph G'(N', A') from the original graph G using the solutions of these optimization variables, such that a multicast tree T_s^* of G' with maximum lifetime is derived.

Note that after our beam discretization procedure, the transmission range and beamwidth θ_v^i of each beam b_v^i at node v is known and the corresponding power p_v^i can be characterized as

$$p_v^i = p\left(\max_u \{r_{vu} \cdot c_{vu}^i\}, \theta_v^i\right),\tag{6}$$

in which r_{vu} is the distance between nodes v and u. Let e_v be the energy supply associated with node v. Given a supporting

multicast tree T_s with a feasible antenna beam assignment at each node, we can obtain the lifetime of node v as

$$t_v = \frac{e_v}{\sum_{i=1}^{|B_v|} p_v^i \cdot \chi_v^i + q_v},$$
(7)

where q_v denotes the reception power at node v. Note that q_v is equal to zero at source v = s. The lifetime of the multicast communication can thus be expressed as follows.

$$t(T_s) = \min_{v \in N(T_s)} t_v = \min_{v \in N(T_s)} \left\{ \frac{e_v}{\sum_{i=1}^{|B_v|} p_v^i \cdot \chi_v^i + q_v} \right\}.$$
 (8)

We consider the family of the trees T_s including all multicast nodes in M. The objective of the multicast lifetime maximization problem is to find a multicast tree T_s^* and the optimal beam assignment at each tree node such that each receiving node in the tree can be covered by an active beam of its transmitting node, each tree node actives at most K beams, and the tree lifetime t^* is maximized, i.e.,

$$t^* \equiv t(T_s^*) = \max_{T_s} t(T_s) = \max_{T_s} \min_{v \in N(T_s)} t_v.$$
(9)

If we consider the reciprocal lifetime ω_v as a weight function at each node v:

$$\omega_v \equiv \frac{1}{t_v} = \sum_{i=1}^{|B_v|} \frac{p_v^i}{e_v} \cdot \chi_v^i + \frac{q_v}{e_v},$$
 (10)

the original problem is equivalent to minimize the objective function ω , which is the bottleneck weight of the multicast tree T_s defined as follows.

$$\omega(T_s) \equiv \max_{v \in N(T_s)} \omega_v. \tag{11}$$

Finally, T_s^* is a multicast tree of *G* with maximum lifetime t^* , which is the reciprocal of the optimal solution ω^* , i.e.,

$$\omega^* \equiv \min_{T_s} \max_{v \in N(T_s)} \omega_v = \left(\max_{T_s} \min_{v \in N(T_s)} t_v \right)^{-1} = \frac{1}{t^*}.$$
 (12)

3.3 Constraint Formulation

To complete the MILP model with the objective function that is to minimize ω , it remains to construct a set of linear constraints that should define a multicast tree with feasible beam configurations in the context of directional multibeam antennas.

The bottleneck constraints guarantee that the objective function ω is equal to the bottleneck weight of the final multicast tree. In other words, its values should be equal to or greater than the lifetime reciprocal of any node in the tree, i.e.,

$$\omega \ge \sum_{i=1}^{|B_v|} \frac{p_v^i}{e_v} \cdot \chi_v^i + \frac{q_v}{e_v}, \forall v \in N.$$
(13)

 $min:\omega$

$$\begin{split} & \omega \geq \sum_{i=1}^{|B_v|} \frac{p_v^i}{e_v} \cdot \chi_v^i + \frac{q_v}{e_v}, \qquad \forall v \in N \\ & \sum_{i=1}^{|B_v|} \chi_v^i \leq K, \qquad \forall v \in N \\ & \sum_{u \neq v} f_{uv}^d - \sum_{u \neq v} f_{vu}^d = \delta_v^d, \quad \forall v \in N, \forall d \in D \\ & f_{vu}^d \leq \sum_{i=1}^{|B_v|} c_{vu}^i \cdot \chi_v^i, \qquad \forall (v, u) \in A, \forall d \in D \\ & \chi_v^i \in \{0, 1\}, \qquad \forall v \in N, \forall i, 1 \leq i \leq |B_v| \end{split}$$

Fig. 3. The MILP model to optimize the maximum-lifetime multicast problem.

Recall that each antenna can support *K* beams at most. This can be translated into the following constraints using the optimization variables χ_v^i as follows.

$$\sum_{i=1}^{|B_v|} \chi_v^i \le K, \forall v \in N.$$
(14)

In order to guarantee the connectivity of the extracted subgraph, we introduce the flow conservation constraints as follows. The formulation using single-commodity flow can be found in the existing work, e.g., in [10], [14]. A more standard and elegant formulation using multiple-commodity flow f_{vu}^d can also achieve the flow conservation constraints:

$$\sum_{u \neq v} f_{uv}^d - \sum_{u \neq v} f_{vu}^d = \delta_v^d, \forall v \in N, \forall d \in D,$$
(15)

in which the demand vector δ_v^d is defined as follows.

$$\delta_v^d \equiv \begin{cases} -1 & v = s \\ 0 & v \in N - \{s, d\} \\ 1 & v = d \end{cases}$$
(16)

Note that these variables f_{vu}^d only represent fictitious flow, instead of meaning multiple copies of same data to be sent to a set of children from a relay node. Using the optimal solution, denoted as $(\cdot)^*$, of each optimization variable obtained from the formulation, we construct a subgraph G'(N', A') of G as follows: $A' = \{(v, u) | \exists d \in D, (f_{vu}^d)^* > 0\}$ and $N' = \{v, u | (v, u) \in A'\}$. The flow conservation constraints in (15) guarantee that there must exist at least one directed path from source to each destination node in G'.

Furthermore, each link in G' should be supported by at least one active beam, i.e.,

$$f_{vu}^{d} \le \sum_{i=1}^{|B_{v}|} c_{vu}^{i} \cdot \chi_{v}^{i}, \forall (v, u) \in A, \forall d \in D.$$

$$(17)$$

Let T_s^* be an arbitrary multicast tree of G'. Because other constrains make beam configuration at each node in T_s^* feasible, the extracted sub-graph T_s^* is an optimal multicast tree of the original graph G with maximum lifetime.

Finally, our derivations on the linear constraints can now allow us to complete the MILP formulation as summarized in Fig. 3 for the maximum-lifetime multicast problem.

4 A GREEDY ALGORITHM

The maximum-lifetime multicast problem has been proven NP-hard [11] even for the networks equipped with singlebeam directional antennas. Although the optimal solutions would be obtained from a mathematical programming approach, e.g., an MILP (mixed integer linear programming) model, the amount of time required for large-size network examples might be excessive. In order to handle such networks, we propose a heuristic algorithm MBLM (Multi-Beam Long-lived Multicast), by the first time, for this multicast lifetime optimization problem in wireless multibeam ad hoc networks.

The basic idea of the MBLM algorithm is to incrementally construct a multicast tree from the source node by including one node at a time in a greedy manner such that the partially constructed tree T_{MBLM} achieves its bottleneck weight $\max\{\omega_v | v \in N(T_{MBLM})\}$ as small as possible until the tree contains all the nodes in M.

We consider a certain iteration round of the tree formation. When determining the new node to be included into the intermediate tree T_{MBLM} , each tree-node v should check each node u outside the tree and calculate the corresponding weight $\omega_v(T_{MBLM}, u)$, which is the weight at node v after T_{MBLM} includes a new link (v, u) and node v reassigns its beams to accommodate the new child u. The final candidate child c_v for the tree-node v is chosen such that the resulting weight at v is minimized, i.e.,

$$c_v = \arg\min_{u \in N-N(T)} \omega_v(T_{MBLM}, u).$$
(18)

After all tree nodes set their candidate child nodes, link (v^*, c_{v^*}) with minimum $\omega_{v^*}(T_{MBLM}, c_{v^*})$ will be chosen to be included into the tree, i.e.,

$$v^* = \arg\min_{v \in N(T_{MBLM})} \omega_v(T_{MBLM}, c_v).$$
(19)

Algorithm 1 The MBLM Algorithm

- 1: Initialize T_{MBLM} by setting $N(T_{MBLM}) = \{s\}$ and $A(T_{MBLM}) = \emptyset$.
- 2: while $M \not\subset N(T_{MBLM})$ do
- 3: Find the arc (v^*, c_{v^*}) using Eqs. (18) and (19).
- 4: Include (v^*, c_{v^*}) into the tree by setting $N(T_{MBLM}) = N(T_{MBLM}) \cup \{c_{v^*}\}$ and $A(T_{MBLM}) = A(T_{MBLM}) \cup \{(v^*, c_{v^*})\}.$
- 5: end while
- 6: Prune the multicast tree T_{MBLM} all transmissions that are not needed to reach the nodes in M.

In order to calculate the weight $\omega_v(T_{MBLM}, u)$, we propose a BRA (Beam Re-Assignment) algorithm. It will return the smallest value of weight at node v under a set of different beam reassignments that can cover the node set $\Lambda_v^+(T_{MBLM}) \cup \{u\}$ by at most K beams. Let λ ($\lambda \leq K$) be the

number of beams at node v before the beam reassignment. We only consider the following three cases for possible beam

Fig. 4. Illustration of the beam reassignment algorithm.

reassignments as illustrated in Fig. 4, in which the left and right parts are the beam assignments before and after the BRA algorithm, respectively.

- Simplex Case: If λ < K, a new beam is assigned to node u using minimum beamwidth as shown in Fig. 4(a).
- 2) *Expanding Case*: Any existing beam can include the new node *u* by widening or/and extending its beam as shown in Fig. 4(b).
- 3) *Merging Case*: Any two existing beams can merge into a single beam and then a new beam is assigned to node *u* using minimum beamwidth as shown in Fig. 4(c).

Eventually, after all destination nodes are reached, the final multicast tree will be obtained by pruning unnecessary links. Note that the post-pruning process shall adjust the antenna configurations at certain nodes as well. The beams should be deactivated if nodes inside are all removed, or shrunk to the minimum one as required by (1) if partially removed. The description of the MBLM algorithm in pseudo code is summarized in Algorithm 1. Note that the beam reassignments are conducted by simple heuristic as show in the above three cases and thus the new weight at node v is expected small, but not necessarily minimum.

The complexity of the MBLM algorithm is analyzed as follows. We notice that the number of possible reassignments for each case is at most 1, λ and $\lambda(\lambda - 1)/2$, respectively. Considering each operation for beam creation, expanding and merging has a complexity of O(1), we conclude the complexity of the BRA algorithm to be $O(\lambda^2)$ or $O(K^2)$. Finally, as we can see from the pseudo code of MBLM, the main loop iterates at most n times and in each iteration step, the BRA algorithm would be invoked by n^2 times at most, resulting in a complexity $O(K^2 \cdot n^3)$ of the MBLM algorithm.

5 THEORETICAL PERFORMANCE OF THE HEURISTIC ALGORITHM

In this section, we study the theoretical performance of the proposed MBLM algorithm in terms of approximation ratio. An algorithm for a minimization problem has an approximation ratio of $\rho(n)$ if, for any input of size n, the cost c of the



solution produced by the algorithm is within a factor of $\rho(n)$ of the cost c^* of an optimal solution: $c/c^* \leq \rho(n)$.

5.1 Preliminary Results

A couple of inequalities shall be developed first as the base for the derivation of the approximation ratio.

Lemma 1. Let r_v denote the longest Euclidean distance between node v and any of its child nodes in a given multicast tree. Then

$$\omega^* \ge \min_{T_s} \max_{v \in N(T_s)} \frac{p(r_v, \theta_{\min}) + q_v}{e_v}.$$
 (20)

Proof. Recall that variable χ_v^i indicates if the beam $b_v^i \in B_v$ is an active beam for each node v in the tree T_s . Let $b_v^j \in B_v$ be the beam containing the child node of node v in T_s with largest distance from v. Then the optimal solution can be bounded below as follows:

$$\begin{split} \omega^* &= \min_{T_s} \max_{v \in N(T_s)} \omega_v \\ &= \min_{T_s} \max_{v \in N(T_s)} \frac{\sum_{i=1}^{|B_v|} p\left(\max_u \{r_{vu} \cdot c_{vu}^i\}, \theta_v^i\right) \cdot \chi_v^i + q_v}{e_v} \\ &\geq \min_{T_s} \max_{v \in N(T_s)} \frac{p\left(\max_u \{r_{vu} \cdot c_{vu}^j\}, \theta_v^j\right) + q_v}{e_v} \\ &= \min_{T_s} \max_{v \in N(T_s)} \frac{p(r_v, \theta_v^j) + q_v}{e_v} \\ &\geq \min_{T_s} \max_{v \in N(T_s)} \frac{p(r_v, \theta_{\min}) + q_v}{e_v}. \end{split}$$

Lemma 2. For any node partition (X, N - X), where $s \in X$ and $D \cap (N - X) \neq \emptyset$,

$$\min_{(v,u)\in C_X} \frac{p(r_{vu},\theta_{\min})+q_v}{e_v} \le \min_{T_s} \max_{v\in N(T_s)} \frac{p(r_v,\theta_{\min})+q_v}{e_v}.$$
 (21)

Proof. Let T_s be a multicast tree. Recalling that there is at least one destination node, e.g., $z \ (z \in D)$, belonging to N - X, we then conclude that there must exist at least one arc, e.g., arc (v', u'), in the tree T_s connecting X and N - X, i.e., $(v', u') \in C_X$, such that the traffic can be propagated from the source s to the destination node z along the links in the tree T_s . Therefore, we can derive the inequality as follows.

$$\begin{split} \min_{(v,u)\in C_X} \frac{p(r_{vu},\theta_{\min})+q_v}{e_v} &\leq \frac{p(r_{v'u'},\theta_{\min})+q'_v}{e'_v} \\ &\leq \frac{p(r'_v,\theta_{\min})+q'_v}{e'_v} \\ &\leq \max_{v\in N(T_s)} \frac{p(r_v,\theta_{\min})+q_v}{e_v}. \end{split}$$

Because the above conclusion is derived based on an arbitrary multicast tree T_s , the inequality (21) holds as well.

By joining equations (20) and (21), we have the following conclusion.



Fig. 5. Illustration of the derivation of the MBLM algorithm's approximation ratio.

$$\omega^* \ge \min_{(v,u)\in C_X} \frac{p(r_{vu}, \theta_{\min}) + q_v}{e_v}.$$
(22)

5.2 An Approximation Ratio Upperbound of MBLM

We now turn our attention to the most interesting and difficult task on deriving the approximation ratio of the MBLM algorithm. Let ω be the solutions obtained from the MBLM algorithm with regards to a multicast request (s, D). Assume that node x becomes the bottleneck node of the final tree after arc (x, y) is added into the tree as shown in Fig. 5. Let $T_{MBLM}^{x,y}$ be the partially constructed multicast tree by the MBLM algorithm before (x, y) to be included. We then define the node partition (X, N - X), in which $X \equiv N(T_{MBLM}^{x,y})$ is the tree node set and N - X as the non-tree node set, respectively. Referring to Fig. 5, we are especially interested in the arc (a, b)that satisfies

$$\frac{p(r_{ab},\theta_{\min})+q_a}{e_a} = \min_{(v,u)\in C_X} \frac{p(r_{vu},\theta_{\min})+q_v}{e_v}.$$
 (23)

Furthermore, we assume that at this moment our BRA algorithm finds node c_a as the candidate child of node a. Note that $c_x = y$. In some instances, but not in all, we also have $c_a = b$. Let χ_w^i be the intermediate beam reassignment for each node $w \in N(T_{MBLM}^{x,y})$ if its candidate child c_w is set by the BRA algorithm. Therefore, the weights at node x and node a can be expressed as (24) and (25), respectively, using the intermediate solutions of beam assignment.

$$\omega_x(T_{MBLM}^{x,y}, c_x) = \sum_{i=1}^{|B_x|} \frac{p_x^i \cdot \chi_x^i}{e_x} + \frac{q_x}{e_x},$$
 (24)

$$\omega_a(T_{MBLM}^{x,y}, c_a) = \sum_{i=1}^{|B_a|} \frac{p_a^i \cdot \chi_a^i}{e_a} + \frac{q_a}{e_a}.$$
 (25)

Recalling the node selection criteria of the MBLM algorithm, given in (18) and (19), we have

$$\omega = \omega_x(T^{x,y}_{MBLM}, c_x) \le \omega_a(T^{x,y}_{MBLM}, c_a).$$
(26)

By applying (26), (25), (23) and (22) sequentially, we derive the analytical expression of ρ in steps as follows.

$$\begin{split} \rho &\equiv \frac{\omega}{\omega^*} \leq \frac{\omega_a(T_{MBLM}^{x,y}, c_a)}{\omega^*} = \frac{\sum_{i=1}^{|B_a|} p_a^i \cdot \chi_a^i / e_a + q_a / e_a}{\omega^*} \\ &= \frac{\sum_{i=1}^{|B_a|} p_a^i \cdot \chi_a^i / e_a + q_a / e_a}{p(r_{ab}, \theta_{\min}) / e_a + q_a / e_a} \cdot \frac{p(r_{ab}, \theta_{\min}) / e_a + q_a / e_a}{\omega^*} \\ &= \frac{\sum_{i=1}^{|B_a|} p_a^i \cdot \chi_a^i + q_a}{p(r_{ab}, \theta_{\min}) + q_a} \cdot \frac{\min_{(v,u) \in C_X} (p(r_{vu}, \theta_{\min}) / e_v + q_v / e_v)}{\omega^*} \\ &\leq \frac{\sum_{i=1}^{|B_a|} p_a^i \cdot \chi_a^i + q_a}{p(r_{ab}, \theta_{\min}) + q_a}. \end{split}$$

Finally, we can achieve the following conclusion based on the above derivations.

Theorem 3. The approximation ratio of the MBLM algorithm is upper bounded by

$$\mu_{\rho} \equiv \frac{\sum_{i=1}^{|B_a|} p_a^i \cdot \chi_a^i + q_a}{p(r_{ab}, \theta_{\min}) + q_a}.$$
(27)

While the upper bound given in (27) is instance specific, it can be used to verify the optimal solutions in some cases. Once the result $\mu_{\rho} = 1$ is achieved; we can conclude that the solution found by the heuristic algorithm is optimal. Actually, in our simulation studies, we have identified many of such scenarios. In particular, a sufficient condition to determine if an obtained solution from MBLM is optimal is given in the following Corollaries.

Corollary 4. The solution obtained from MBLM is optimal if

$$\Lambda_a^+(T_{MBLM}^{x,y}) = \emptyset.$$
⁽²⁸⁾

- **Proof.** If the downlink node set of *a* in the intermediate tree $T_{MBLM}^{x,y}$ is empty, we can conclude that its candidate child must be *b* from (18) and (23), i.e., $c_a = b$ as shown in Fig. 5. Therefore, we have $\sum_{i=1}^{|B_a|} p_a^i \cdot \chi_a^i = p(r_{ab}, \theta_{\min})$, resulting in $\mu_{\rho} = 1$.
- **Corollary 5.** The solution obtained from MBLM is optimal if omnidirectional antenna is used.
- **Proof.** When each beam is omni-directional, i.e., $\theta_{\min} = \theta_{\max}$ and K = 1, $c_a = b$ is always true since node b defined by (23) satisfies the node selection criterion of Algorithm 1 given in (18). Therefore, we have $\sum_{i=1}^{|B_a|} p_a^i \cdot \chi_a^i = p(r_{ab}, \theta_{\min})$, resulting in $\mu_{\rho} = 1$.

5.3 A Tighter Upperbound of Approximation Ratio

The efforts on deriving the upper bound of approximation ratio given in Theorem 3 aim for an explicit expression, showing that the MBLM algorithm has guaranteed performance, i.e., with a bounded approximation ratio. Now we further investigate such theoretical results and obtain an important result that achieves a tighter approximation ratio upperbound of our MBLM algorithm. We first define:

$$\omega_0 \equiv \min_{T_s} \max_{v \in N(T_s)} \left\{ \frac{p(r_v, \theta_{\min}) + q_v}{e_v} \right\}.$$
 (29)

Lemma 6. Given a network topology G, the value ω_0 defined in (29) can be found in polynomial time.

Proof. If we consider a fictitious weight function on each arc (v, u) of the graph *G* as follows

$$\omega_{vu} \equiv \frac{p(r_{vu}, \theta_{\min}) + q_v}{e_v},\tag{30}$$

equation (29) can thus be rewritten as

$$\omega_0 \equiv \min_{T_s} \max_{v \in N(T_s)} \frac{p(r_v, \theta_{\min}) + q_v}{e_v}$$
$$= \min_{T_s} \max_{(v,u) \in A(T_s)} \frac{p(r_{vu}, \theta_{\min}) + q_v}{e_v}$$
$$= \min_{T_s} \max_{(v,u) \in A(T_s)} \omega_{vu}, \qquad (31)$$

which is equivalent to find a bottleneck multicast tree of the graph *G*. This optimization problem has already been comprehensively studied and some of the algorithms, e.g., the variant Dijkastra's algorithm [8] or the Prim's algorithm [9], can find the optimal solution ω_0 in polynomial time.

The value ω_0 is useful for computing a tight approximation ratio μ'_{ρ} of Algorithm 1, where μ'_{ρ} is defined as

$$\mu'_{\rho} \equiv \frac{\sum_{i=1}^{|B_x|} p_x^i \cdot \chi_x^i + q_x}{e_x \cdot \omega_0}.$$
(32)

Theorem 7.

$$\rho \leq \mu'_{\rho} \leq \mu_{\rho}.$$

Proof. By applying (24), (20) and (29), we get

$$\begin{split} \rho &= \frac{\omega_x(T^{x,y}_{MBLM},y)}{\omega^*} \\ &= \frac{\sum\limits_{i=1}^{|B_x|} p_x^i \cdot \chi_x^i + q_x}{e_x \cdot \omega_0} \cdot \frac{\omega_0}{\omega^*} \\ &\leq \frac{\sum\limits_{i=1}^{|B_x|} p_x^i \cdot \chi_x^i + q_x}{e_x \cdot \omega_0} = \mu'_\rho. \end{split}$$

Furthermore, because the two upper bounds μ'_{ρ} and μ_{ρ} can be rewritten as

$$\mu'_{\rho} = \frac{\omega_x(T_{MBLM}^{x,y}, y)}{\min_{T_s} \max_{v \in N(T_s)} (p(r_v, \theta_{\min}) + q_v)/e_v},$$
(33)

$$\mu_{\rho} = \frac{\omega_a(T_{MBLM}^{x,y}, c_a)}{\min_{(v,u) \in C_{N(T_{MBLM}^{x,y})}} (p(r_{vu}, \theta_{\min}) + q_v)/e_v}.$$
 (34)

$$min: \quad \sum_{i=1}^{|B_x|} p_x^i \cdot \chi_x^i$$

s.t.
$$\sum_{i=1}^{|B_x|} \chi_x^i \le K$$

$$\sum_{i=1}^{|B_x|} c_{xu}^i \cdot \chi_x^i \ge 1, \quad \forall u \in \Lambda_x^+(T_s)$$

$$\chi_x^i \in \{0, 1\}, \qquad 1 \le i \le |B_x|$$

Fig. 6. The IP model to optimize the beam reconfiguration problem.

respectively, the conclusion $\mu'_{\rho} \leq \mu_{\rho}$ is straightforward to be achieved from Eqs. (26) and (21).

This result provides another computationally efficient method as given in Corollary 8, with polynomial time complexity, to verify if a solution from MBLM is optimal.

Corollary 8. The solution obtained from MBLM is optimal if

$$\frac{\sum_{i=1}^{|B_x|} p_x^i \cdot \chi_x^i + q_x}{e_x} = \omega_0.$$
(35)

Proof. The conclusion is obtained immediately from Theorem 7 because the upper bound μ'_{ρ} under this condition is equal to 1.

Note that since both (28) and (35) are just sufficient conditions for optimality checking, either could be used for rechecking when the other fails.

6 AN ENHANCED APPROXIMATION ALGORITHM

The new bound in a close-form eventually gives us an insight to design enhanced approximation algorithm with better performance. The upper bound of approximation ratio in a close-form as shown in Theorem 7 guides us to design new heuristic algorithm that can minimize the value μ'_{ρ} and thus achieve better performance.

Now we know that to minimize the upper bound μ'_{ρ} given in (32) is equivalent to minimize $\sum_{i=1}^{|B_x|} p_x^i \cdot \chi_x^i$, in which x is the bottleneck node of the multicast tree T_s with the beam assignment χ_x^i obtained by the MBLM algorithm. Therefore, it can be formulated as an IP problem as given in Fig. 6.

The first constraint in Fig. 6 guarantees that the number of active beams at node v not to exceed K. The second constraint means that for any child node u of x, it must be covered by at least one active beam. Let $k \equiv |\Lambda_x^+(T_s)|$ be the number of child nodes of x in the tree T_s . From the results of our beam enumeration method discussed in Section 3.1, this problem can be solved in $O(k^{3K})$ time even by exhaustive search. Due to few constraints in the IP model, only O(k), the optimal solution can always be obtained in a timely manner by an IP solver as we observed in our simulation studies.

We notice that this minimization problem is very relevant to the well-known *weighted set cover* problem [16], formulated as follows. Let U be a finite set of elements and $V = \{V_1, V_2, \ldots\}$ be a collection of subsets of U such that $\bigcup_{V_i \in V} V_i = U$. Each element V_i in V associates a weight $c(V_i)$. The weighted set cover problem is to find a subset $C \subseteq V$ such that it covers all the elements in U, i.e., $\bigcup_{V_i \in C} V_i = U$ and $\sum_{i:V_i \in C} c(V_i)$ is minimized. When the maximum number of active beams is arbitrary up to $|\Lambda_x^+(T_s)|$, the problem to minimize $\sum_{i=1}^{|B_x|} p_x^i \cdot \chi_x^i$ is exactly a weighted set cover problem if we made a mapping of the two optimization problems as follows.

$$U = \Lambda_x^+(T_s),$$

$$V_i = b_x^i, i = 1, 2, \dots, |B_x|,$$

$$c(V_i) = p_x^i, i = 1, 2, \dots, |B_x|,$$

$$C = \{b_x^i | \chi_x^i = 1\}.$$

Finally, an improved approximation algorithm EMBLM (Enhanced MBLM) based on our MILP formulation is achieved. The description of the algorithm in pseudo code is given in Algorithm 2.

Another possible enhancement to Algorithm 1 is that we calculate the weight $\omega_v(T_{MBLM}, u)$ by solving the corresponding IP problem as formulated in Fig. 6 instead of invoking the BRA algorithm. Although this variant algorithm is likely to provide better solutions, it is not included in our study since it, contrary to MBLM/EMBLM, does not lead to criteria for verifying optimality.

Algorithm 2 The EMBLM Algorithm

1: The initial tree T_s is obtained from MBLM.

2: repeat

- 3: Find the bottleneck node x of T_s .
- 4: Optimize the beam assignment at node *x* using the IP model given in Fig. 6.
- 5: until no more beam reassignment could be made.

7 PERFORMANCE EVALUATION

In this section, we would like to explore the multicast lifetime capacity of WANETs using multi-beam directional antennas by evaluating the performance of both exact algorithm based on our MILP model given in Fig. 3. Furthermore, we shall evaluate the performance of heuristic algorithms MBLM and EMBLM, in terms of providing longest-lived multicast lifetime, in multi-channel WANETs with directional antennas. In order to guide the practical deployment of multi-beam technology in WANETs, we shall also evaluate the tradeoff between the hardware costs, in terms of maximum number of beams that the networks can support for each node, and the lifetime improvement by using multiple beam antennas.

In our network settings, a number of nodes are randomly generated within a square region 10×10 . The energy supply at each node is uniform distributed across e_{\min} and e_{\max} and the reception power is set to zero. The maximum number of active beams includes K = 1, 2 and 3, which are sufficient to learn the performance trend as we shall see later. The sub-problem formulated in Fig. 6 has shown to be solved very fast for all the cases in our experiments using CPLEX. The simulation parameters are summarized in Table 3, in which the units of parameters are all consistent with each other. We randomly generated 50 network examples for each network setting and we present here the average over those examples.

TABLE 3 Parameter Values for Simulation

n	20, 100 and 500
m/n	0.25, 0.5 and 1.0
$ heta_{\min}$	$15^{\circ}, 30^{\circ}, 60^{\circ}, 90^{\circ}$ and 180°
(p_{\max}, p_{\min})	(10, 1)
(e_{\max},e_{\min})	(500, 10)
K	1, 2 and 3
α	2

7.1 Approximation Ratio Bounds

The first set of experiments is to evaluate our derived theoretical bounds of approximation ratio μ_{ρ} and μ'_{ρ} given in Theorem 3 and Theorem 7, respectively, based on 100-node network examples.

The experimental results using directional multibeam antennas for K = 2, n = 100 and m = 50 with minimum beamwidth $\theta_{\min} = 60^{\circ}$ are given in the Fig. 7. The derived bound given by Theorems 3 and 7 is equal to one in 37 and 43, respectively, out of 50 randomly generated network instances. Those instances indicate that our proposed MBLM algorithm achieves optimum. The similar results have been observed for many other scenarios as well and thus are omitted here.

7.2 EMBLM vs. MBLM

We use the *normalized performance* to evaluate the performance improvement of EMBLM over MBLM. It is defined as the ratio of bottleneck node weight ω of the multicast tree obtained by EMBLM to the one by MBLM.

Fig. 8(a)–(c) depict graphically the normalized performance under K = 2 and various multicast sizes in networks with 20, 100 and 500 nodes, respectively. The x-axis represents the minimum beamwidths 15°, 30°, 60°, 90° and 180°, corresponding to the numbers 1–5 on the x-axle. We observe that the enhanced algorithm EMBLM improves the MBLM algorithm significantly when the minimum beamwidth is small. On the other hand, once the minimum beamwidth increases (greater than 90°), both algorithms consistently converge to the optimal solutions, a degenerate version [8], [9] for the omnidirectional antenna case.

7.3 Multi-Beam vs. Single-Beam

In this section, we would like to evaluate the performance improvement by using multiple beams over the configuration of just using single beams. Let t_K denotes the tree lifetime obtained from exact or heuristic algorithm when the maximum beam number is set as K. We use the metric t_K/t_1 to facilitate the comparisons of performance improvements under various values of K to the traditional case using singlebeam antennas over a wide range of network examples.

Table 4 presents the performance results on 20-node network examples by exact algorithm based on our MILP model using CPLEX under various multicast group sizes and minimal antenna beamwidths. We list mean and variance of the performance metrics in a format (mean, variance) for each K = 2 and 3 in the tables. We observe that, for all the cases, using multiple-beam directional antennas can improve the multicast lifetime significantly, in particular for smart array antennas that can tune narrow beams. For example, when



Fig. 7. The upper bounds of μ_{ρ} and μ'_{ρ} .

 $\theta_{\min} = 15^{\circ}$, the communication time using two-beam antennas is up to 2.13, 2.45 and 2.61 times, on an average, of the one using traditional single-beam antennas for multicast m = 5, 10 and 20, respectively.

In order to study the performance gain of multi-beam antennas in large networks, where the optimal solution based on our MILP model is intractable, we evaluate the ratio of lifetime obtained by EMBLM using multi-beam antennas to the one by the heuristic algorithm proposed in [9] using single-beam antennas. In the same format as in Table 4, the experimental results on 100-node and 500-node network examples are summarized in Tables 5 and 6, respectively. We observe similar phenomena that using two-beam directional antennas can improve the multicast lifetime significantly, while the additional improvements are marginal when increasing the antenna array elements to support more beams. We attribute it to the fact that, while more beams more total energy savings for multicast, two-beam antennas can save most energy at the bottleneck node.

7.4 Directional vs. Omnidirectional Antennas

Finally, we compare the performance of the EMBLM algorithm under various directional and omnidirectional antennas. In addition to the lifetime, other comparison metrics include power and delay of a multicast tree, in which the former is defined as the overall energy consumption in a unit time to multicast messages to all destinations in the multicast tree, while the latter as the maximum hops from the source to any destination in the multicast tree. The normalized experimental results, i.e., the ratios of the performance of EMBLM under directional antennas to the corresponding one under omnidirectional antennas, from 100-node and 500-node network instances are presented in Figs. 9 and 10, respectively.

Fig. 9(a)–(c) illustrate the normalized lifetime, power and delay, respectively, under various maximum beam numbers (*K*) and multicast sizes (*m*) in networks with 100 nodes. We first investigate the performance in single-beam (*K* = 1) network examples. We observe that using directional antennas can significantly improve the lifetime for both multicast (*m* = 50) and broadcast (*m* = 100) as shown in Fig. 9(a), especially when the minimum beamwidth θ_{\min} is small. Considering the overall power consumption, we notice that directional antennas save energy as well shown in Fig. 9(b). For example, in typical network examples with $\theta_{\min} = 60^{\circ}$, only 20% of the total energy consumption is required compared to using omnidirectional antennas. The results of



Fig. 8. Performance improvement of EMBLM over MBLM as a function of the minimum beamwidths 15°, 30°, 60°, 90° and 180° (corresponding to the numbers 1-5 on the x-axle) under various multicast sizes.

maximum delay in a multicast tree shown in Fig. 9(c) indicate that using directional antennas tends toward increasing the delay, but not too much and only when the minimum beamwidth is small. While the increased delay is about 50% when θ_{\min} is equal to 15°, negligible delay is observed when θ_{\min} is 60° or larger.

In the multi-beam directional antenna (K = 2) case as shown in Fig. 9(a), the lifetime normalized to the result achieved by omnidirectional antennas is further improved as revealed already in the previous section. Furthermore, the overall power consumption and maximum delay are both reduced as illustrated in Fig. 9(b) and (c), respectively. We attribute this desired performance to the fact that using multibeam directional antennas inclines to cover more down-link nodes at each transmission node in the multicast tree, while keeping the energy consumption relatively low at the same time. In large networks with 500-nodes, similar experimental results are obtained as given in Fig. 10.

8 SKETCH OF PRACTICAL ISSUES

The heuristic algorithms presented in Sections 4 and 6 are essentially centralized. To be deployed with low complexity

TABLE 4 Performance Improvement Using Multi-Beam vs. Single-Beam in 20-Node Networks

K	m	$\theta_{\min} = 15^{\circ}$	$\theta_{\rm min} = 30^{\circ}$	$\theta_{\rm min} = 60^{\circ}$	$\theta_{\rm min} = 90^{\circ}$
2	5	2.13, 0.847	1.52, 0.196	1.16, 0.079	1.04, 0.012
2	10	2.45, 0.750	1.67, 0.231	1.20, 0.077	1.06, 0.011
2	20	2.61, 0.905	1.71, 0.461	1.22, 0.056	1.06, 0.009
3	5	2.51, 0.745	1.59, 0.315	1.18, 0.069	1.04, 0.012
3	10	2.66, 0.893	1.70, 0.366	1.22, 0.068	1.06, 0.011
3	20	2.81, 0.878	1.79, 0.410	1.24, 0.056	1.06, 0.009

TABLE 5 Performance Improvement Using Multi-Beam vs. Single-Beam in 100-Node Networks

K	m	$\theta_{\min} = 15^{\circ}$	$\theta_{\min} = 30^{\circ}$	$\theta_{\min} = 60^{\circ}$	$\theta_{\min} = 90^{\circ}$
2	25	1.37, 0.319	1.35, 0.103	1.08, 0.035	1.04, 0.010
2	50	1.43, 0.442	1.38, 0.147	1.09, 0.029	1.04, 0.009
2	100	1.50, 0.677	1.42, 0.205	1.11, 0.022	1.05, 0.006
3	25	1.41, 0.331	1.38, 0.124	1.08, 0.035	1.04, 0.010
3	50	1.46, 0.483	1.40, 0.151	1.09, 0.029	1.04, 0.009
3	100	1.52, 0.630	1.43, 0.198	1.11, 0.022	1.05, 0.006

in real networks, especially in large-scale networks, they are desired to be implemented in a distributed manner. Two procedures are considered in the distributed implementation: the beam discretization and multicast tree construction.

In our beam discretization method proposed in Section 3.1, the location information of all neighbors at each node is required. In recent years, we have seen tremendous efforts devoted to building localization systems for wireless networks using signal strength, e.g., [24]–[27]. The advantage of such an approach is that using the same radio hardware for both communication and localization would enable a tremendous savings over deployment of a specific localization infrastructure. Furthermore, directional antennas can help improving the accuracy of localization as suggested in the literature [28], [29], e.g., lateration and angle measurements using directional antennas. The former requires multiple neighbors with knowledge of location to pinpoint the target node, while the latter uses orientation of antennas in addition to distance. Even if lateration is used based on the distance estimates with the directional antennas, directionality of the antennas is useful since it leads to more accurate estimates of individual distances [28]. This is attributed to the fact that in omni-directional antennas, there are more multi-path effects leading to more interference and more fluctuation in received signal strength. To deal with some inaccuracy of existing localization techniques, a calibration process will be performed to achieve the consistency between the theoretical and realistic results on beam configurations. Once a beam is chosen for supporting the multicast tree obtained from our heuristic algorithms, its maximum transmission range, orientation and beamwidth will be slightly adjusted such that the connectivity of the multicast tree is guaranteed via probing messages.

TABLE 6 Performance Improvement Using Multi-Beam vs. Single-Beam in 500-Node Networks

K	m	$\theta_{\min} = 15^{\circ}$	$\theta_{\min} = 30^{\circ}$	$\theta_{\min} = 60^{\circ}$	$\theta_{\min} = 90^{\circ}$
2	125	1.41, 0.282	1.37, 0.089	1.10, 0.026	1.05, 0.008
2	250	1.48, 0.408	1.39, 0.133	1.12, 0.018	1.05, 0.004
2	500	1.52, 0.582	1.43, 0.181	1.13, 0.016	1.05, 0.004
3	125	1.45, 0.312	1.39, 0.128	1.10, 0.026	1.05, 0.008
3	250	1.51, 0.471	1.41, 0.187	1.12, 0.018	1.05, 0.004
3	500	1.53, 0.606	1.44, 0.234	1.13, 0.016	1.05, 0.004



Fig. 9. Normalized lifetime, power and delay as a function of the minimum beamwidths 15°, 30°, 60°, 90° and 180° (corresponding to the numbers 1–5 on the x-axle) in 100-node networks under various multicast sizes.



Fig. 10. Normalized lifetime, power and delay as a function of the minimum beamwidths 15°, 30°, 60°, 90° and 180° (corresponding to the numbers 1–5 on the x-axle) in 500-node networks under various multicast sizes.

The above beam discretization also functions as a neighbor discovery process that allows each node to be aware of the existence and knowledge of all its neighbors. It will serve as the base for the following distributed implementation for multicast tree construction. Note that being a neighbor of any node v implies that it can be covered by at least one beam of v.

A distributed algorithm with low communication complexity has been proposed in [23] for the bottleneck multicast tree problem. While it provides an optimal solution to the multicast tree lifetime maximization problem when omnidirectional antenna is used, it can be extended to the similar problem in networks with multi-beam directional antennas. The basic idea is that whenever there is a multicast session request (s, D) in the network but no route information is known, the source will initiate the MBLM algorithm by running the distributed Search-and-Grow procedure iteratively. The Search phase of the procedure finds the minimum weight of arcs crossing the tree node set and the non-tree node set, denoted as w_{\min} . In the subsequent Grow phase, the intermediate tree grows by including non-tree links with a weight at most w_{\min} into the multicast tree. The only difference for the multi-beam directional antenna case is that the minimum weigh $w_{\min} = \omega_{v^*}(T_{MBLM}, c_{v^*})$ is obtained from Eqs. (18) and (19) by running the BRA algorithm at each tree node.

For the distributed implementation of the EMBLM algorithm, the initial multicast tree is obtained using exactly the same procedure above with a communication complexity of O(|A|) [23]. After that, the source will inform the bottleneck node, by sending notification message along the multicast

tree, to reconfigure its beam assignment using the IP model given in Fig. 6. This will be iterated until the lifetime of the bottleneck node cannot be prolonged further. Each improvement step has a communication complexity of O(|N|).

9 CONCLUSION

In this paper, we have systematically studied the fundamental problem associated with multicast lifetime optimization in WANETs with directional multibeam antennas. Our new proposed MILP formulation provides exact algorithm for small sized networks. To our best knowledge, this formulation is the first work that can accommodate both continuous and discrete multibeam antenna types. Many application scenarios can be solved efficiently based on this formulation using branch-and-cut or cutting planes techniques. In order to explore the multicast lifetime capacity in large-scale networks, we also propose a couple of heuristic algorithms with theoretical performance analysis. Our proofs show that the approximation ratios of the proposed algorithms are bounded by a constant number. In particular, we can claim in some cases that these heuristic algorithms achieve optimum based on the close-form of the approximation ratio bounds that we derived. The multicast lifetime capacity in such network settings is then quantitatively studied by evaluating the proposed performance-guaranteed heuristic algorithms using simulations. Finally, we have found from the experiments that using two-beam directional antennas can exploit most lifetime capacity of the networks for multicast communications.

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