Due: 2021/3/6

Homework 2 - Inductive Proof

∗ If there is any problem, please contact TA.
Name:________ Student ID:________ Email: ___________

Problem 1. 20 points

(a) Please look at page 21 in slide "inductive-proof". In the proof of the second case $\frac{n\ nat}{S(n)\ nat}$, what is the assumption in this case and what is the difference between assumption and I.H.?

(b) We define a judgement form $IsNat\ x\ a$.

$$\frac{x\ nat}{IsNat\ x\ true}NatRule \qquad \frac{x\ list}{IsNat\ x\ false}ListRule \qquad \frac{x\ tree}{IsNat\ x\ false}TreeRule$$

For which rule we can use its inversion rule? Give an explanation.

Problem 2. (20 points)

(a) Give an inductive definition of the judgement form max $n_1\ n_2\ n_3$, which indicates the max number between n_1 and n_2 is n_3.

(b) Prove by induction: if max $n_1\ n_2\ n_3$, then max $n_2\ n_1\ n_3$.

Problem 3. (30 points)

$$\frac{}{nil\ list} \qquad \frac{n\ nat \quad l\ list}{n :: l\ list}$$

(a) Define $append\ n\ l\ l' \iff l'$ is l appended with natural number n

(b) Define $reverse\ l\ l' \iff l'$ is the reverse of list l

(c) Define $sum\ l\ n \iff$ the sum of all elements in list l is n

(d) (20 points) Prove: If $sum\ l\ n$, and $reverse\ l\ l'$, then $sum\ l'\ n$.

Problem 4. (30 points) Recall the definition of natural numbers by n nat judgement taught in the lecture.

(a) Give an inductive definition of the judgement form fib $n_1\ n_2$, which indicates the n_1^{th} Fibonacci number is n_2.

(b) Give an inductive definition of the judgement form fibsum $n_1\ n_2$, which indicates the sum of the first n_1 Fibonacci numbers is n_2.

(c) Prove by induction: If fibsum n m then fib succ(succ(n)) succ(m), that is

$$\sum_{i=1}^{n} F_i = F_{n+2} - 1.$$

Remark: You just need to send your .pdf file to likaijian@sjtu.edu.cn. Email Subject line Format(also the pdf file name): HW_X_Name_StudentID