Due: 2021/3/20

Homework 4 - Typed

∗ If there is any problem, please contact TA.
Name:_____ Student ID:_____ Email: _____

Problem 1. (40 points) Please define following terms using lambda calculus:

1. iszero n
2. subtraction m n
3. equal m n
4. factorial n (One way to define it is to use pair)

Suppose *pred n* (predecessor of n) is given as follow:
$pred = \lambda nfx.n\ (\lambda gh.h\ (g\ f))\ (\lambda u.x)\ (\lambda u.u)$ (where λnfx means $\lambda n.\lambda f.\lambda x$)
(You can directly use the definition in the slides and the last homework, like add, tru and fls)

Problem 2. (40 points) Prove the **exchange lemma**: If $\Gamma, x : t_1, y : t_2, \Gamma' \vdash e : t$, then $\Gamma, y : t_2, x : t_1, \Gamma' \vdash e : t$. (proof by induction on derivation of $\Gamma, y : t, x : t, \Gamma' \vdash e : t$).

Problem 3. (20 points) **Preservation Theorem**: If $\Gamma \vdash e : t$ and $e \to e'$, then $\Gamma \vdash e' : t$. Is the inversion of preservation theorem right? Which is: If $\Gamma \vdash e' : t$ and $e \to e'$, then $\Gamma \vdash e : t$. Give a counter example if it's wrong or prove it if it's right.

Remark: You just need to send your .pdf file to likaijian@sjtu.edu.cn. Email Subject line Format(also the pdf file name): HW_X_Name_StudentID