Due: 2020/4/3

Homework 6 - Extend2

∗ If there is any problem, please contact TA.

Name:_____ Student ID:_____ Email: _____

Problem 1. Use *Fix* to write a lambda function called *fib*: int → int to compute the n-th Fibonacci number ($F_0 = 0, F_1 = 1, F_i = F_{i-1} + F_{i-2}$).

Problem 2. Consider adding trees with leaf node elements of type t (a "t tree") to the extended simply-typed language we have defined up to Lecture 6. The additional types, expressions, and values are as follows:

$t ::=$ ... (as already defined) ... $\mid t$ `tree`

$e ::=$ ... (as already defined) ... $\mid$ `Lf`$(e) \mid$ `Br`$(e_1, e_2) \mid$ `case` e `of` $(\texttt{Lf}(x) \Rightarrow e_1 \mid \texttt{Br}(x, y) \Rightarrow e_2)$

$v ::=$ ... (as already defined) ... $\mid$ `Lf`$(v) \mid$ `Br`(v_1, v_2)

Here are the operational rules:

$$\frac{e \to e'}{\texttt{Lf}(e) \to \texttt{Lf}(e')} \quad \text{(L)}$$

$$\frac{e_1 \to e_1'}{\texttt{Br}(e_1, e_2) \to \texttt{Br}(e_1', e_2)} \quad \text{(B1)}$$

$$\frac{e_2 \to e_2'}{\texttt{Br}(v_1, e_2) \to \texttt{Br}(v_1, e_2')} \quad \text{(B2)}$$

$$\frac{e \to e'}{\texttt{case } e \texttt{ of } (\texttt{Lf}(x) \Rightarrow e_1 \mid \texttt{Br}(x, y) \Rightarrow e_2) \to \texttt{case } e' \texttt{ of } (\texttt{Lf}(x) \Rightarrow e_1 \mid \texttt{Br}(x, y) \Rightarrow e_2)} \quad \text{(C1)}$$

$$\frac{}{\texttt{case Lf}(v) \texttt{ of } (\texttt{Lf}(x) \Rightarrow e_1 \mid \texttt{Br}(x, y) \Rightarrow e_2) \to e_1[v/x]} \quad \text{(C2)}$$

$$\frac{}{\texttt{case Br}(v_1, v_2) \texttt{ of } (\texttt{Lf}(x) \Rightarrow e_1 \mid \texttt{Br}(x, y) \Rightarrow e_2) \to e_2[v_1/x][v_2/y]} \quad \text{(C3)}$$

(a) Give the typing rules for `Lf`, `Br` and `case` expressions.
(b) Define a function *height* : *t tree* → *int*, which returns the height of a given tree, using the fix-point combinator.

Problem 3. Using the environment model for lambda calculus with let, write the detailed multi-step evaluation steps of the following λ expression. (Please define closures carefully just like in the lecture slides.)

```
let x = 2 in
  let y = 4 in
    let f1 = \x.\y.x+2*y in
      let f2 = \x.\y.2*x-y in
      f2 (f1 y x) 3
```

Remark: You just need to send your .pdf file to likaijian@sjtu.edu.cn. Email Subject line Format(also the pdf file name): HW_X_Name_StudentID