

Homework 9 - Inference

* If there is any problem, please contact TA.

Name:_____ Student ID:_____ Email: _____

Problem 1. Write down the principal solutions for the following sets of constraints:

- (a) $\{X = Int, Y = X \rightarrow X\}$
- (b) $\{Int \rightarrow Int = X \rightarrow Z\}$
- (c) $\{X \rightarrow Y = Y \rightarrow Z, Z = U \rightarrow W\}$
- (d) $\{Int = Int \rightarrow X\}$
- (e) $\{X = Int \rightarrow X\}$
- (f) $\{\}$

Problem 2. A language P is defined as follows.

$e ::= x \mid n \mid true \mid false \mid succ \mid pred \mid iszero \mid$
 $if\ e\ then\ e\ else\ e \mid fn\ x\ =>\ e \mid e\ e \mid rec\ x\ =>\ e \mid (e)$

The above grammar is quite ambiguous. We can resolve ambiguities by adopting the following conventions:

- Function application associates to the left, e.g. $e\ f\ g$ is $(e\ f)\ g$, not $e\ (f\ g)$.
- Function application binds tighter than `if`, `fn`, and `rec`, e.g. $fn\ f\ =>\ f\ 0$ is $fn\ f\ =>\ (f\ 0)$, not $(fn\ f\ =>\ f)\ 0$.

Evaluation Rules

(1) $n \Rightarrow n$, for any non-negative integer literal n

(2) $true \Rightarrow true$ $false \Rightarrow false$

(3) $error\ s \Rightarrow error\ s$

(4) $succ \Rightarrow succ$ $pred \Rightarrow pred$ $iszero \Rightarrow iszero$

(5)
$$\frac{b \Rightarrow true \qquad e1 \Rightarrow v}{if\ b\ then\ e1\ else\ e2 \Rightarrow v}$$

$$(6) \quad \frac{b \Rightarrow \text{false} \quad e2 \Rightarrow v}{\text{if } b \text{ then } e1 \text{ else } e2 \Rightarrow v}$$

$$(7) \quad \frac{e1 \Rightarrow \text{succ} \quad e2 \Rightarrow n}{e1 \ e2 \Rightarrow n+1}$$

$$(8) \quad \frac{e1 \Rightarrow \text{pred} \quad e2 \Rightarrow 0}{e1 \ e2 \Rightarrow 0} \quad \frac{e1 \Rightarrow \text{pred} \quad e2 \Rightarrow n+1}{e1 \ e2 \Rightarrow n}$$

$$(9) \quad \frac{e1 \Rightarrow \text{iszero} \quad e2 \Rightarrow 0}{e1 \ e2 \Rightarrow \text{true}} \quad \frac{e1 \Rightarrow \text{iszero} \quad e2 \Rightarrow n+1}{e1 \ e2 \Rightarrow \text{false}}$$

$$(10) \quad (\text{fn } x \Rightarrow e) \Rightarrow (\text{fn } x \Rightarrow e)$$

$$(11) \quad \frac{e1 \Rightarrow (\text{fn } x \Rightarrow e) \quad e2 \Rightarrow v1 \quad e[x:=v1] \Rightarrow v}{e1 \ e2 \Rightarrow v}$$

$$(12) \quad \frac{e[x:=(\text{rec } x \Rightarrow e)] \Rightarrow v}{(\text{rec } x \Rightarrow e) \Rightarrow v}$$

Typing

$$t ::= 'a \mid \text{int} \mid \text{bool} \mid t \rightarrow t \mid (t)$$

Here $'a$ is a *type variable*, used for polymorphic types.

$$(ID) \quad \frac{E(x) = t}{E \mid - x : t}$$

$$(NUM) \quad E \mid - n : \text{int}$$

$$(BOOL) \quad E \mid - \text{true} : \text{bool}$$

$$E \mid - \text{false} : \text{bool}$$

$$(BASE) \quad E \mid - \text{succ} : \text{int} \rightarrow \text{int}$$

$E \vdash \text{pred} : \text{int} \rightarrow \text{int}$

$E \vdash \text{iszero} : \text{int} \rightarrow \text{bool}$

(IF)
$$\frac{E \vdash e_1 : \text{bool} \quad E \vdash e_2 : t \quad E \vdash e_3 : t}{E \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : t}$$

(\rightarrow INTRO)
$$\frac{E[x : t_1] \vdash e : t_2}{E \vdash \text{fn } x \Rightarrow e : t_1 \rightarrow t_2}$$

(\rightarrow ELIM)
$$\frac{E \vdash e_1 : t_1 \rightarrow t_2 \quad E \vdash e_2 : t_1}{E \vdash e_1 e_2 : t_2}$$

(REC)
$$\frac{E[x : t] \vdash e : t}{E \vdash \text{rec } x \Rightarrow e : t}$$

(a) Infer the principal type of

$\text{rec } m \Rightarrow \text{fn } x \Rightarrow \text{fn } y \Rightarrow \text{if } \text{iszero } y \text{ then } x$
 $\text{else } m \text{ (pred } x \text{) (pred } y \text{)}$

(Find the constraint set, get principal solution and principal type)

Remark: You just need to send your .pdf file to likaijian@sjtu.edu.cn. Email Subject line Format(also the pdf file name): **HW_X_Name.StudentID**