Relational Database Design Theory (I)

March 24, 2023



- Assignment (II) due: April 2, 2023
- The first quiz: April 7, 2023



- Functional dependency theory (this lecture)
- NF's and decomposition algorithms (next lecture)

Functional Dependency Theory

Functional dependencies

Let $X = \{A_1, \dots, A_n\}$ and $Y = \{B_1, \dots, B_m\}$ be sets of attributes.

Definition

[Functional dependency]

A functional dependency (FD) is of the form

 $X \to Y$

that requires the attributes of X functionally determining the attributes Y.

In particular, a relation R satisfies $X \to Y$ if for every two tuples t_1 and t_2 of R

$$\wedge_{i=1}^{n} t_1[A_i] = t_2[A_i] \rightarrow \wedge_{j=1}^{m} t_1[B_j] = t_2[B_j].$$

- FD's are unique-value constraints.
- A FD $X \rightarrow Y$ holds on a relational schema R if every instance of R satisfies $X \rightarrow Y$.
- If $Y \subseteq X$, then $X \to Y$ is trivial.

Notation convention

- $A_1 \dots A_n$ represents $\{A_1, \dots, A_n\}$.
- Attributes: A, B, C, D, E
- Sets of attributes: X, Y, Z
- XY represents $X \cup Y$



sid	cid	cname	room	grade
123	Al-3613	Database	1-108	A+
223	Al-3613	Database	1-108	B+
123	CS-101	CS Intro.	3-325	А
334	CS-101	CS Intro.	3-325	A-
345	ICE-1404P	Database	2-203	А

Table: R(sid, cid, cname, room, grade)

- $\bullet \ \mathsf{cid} \to \mathsf{cname}$
- $\bullet \ \mathsf{cid} \to \mathsf{room}$
- cid \rightarrow {cname, room}
- $\bullet \ \text{sid}, \text{cid} \rightarrow \text{grade}$



A set X of attributes is a candidate key for relation R if

- X functionally determines all other attributes of R, i.e., X is a superkey.
- No proper subset of X functionally determines all other attributes of R. -That is, X is minimal.

A motivation to study FD's

sid	cid	cname	room	grade
123	Al-3613	Database	1-108	A+
223	Al-3613	Database	1-108	B+
123	CS-101	CS Intro.	3-325	А
334	CS-101	CS Intro.	3-325	A-
345	ICE-1404P	Database	2-203	А

Table: R(sid, cid, cname, room, grade)

- Data redundancy
- Update/insertion/deletion anomaly

Lossless join decomposition

Goal: Decompose R into R_1 and R_2 s.t.

 $\mathbf{R} = \mathbf{R}_1 \bowtie \mathbf{R}_2$

sid	cid	cname	room	grade
123	Al-3613	Database	1-108	A+
223	AI-3613	Database	1-108	B+
123	CS-101	CS Intro.	3-325	А
334	CS-101	CS Intro.	3-325	A-
345	ICE-1404P	Database	2-203	А

Table: R(sid, cid, cname, room, grade)

s_id	c_id	grade
123	Al-3613	A+
223	Al-3613	B+
123	CS-101	А
334	CS-101	A-
345	ICE-1404P	А

Table: R₁(sid, cid, grade)

c_id	cname	room
AI-3613	Database	1-108
CS-101	CS Intro.	3-325
ICE-1404P	Database	2-203

Table: R₂(cid, cname, room)

- $R_1 \cap R_2 = {\text{cid}}.$
- cid is a superkey of R_2 , i.e., cid \rightarrow {cid, cname, room}.

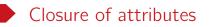
• Reasoning about FD's

Definition

- A set F of FD's logically implies a set G of FD's if every relation instance that satisfies all the FD's in F also satisfies all the FD's in G.
- F and G are equivalent if (i) F logically implies G and (ii) G logically implies F.

Example

- $\{A \rightarrow B\}$ logically implies $\{AC \rightarrow BC\}$.
- $\{A \rightarrow B, B \rightarrow C\}$ logically implies $\{A \rightarrow C\}$.
- $\{A \rightarrow B, B \rightarrow C\}$ is equivalent to $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$.
- $\{A_1A_2 \rightarrow B_1B_2B_3\}$ is equivalent to $\{A_1A_2 \rightarrow B_1, A_1A_2 \rightarrow B_2, A_1A_2 \rightarrow B_3\}$.



[Attribute closure]

Let X be a set of attributes and F be a set of FD's. The closure of X under F, written as X_F^+ , is the set of all attributes B such that F logically implies $X \to B$.

- We omit the subscript F and write X^+ if F is clear from the context.
- To determine whether F logically implies $X \to Y$ it suffices to check whether $Y \subseteq X^+$.
- To see if X is a superkey of R, it suffices to check if X^+ contains all the attributes of R.

Computing attribute closure

Figure: Computing attribute closure

- $F = \{A \rightarrow B, A \rightarrow C, CD \rightarrow E, CD \rightarrow K, B \rightarrow E\}$
- What is {A, D}_F^+?
- Is {A, D} a superkey/candidate key?

Algorithm correctness

Correctness. $\hat{X}_{F}^{+} = X_{F}^{+}$, where \hat{X}_{F}^{+} is the set of attributes computed by the algorithm.

- $\hat{X_F^+} \subseteq X_F^+$. $X \subseteq X_F^+$ and by I.H. every new element introduced in line 4 is also in X_F^+ .
- $X_F^+ \subseteq X_F^+$. Let B be an attribute not in X_F^+ . It suffices to show that F cannot imply $X \to B$. That is, there is a table R s.t.(i) R satisfies F, and (ii) R does not satisfy $X \to B$.

Let
$$\hat{X_F^+} = \{A_1, A_2, \dots, A_n\}$$
 and $\overline{X_F^+} = \{B_1, B_2, \dots, B_m\}$. We define R as

A_1	A_2	 An	B ₁	B ₂	 B _m
1	1	 1	1	1	 1
1	1	 1	0	0	 0

It should be clear that R does not satisfy $X \rightarrow B$. It remains to verify that R satisfies F.

Claim. R satisfies F.

We prove it by contraction. Let $X' \to Y'$ be an FD in F that R does not satisfy. By construction, we must have $X' \subseteq \{A_1, A_2, \ldots, A_n\}$ and $Y' \cap \{B_1, B_2, \ldots, B_m\} \neq \emptyset$. It follows that all the attributes in Y' should also be included in \hat{X}_F^+ (lines 3-4). This contradicts to $Y' \cap \{B_1, \ldots, B_m\} \neq \emptyset$.



The closure of F, denoted by F^+ , is the set of all FD's logically implied by F.

Question. Given a set of FD's F, how to decide whether $X \rightarrow Y \in F^+$?

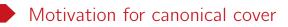
- Approach 1: compute X^+ and check whether $Y \subseteq X^+$.
- Approach 2: use Armstrong's axioms.

Armstrong's axioms

- Reflexivity: If $Y \subseteq X$, then $X \to Y$.
- Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$.
- Transitivity: If $X \to Y$ and $Y \to Z$, then $X \to Z$.

Theorem (Armstrong '74). The Armstrong's axioms are both sound and complete.

- Soundness: Only correct FD's are derived.
- Completeness: Every FD in F⁺ can be derived by using the axioms.



- A set of FD's F defines a set of unique-value constraints.
- We want a minimal set F' of FD's to reduce constraint checking cost.
- F' should be equivalent to F to ensure correctness.

A canonical cover $F_{\rm c}$ of F is a minimal set of FD's equivalent to F.

Extraneous attributes

An attribute of a FD $X \rightarrow Y$ in FD is extraneous if we can remove it without changing F⁺.

- An attribute $A \in X$ is extraneous and can be removed from the LHS of $X \to Y$ if F logically implies $(F \setminus \{X \to Y\}) \cup \{(X \setminus \{A\}) \to Y\}.$
- Example. $F = \{AB \rightarrow C, A \rightarrow D, D \rightarrow C\}$
- An attribute $B \in Y$ is extraneous and can be removed from the RHS of $X \to Y$ if $(F \setminus \{X \to Y\}) \cup \{X \to (Y \setminus \{B\})\}$ logically implies F.
- Example. $F = \{A \rightarrow CD, D \rightarrow C\}$

Extraneous attributes (cont'd)

Let $X \to Y$ be a FD in F.

• $A \in X$ is extraneous and can be removed from the LHS of $X \to Y$ if F logically implies $(F \setminus \{X \to Y\}) \cup \{(X \setminus \{A\}) \to Y\}$ $\iff Y \subseteq (X \setminus \{A\})_F^+.$

- Example. $F = \{AB \rightarrow C, A \rightarrow D, D \rightarrow C\}$. B can be removed from $AB \rightarrow C$ since $\{A\}_F^+ = \{A, C, D\}$ and $\{C\} \subseteq \{A\}_F^+$.
- $B \in Y$ is extraneous and can be removed from the RHS of $X \to Y$ if $(F \setminus \{X \to Y\}) \cup \{X \to (Y \setminus \{B\})\}$ logically implies F. $\iff B \in X_{F'}^+$ where $F' = (F \setminus \{X \to Y\}) \cup \{X \to (Y \setminus \{B\})\}.$
- Example. $F = \{A \rightarrow CD, D \rightarrow C\}.$

C can be removed from $A \to CD$ since $C \in \{A\}_{F'}^+$ where $F' = \{A \to D, D \to C\}$.



A canonical cover F_c for F is a set of FD's equivalent to F such that

- $\bullet\,$ No FD in F_c contains an extraneous attribute.
- Each LHS of a FD in F_{c} is unique.

Computing canonical cover

Figure: Computing canonical cover

Canonical cover example

Let
$$F = \{A \to BC, B \to C, A \to B, AB \to C\}.$$

• $F_0^0 = \{A \to BC, B \to C, AB \to C\}$

•
$$F_c^1 = \{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$$

• $F_c^1 = \{A \rightarrow B, B \rightarrow C, AB \rightarrow C\}$
• $F_c^2 = \{A \rightarrow B, B \rightarrow C\}$

Let $F = \{A \rightarrow BC, B \rightarrow AC, C \rightarrow AB\}.$

•
$$F_c = \{A \rightarrow B, B \rightarrow C, C \rightarrow A\}.$$

• $F_c = \{A \rightarrow C, C \rightarrow B, B \rightarrow A\}.$
• $F_c = \{A \rightarrow C, B \rightarrow C, C \rightarrow AB\}.$