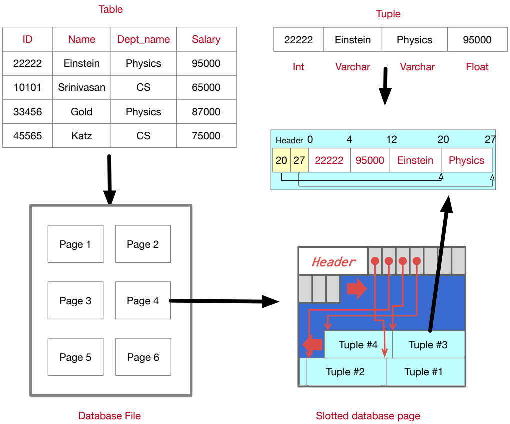
Indexing

Spring, 2024

Data storage structures (review)



Table

ID	Name	Dept_name	Salary
22222	Einstein	Physics	95000
10101	Srinivasan	CS	65000
33456	Gold	Physics	87000
45565	Katz	CS	75000

Tuple

22222	Einstein	Physics	95000
Int	Varchar	Varchar	Float

Database File

Slotted database page

- Tables are stored in database files.
- Each database file consists of a collection of pages.
- Each page holds a collection of tuples.

DBMS: Access method

Purpose: Support DBMS's execution engine to read/write data from pages more efficiently.

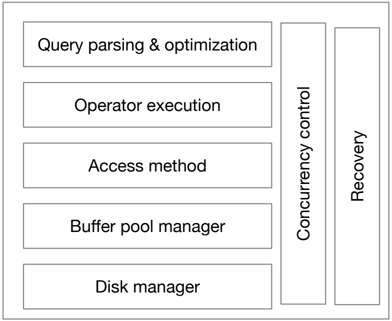
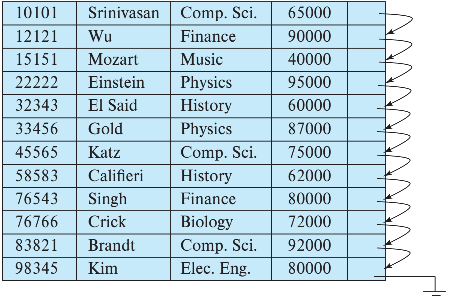


Figure: DBMS architecture

Indexing basics

Example

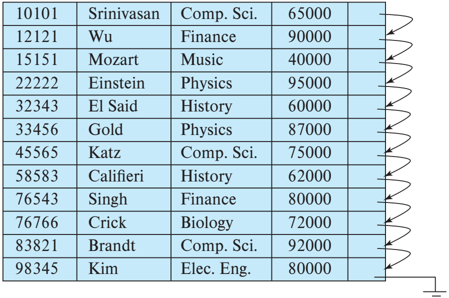


10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
32343	El Said	History	60000	
33456	Gold	Physics	87000	
45565	Katz	Comp. Sci.	75000	
58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

- Table instructor uses sequential file organization based on search key ID.
  – Records are ordered according to the attribute ID.
- Total number of pages of table instructor: 1,000 pages.
- Estimate the number of pages to read from disk for the query

```
SELECT * FROM instructor WHERE ID = '22222';
```

Example (cont'd)



10101	Srinivasan	Comp. Sci.	65000
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76543	Singh	Finance	80000
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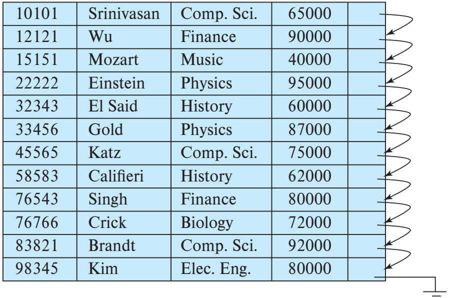
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

- Sequential scan: requires reading 1000 pages in the worst case.

Example (cont'd)



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22222	Einstein	Physics	95000
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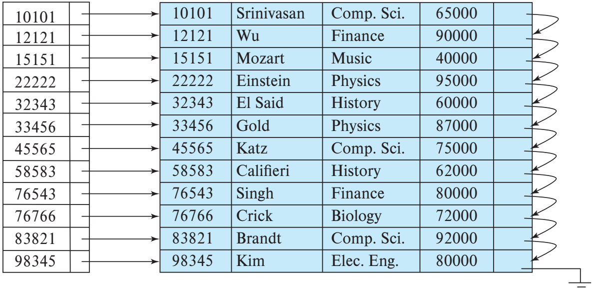
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

- Sequential scan: requires reading 1000 pages in the worst case.
- Binary search: $\lceil \log_2 1000 \rceil = 10$.

Example (cont'd)



10101	
12121	
15151	
22222	
32343	
33456	
45565	
58583	
76543	
76766	
83821	
98345	

10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
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58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

- Sequential scan: requires reading 1000 pages in the worst case.
- Binary search: $\lceil \log_2 1000 \rceil = 10$.
- Index scan: 3 pages + 1 page (assuming that the index files uses 3 pages).
- Index scan is also effective if the table is organized as a heap file.

Index data structure

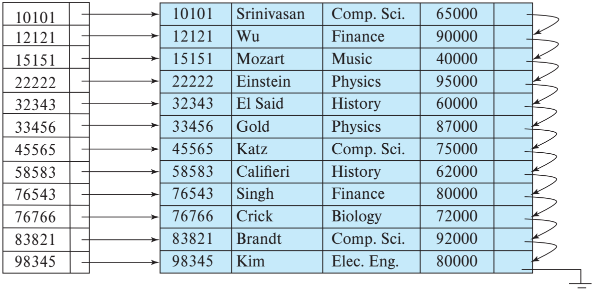
- Search key: an attribute or a set of attributes used to look up records in a file.
- An index file consists of records (called index entries) of the form

search key	pointer

- An index file is usually much smaller than the original file.
- We will only consider ordered indexes in this lecture.
    - Ordered indexes: search keys are organized in sorted order.
    - Hash indexes: search keys are distributed uniformly across buckets via a has function.

Dense indexes

- One index entry for each search key value.



10101
12121
15151
22222
32343
33456
45565
58583
76543
76766
83821
98345

10101	Srinivasan	Comp. Sci.	65000
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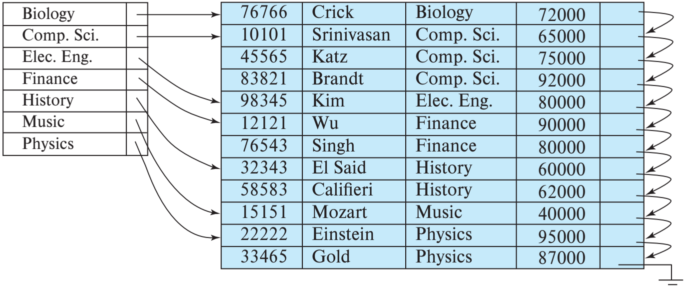
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

Figure: Dense index on attribute ID of table instructor

Dense indexes

- One index entry for each search key value.



Biology
Comp. Sci.
Elec. Eng.
Finance
History
Music
Physics

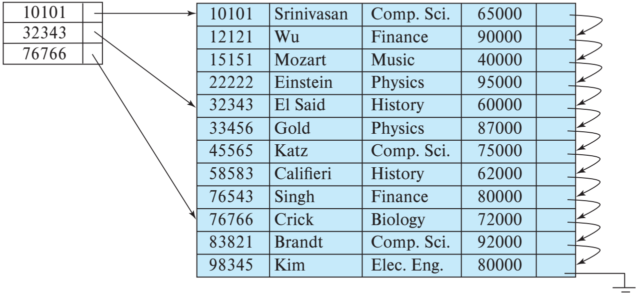
76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000

Figure: Dense index on attribute dept_name of table instructor

- It is possible that one index entry may point to multiple records.

Spare indexes

- Index entries for only some search key values.
  - Typically one index entry for each block.



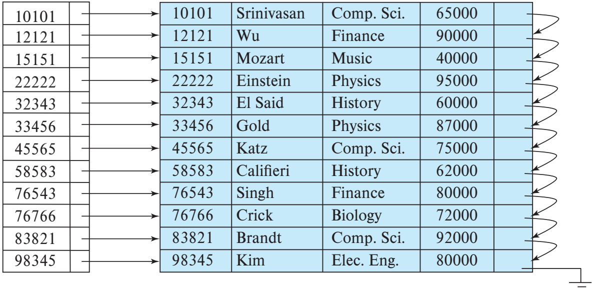
10101
32343
76766

10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

Figure: Sparse index on attribute ID of table instructor

- Applicable only when records are ordered by the search key. Why?

Clustering indexes



10101	
12121	
15151	
22222	
32343	
33456	
45565	
58583	
76543	
76766	
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10101	Srinivasan	Comp. Sci.	65000	
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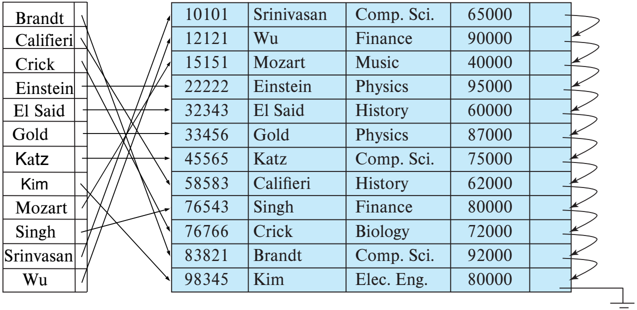
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

- Recall that index entries are sorted on the search key in an ordered index.
- Clustering index: search key order also defines the sequential order of data records.
- A clustering index is also known as a primary index.

Non-clustering index



Brandt
Califieri
Crick
Einstein
El Said
Gold
Katz
Kim
Mozart
Singh
Srinvasan
Wu

10101	Srinivasan	Comp. Sci.	65000
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- Non-clustering index: search key order differs from the sequential order of data records.
- A non-clustering index is also known as a secondary index.
- Secondary index is always dense. Why?

Recap

- An index is a data structure that improve the speed of data retrieval.
- Each index entry includes a search key and a pointer to a specific record.
- An index file is typically much smaller than the actual data files.
- Dense indexes vs. sparse indexes
- Clustering indexes vs. non-clustering indexes.

B^+-tree

B$^+$-tree

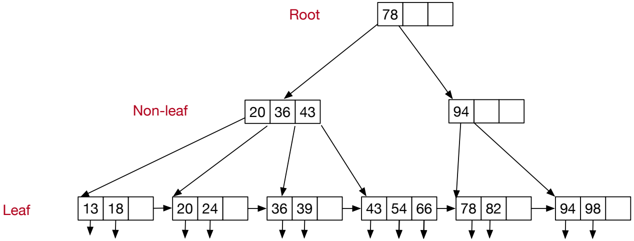


Figure: A sample B$^+$-tree with max_fanout$= 4$

A B$^+$-tree in a self-balancing search tree with following properties.

- Perfectly balanced; search, insertions, and deletions are in logarithmic time.
- Optimized for disk-based DBMS: one node per block/page, large fan-out.

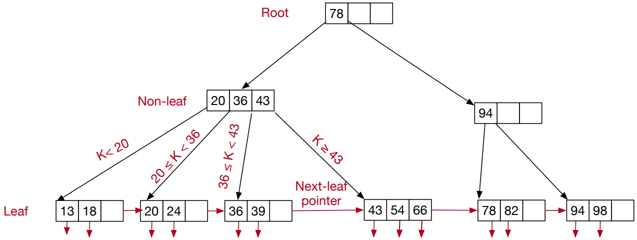
B^+-tree node

- Each B^+-tree node contains at most $n-1$ search keys and n pointers.
  - n is referred to as the max_fanout parameter.
- Search keys are arranged in sorted order:

$$K_1 < K_2 < \cdots < K_m < \ldots$$

- Every active pointer P_i points to a node in the next level.
- In practice, n can be hundreds, i.e., large fan-out.

B^+-tree node



- P_i points the sub-tree of search keys K with

$$K_{i-1} \leqslant K < K_i.$$

- Leaf nodes are chained up by the last pointer P_n, i.e., next-leaf pointer.
- Other active pointers P_i in leaf nodes point to the data page corresponding to key K_i.
- Index entries to data pages are stored in leaf nodes only.

B^+-tree invariant

- Balance invariant: all leaves are at the same level.
- Occupancy invariant: all nodes (except root) are at least half-full.

	Min #(Active pointers)	Min #(Keys)
Root	2	1
Internal node	$\lceil n/2 \rceil$	$\lceil n/2 \rceil - 1$
Leaf node	$\lfloor n/2 \rfloor$	$\lfloor n/2 \rfloor$

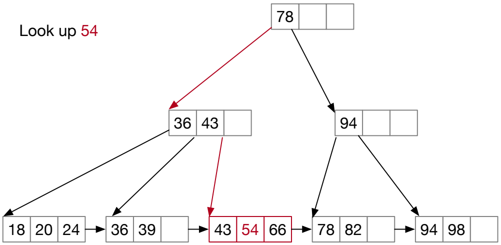
Table: Half-full constraint for B^+-trees

Claim. The height of a B^+-tree with N search keys is at most $\lceil \log_{\lceil n/2 \rceil} N \rceil$.

B^+-tree in practice

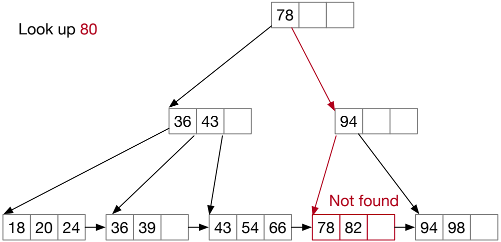
- $N = 1,000,000$.
- Page size: 4k bytes, index entry size 40 bytes.
- $n = 100$.
- $\lceil \log_{\lceil n/2 \rceil} N \rceil = 4$. That is, at most 4 I/O's for every lookup.
- If we cache the root node in buffer pool, then at most 3 I/O's are needed.

Query (1)



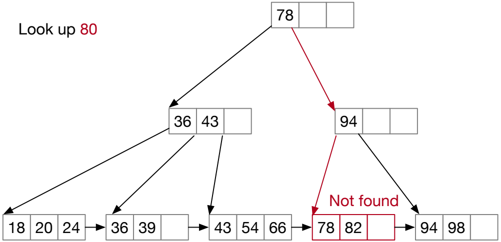
- `SELECT * FROM R WHERE K=54;`

Query (1)



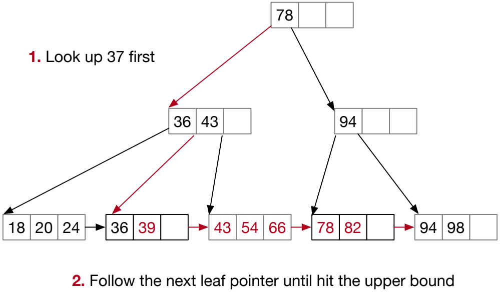
- `SELECT * FROM R WHERE K=54;`
- `SELECT * FROM R WHERE K=80;`

Query (1)



- `SELECT * FROM R WHERE K=54;`
- `SELECT * FROM R WHERE K=80;`
- This type of query is known as point query.

Query (2)



1. Look up 37 first

2. Follow the next leaf pointer until hit the upper bound

- `SELECT * FROM R WHERE k >= 37 AND K <= 90;`
- This type of query is known as range query.

Insertion (1)

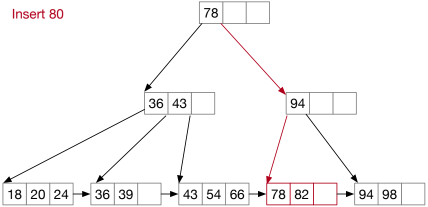


Figure: Insert key 80 ($n = 4$)

- Locate the leaf node for the key to be inserted.
- Insert the key directly when the target node has enough space.

Insertion (1)

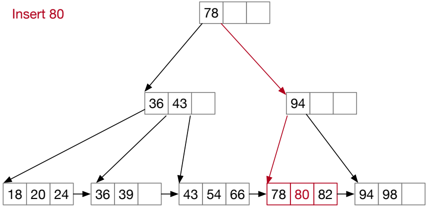


Figure: Insert key 80 ($n = 4$)

- Locate the leaf node for the key to be inserted.
- Insert the key directly when the target node has enough space.

Insertion (2)

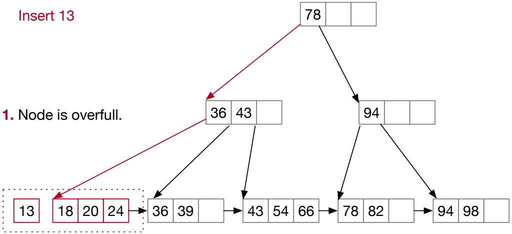


Figure: Insert key 13 ($n = 4$)

- Split the target node if the insertion make it overfull.

Insertion (2)

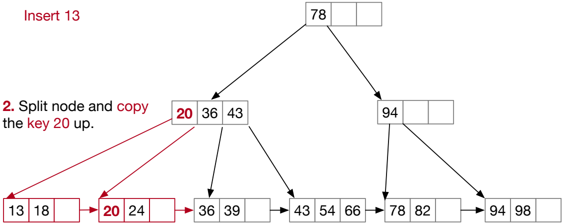
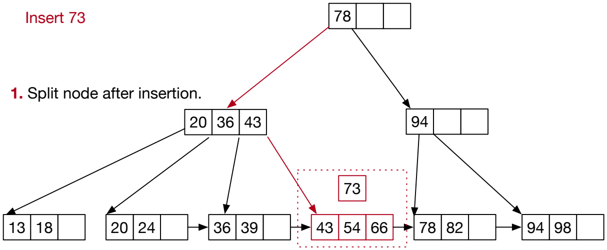


Figure: Insert key 13 ($n = 4$)

- Split the target node if the insertion make it overfull.
- Need to copy the middle key up and adjust the pointers accordingly.

Insertion (3)


Insert 73

1. Split node after insertion.

Figure: Insert key 73 ($n = 4$)

- Node splitting can propagate recursively.

Insertion (3)

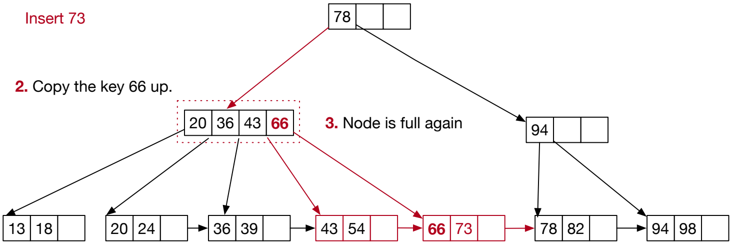


Figure: Insert key 73 ($n = 4$)

- Node splitting can propagate recursively.

Insertion (3)

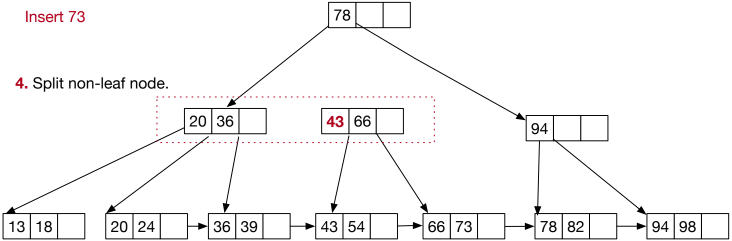


Figure: Insert key 73 ($n = 4$)

- Node splitting can propagate recursively.
- When splitting a non-leaf node, the the middle key is push up rather than copied.

Insertion (3)

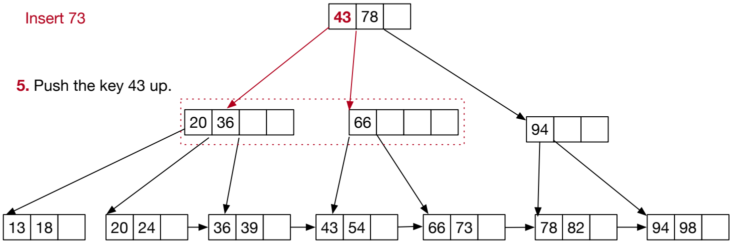


Figure: Insert key 73 ($n = 4$)

- Node splitting can propagate recursively.
- When splitting a non-leaf node, the the middle key is push up rather than copied.
- In the worst case, the root is split and a new root is created, linking to the split nodes.
  - Consequently, the tree height increases by one.

Insertion recap

1. Find the correct leaf L for the given key to be inserted.
2. Add a new entry into L in sorted order.
   - If L has enough space, the operation is done.
   - If L becomes overfull, then
     (a) Split L into two nodes L and L'.
     (b) Redistribute entries evenly and copy up the middle key.
     (c) Adjust the pointers accordingly, including
     (i) next-leaf pointers, and (ii) a pointer from parent of L to L'.
3. To split a non-leaf node, redistribute entries evenly and push up the middle key.
4. Process the nodes recursively until all nodes are half-full.

Deletion (1)

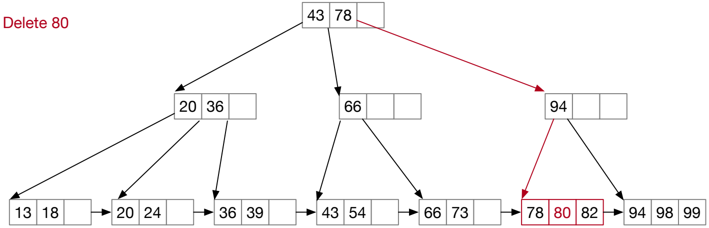


Figure: Delete key 80 ($n = 4$)

Deletion (1)

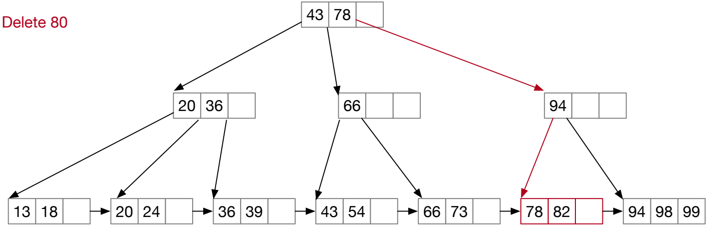
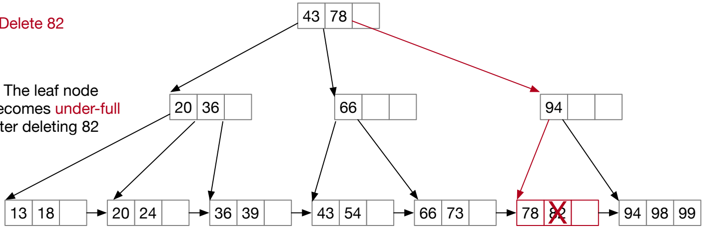


Figure: Delete key 80 ($n = 4$)

Deletion (2)



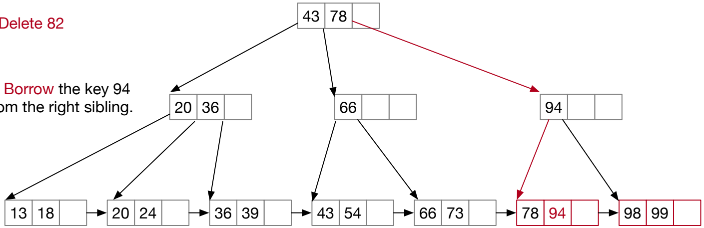
Delete 82

1. The leaf node becomes under-full after deleting 82

Figure: Delete key 82 ($n = 4$)

- If the target node is underfull after a deletion, then try to borrow one key from siblings.

Deletion (2)



Delete 82

2. Borrow the key 94 from the right sibling.

Figure: Delete key 82 ($n = 4$)

- If the target node is underfull after a deletion, then try to borrow one key from siblings.

Deletion (2)

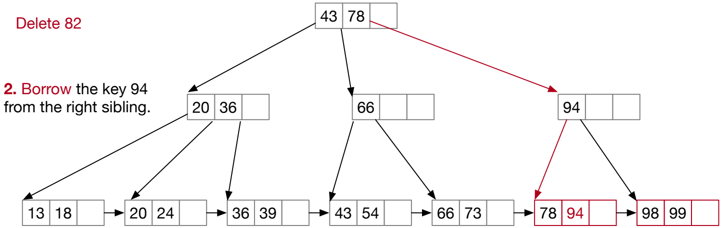


Figure: Delete key 82 ($n = 4$)

- If the target node is underfull after a deletion, then try to borrow one key from siblings.
- Remember to fix the key in the affected parent node.
  - Replace the affected key with the middle key of the two updated children.

Deletion (2)

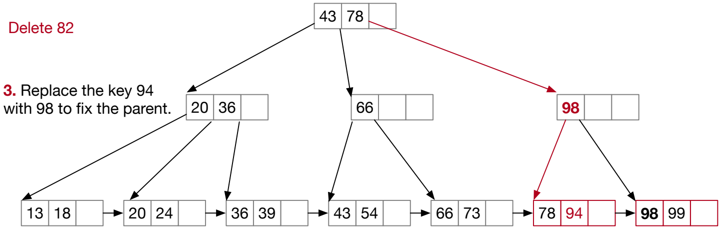


Figure: Delete key 82 ($n = 4$)

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Deletion (3)

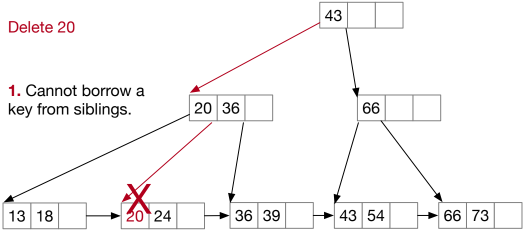


Figure: Delete key 20 ($n = 4$)

- If borrowing is not possible, merge the affected node with one sibling.

Deletion (3)

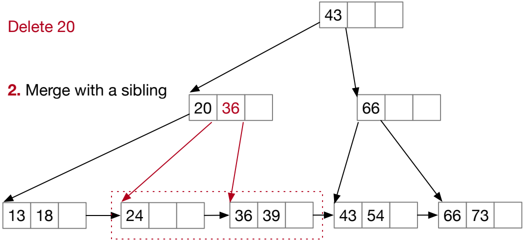


Figure: Delete key 20 ($n = 4$)

- If borrowing is not possible, merge the affected node with one sibling.
- When merging leafs, remove the key associated with the merged nodes from the parent.

Deletion (3)

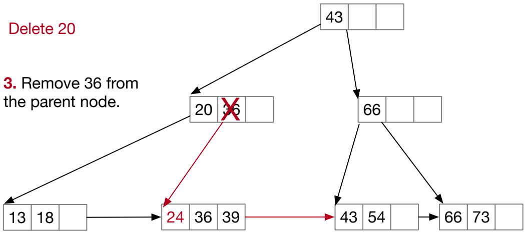


Figure: Delete key 20 ($n = 4$)

- If borrowing is not possible, merge the affected node with one sibling.
- When merging leafs, remove the key associated with the merged nodes from the parent.

Deletion (3)

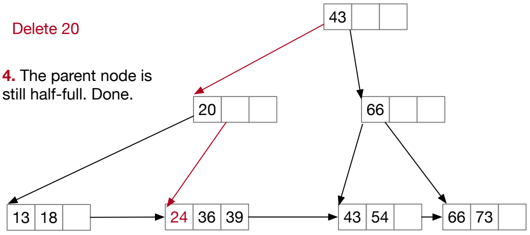


Figure: Delete key 20 ($n = 4$)

- If borrowing is not possible, merge the affected node with one sibling.
- When merging leafs, remove the key associated with the merged nodes from the parent.

Deletion (4)

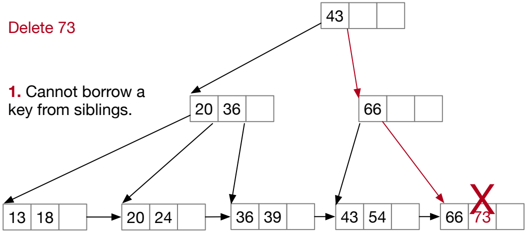


Figure: Delete key 73 ($n = 4$)

Deletion (4)

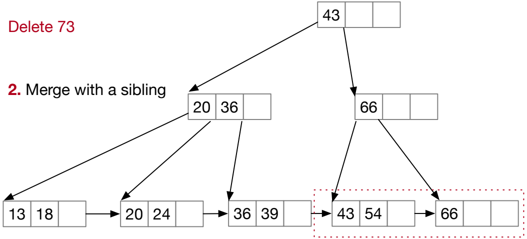
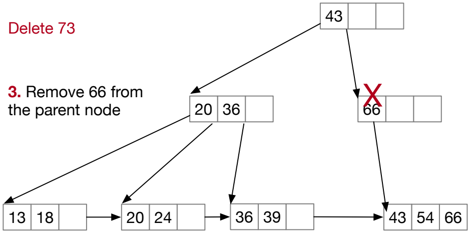


Figure: Delete key 73 ($n = 4$)

Deletion (4)

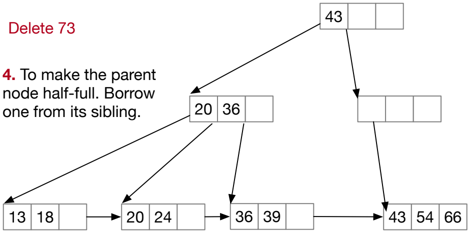


Delete 73

3. Remove 66 from the parent node

Figure: Delete key 73 ($n = 4$)

Deletion (4)



Delete 73

4. To make the parent node half-full. Borrow one from its sibling.

Figure: Delete key 73 ($n = 4$)

Deletion (4)

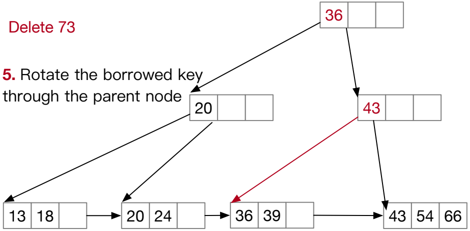


Figure: Delete key 73 ($n = 4$)

- When borrowing from an internal node, rotate the borrowed key through its parent.

Deletion (5)

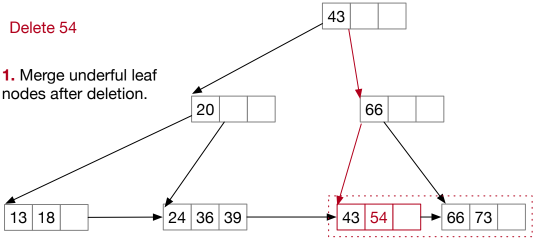


Figure: Delete key 54 ($n = 4$)

- Deletion can be propagated up all the way to root.

Deletion (5)

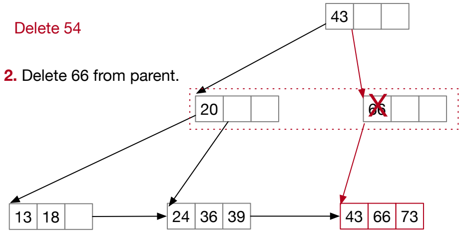


Figure: Delete key 54 ($n = 4$)

- Deletion can be propagated up all the way to root.

Deletion (5)

Delete 54

3. Merge two non-leaf nodes. Pull 43 down.

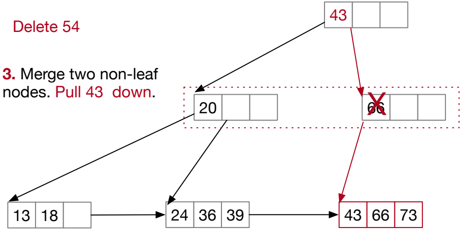
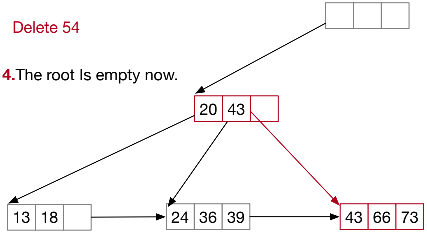


Figure: Delete key 54 ($n = 4$)

- Deletion can be propagated up all the way to root.
- When merging two non-leaf nodes, we need to pull a key down from the parent.

Deletion (5)


Delete 54

4.The root Is empty now.

Figure: Delete key 54 ($n = 4$)

- Deletion can be propagated up all the way to root.
- When merging two non-leaf nodes, we need to pull a key down from the parent.
- When root becomes empty, remove it and make its child as the new root.

Deletion (5)

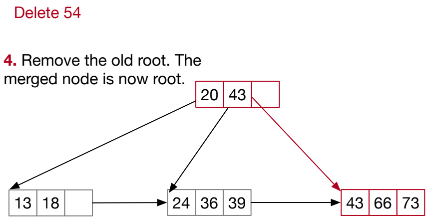


Figure: Delete key 54 ($n = 4$)

- Deletion can be propagated up all the way to root.
- When merging two non-leaf nodes, we need to pull a key down from the parent.
- When root becomes empty, remove it and make its child as the new root.

Deletion Recap

1. Find the correct leaf L.
2. Remove the entry from L for the given key.
    - If L is still half-full, the operation is done.
    - If L becomes under-full, then
        (a) First try to redistribute by borrowing one from siblings.
        (b) If redistribution fails, then merge L and a sibling.
3. When merging two leaf nodes, remove from the parent the key associated with the two leaf nodes to be merged.
4. When merging two non-leaf nodes, pull down the associated key instead.
5. When borrowing from internal nodes, rotate the borrowed key through the parent node.
6. Process the nodes recursively until all nodes are half-full.

Performance analysis

	I/O Cost
Query	$\log_{\lceil n/2 \rceil} N$
Insertion	$\log_{\lceil n/2 \rceil} N$
Deletion	$\log_{\lceil n/2 \rceil} N$

B^+-tree vs. B-tree

- B^+-trees store data entries in leaf nodes only.
  – All key lookups require the same number of I/O's.
- B-trees store data entries in both leaf and non-leaf nodes.
  – Records in non-leaf nodes can be accessed with fewer I/O's.

Problems with B-tree in disk-based DBMS:

1. Storing more data in non-leaf nodes decreases fanout and increases the tree height.
2. It takes more I/O's to access records in leaves, and the majority of records are in leaves.
3. Range query is more complicated in B-trees.