Node Buffer Size in Intermittently Connected Mobile Wireless Networks with Infrastructure Support

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Abstract

We study the fundamental lower bond for node buffer size in intermittently connected mobile wireless networks with Base Station. In this kind of networks, all the mobility is independent and identically distributed(i.i.d.), and the linkage between two nodes is intermittent because of external constraints. There exists Base Stations in this network following the uniform distribution. Given the condition that each node has the same probability $1 - p$ *to be inactive during each time slot, there exists a critical value* $p_c(\lambda)$ *for this probability from a percolation-based perspective. When* $p < p_c(\lambda)$*, the network is in the subcritical case, and the occupied buffer size is* $\Theta(\frac{n}{m})$*, where* n *is the number of nodes in the network, and* m *is the number of base stations. If* $p > p_c(\lambda)$*, the network is in the supercritical case, and the occupied buffer size is* $\Theta(1)$*. Our work also show that in subcritical case, for static networks without base stations (BSs), the achievable lower bound for occupied buffer size is* $\Theta(\sqrt{\frac{n}{m}})$*, and for mobile networks without base stations, there is a achievable lower bound* $\Theta(n)$ *for buffer occupation. For both the networks there is a achievable lower bound for occupied buffer size* $\Theta(1)$ *in supercritical case.*

1. Introduction

Since the field of large-scale wireless network has received great attention in the past several years, a lot of studies about scaling properties of capacity,connectivity and delay in this kind of network have been done since the seminal work by Gupta and Kumar. However, most of the previous works focus on the assumption of maintaining always full connectivity, which does not match the case in real world. In practical networks, only intermittent connectivity is guaranteed. For example, the mobility of users, the sleep model of terminals would cause the inactivity of nodes in the network. Some This type of networks are referred to as Delay/Disruption Tolerant Networks (DTNs), and some related works has been done by [1]. In [2], O. Dousse et al. studied the latency of wireless sensor networks with uncoordinated power saving mechanisms, where constraint on the network is limited node energy and nodes switch between active (on) mode and inactive (off) mode. In [3], W. Ren and Q. Zhao considered a cognitive radio network where secondary users should keep inactive until the availability of wireless channel, and constraint for the secondary networks is the existence of primary users. In [4], Z. Kong and E. M. Yeh studied a mobile wireless network where the link between two nodes might break (turn inactive) when distance between them is out of the transmission range. One common feature of the above three papers is that they are all based on the theory of percolation (see [5], [6], [7]), which will also be used in this work.

Since the inactivity of nodes, buffer size in necessary in this kind of networks. In the multi-hop networks, packets have to be stored in the relay node if the next neighbor is inactive, which means that the minimum buffer size requirements do not approach zero. Throughput capacity of mobile wireless networks with limited node buffer has been investigated by J. D. Herdtner and E. Chong in [8]. In [9], S. Bodas et al. have studied scheduling methods in multichannel wireless networks in the small-buffer regime.

In [10], Yuanzhong Xu has showed that the achievable lower bound for buffer size occupied in intermittently connected static wireless networks is $\Theta(1)$ or $\Theta(\sqrt{n})$, according to the probability p for a node to be active. There exists a critical value $p_c(\lambda)$ in the network with the density of λ. When $p > p_c(\lambda)$, the network is in the supercritical case, and the achievable lower bound for the occupied buffer size is $\Theta(1)$, which is independent of the size of the network. If $p < p_c(\lambda)$, the network is in the subcritical case, and the tight lower bound for the buffer occupation is $\Theta(\sqrt{n})$, where n is the number of nodes in the network.

However, the previous work studies on static network only. In the real networks such ad-hoc network, terminals such as personal computers are supposed to be moving,

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so it is necessary to take the mobility of nodes into consideration. Moreover, the properties of occupied buffer size in intermittently connected mobile wireless networks with infrastructure support such as base stations is remain to be studied.

In this paper, we focus on the intermittently connected networks in which the mobility of each nodes is independent and identically distributed(i.i.d.). Moreover, the impact of base stations are also taken into account.

In our study, we have learned that in the subcritical case , the achievable lower bound for buffer occupation is $\Theta(\sqrt{\frac{n}{m}})$ in the static network with BSs, $\Theta(n)$ in the mobile network without BSs, and $\Theta(\frac{n}{m})$ in the mobile network with BSs. On the other hand, in the supercritical case, the achievable lower bound for buffer occupation is $\Theta(1)$ for all the three network models.

The study of buffer occupation is crucial in large scale network because the resource of a single node is always limited. In wireless sensor networks (WSNs), for example, the available buffer size for each node should be considered during designation. There are also some other interesting results. There is no use to expend the radius of BSs to cut down the occupied buffer size if the number of channels is limited. The probability for a malicious relay node to decode the packets from a source is independent from the number of users or BSs in both mobile networks and static network with BSs, while the security of network coding is improved by the increasing number of BSs in mobile networks with BSs.

This paper is organized as follows. In Section 2, we present the network model and some basic assumptions. In Section 3, we study the static network with BSs. In Section 4, we study the buffer size requirements in mobile networks. In Section 5, we study buffer occupation in mobile networks with BSs. In Section 6, we discuss the effects of the radius of BSs and the limitation of channels. The effects of the number of users on the security of network coding is also discussed in this Section. Finally, we conclude this paper in Section 7.

2. Network Model and Assumptions

2.1. Node Locations and Direct Links

We consider the network with the size of $L \times L$, with constant node density λ. Since the number of nodes in the network is n, we have $L = \sqrt{\frac{n}{\lambda}} = \Theta(\sqrt{n})$. Locations of nodes follows uniform distribution.

Each node covers a disk shaped area with radius r. To simplify the analysis, r is treated as a same constant for all nodes. Let $X_i (1 \leq i \leq n)$ denote the random position of node v_i. Two nodes v_i and v_j are directly connected via a direct link if and only if $||X_i - X_j|| \leq 2r$, where $||X_i - X_j||$ is the Euclidean distance between v_i and v_j . The set of all nodes in the network is denoted by $N(\lambda, L)$. When $L \to \infty$ (or $n \to \infty$ equally), the network is denoted by $N_\infty(\lambda)$.

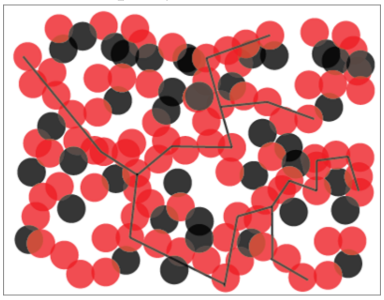

Figure 1. The Active Giant Cluster.

According to continuum percolation theory, there is a critical value for λ, λ_c. Only when $\lambda > \lambda_c$,there exists a unique infinite connected cluster in $N_\infty(\lambda)$ (*giant cluster* , denoted by $CN_\infty(\lambda)$. Since in this paper, we focus on the network in which giant cluster is able to exist, the following assumption is made.

*Assumption 1 (On Node Density):*Node density in the network is large enough to guarantee percolation, i.e. $\lambda > \lambda_c$.

In the case of static network, we mainly analyze communications of nodes within the giant cluster. However In the case of mobile network, there is no need for a nodes to be in the giant cluster, but taking advantage of the giant cluster will cut down the buffer size in need, which will be shown in the following analysis. So in this case, the giant cluster is also assumed to be exist. We denote the nodes belonging to the giant cluster by the term *connected nodes*.

When n is not infinite, we define the giant cluster as the largest connected cluster in the network. According to continuum percolation theory again, the number of connected nodes in $N(\lambda, L)$, n_c, approaches to a constant proportion of n.Since we consider connected nodes only, we let n denotes n_c in the following analysis, without loss of generality.

2.2. External Constraints and Node Inactivity

In this paper, taking external constraints into consideration, we assume that each node switches between active state and inactive state randomly. During active state, a node can transmit or receive messages, while during inactive state it can neither transmit nor receive messages. Transmission between two nodes is possible only if both the transmitter and the receiver are active.

We assume the external constraints in the network are in a synchronized time-slotted manner with a slot length T_{EC},which implies that the state of each node changes only at the beginning of a time slot. Further, the effects of external constraints satisfy the following assumptions:

1) States of each active nodes vary from one time slot to another, and are i.i.d. among different time slots.

2) The probability to be active is a constant p for all nodes in the network.

3) States of different nodes are i.i.d.

The network with external constraints is denoted by $CN(\lambda, L, p)$ (or $CN_{\infty}(\lambda, p)$ if $L \to \infty$).Since the possibility of node inactivity, we cannot guarantee a complete path connecting an arbitrarily selected pair of nodes all the time. Hence, the network is *intermittently connected.*

2.3. Base Station

The number of BSs in the network is denoted by m. Uniform distribution is applied to BS in this paper. Then the network is divided into $\frac{n}{m}$ cells. In each cell there is only one BS in the center of it, as shown in Figure 2.

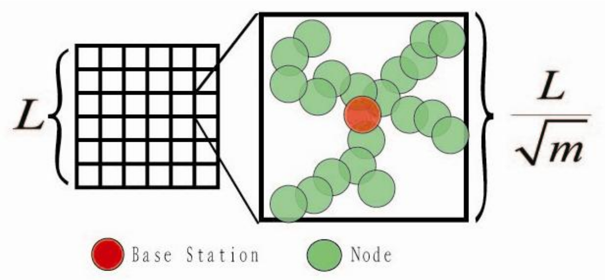

Figure 2. The Cells and Base Stations.

R denotes the radius of a BS. If the distance between a node and a BS is smaller than $r_n + R$, it is regarded that the node is connected to the BS successfully. All the BS in the network are connected to each other with wired linkage, and they are always active.

The limit of channel or bandwidth is not taken into account before Section 6. The effects of R will also be discussed in Section 6, before which we let $R = r$ to simplify expression.

2.4. Mobility

The mobility of each nodes follows i.i.d mobility model. The positions of all the nodes vary from one time slot to another and are i.i.d. among different time slots. This process happens at the beginning of each time slot only, which means that the positions of all the nodes do not change during one time slot. The range of the location of each nodes is assumed to be in the whole network.

In the case that mobility and BSs are both applied in the network, we assume that every mobile node has a *home cell.* At any time slot, the probability for a node to jump outside its home cell is q. If the node cannot jump outside the home cell, it chooses a new position in the cell at the time slot. If it moves out of the cell successfully, it chooses a new position in the whole network randomly (including the home cell), but at the next time slot, it has the probability $1-q$ to return to its home cell.

2.5. Traffic Pattern and Buffering

*Traffic Pattern of Connected Nodes:*For each connected node in the network, as a source, it randomly chooses a permanent destination among other connected nodes, and this source-destination relationship does not change over time. Each connected node generates messages to its corresponding destination node in a multi-hop fashion at a constant rate, r_g, which does not vary among different nodes.

*Buffering:*In each hop, if the transmitter or the receiver is inactive, the message should be kept in the buffer of the transmitter until both nodes are active. As we define before, if a node (as a source) or its first intermediate node toward destination is inactive, it cannot send any message actually. Yet we can still assume the source node "sends" messages at rate r_g but temporarily stored in the buffer of itself.

We define the *per-node throughput capacity* as the maximum bits per second each connected node can send to its chosen destination node. Now we give a basic assumption on channel capacity and per-node throughput capacity in this paper.

*Assumption 2 (On Capacity and Processing Speed):*First, channel capacity for every directly connected nodes is large enough to be viewed as infinity, compared to the actual transmission rate of each node. As a result, per-node throughput capacity can also be viewed as infinity compared to r_g. Second, node processing speed is also infinitely large, compared to the state-switching frequency $\frac{1}{T_{EC}}$.

As Assumption 2 states, the capacity of the network and processing speed is infinity, which implies that once a node and its next hop turn active, they can transmit and receive message without delay. In static network, if all nodes in one path are active, the message can be transmitted from one end to the other without delay. This helps us focus on the limits posed by external constraints on node buffer size in the network.

Maximum Buffer Occupation in Each Time Slot: Since the capacity is infinity, buffered messages in each node are transmitted only at the beginning of each time slot within a very small time interval. On the other hand, the message generation rate r_g is finite and constant, and thus smooth message buffering could happen during each time interval. Therefore, in each time slot, the size of occupied buffer in each node is maximum at the end of the time slot. For a connected node w, we use S_w^L to denote the average occupied buffer size of w at the end of time slot, and S_L to denote the average occupied buffer size of one node in the network.

Message Slot: We call the messages generated by u during one time slot whose destination is v a *message slot* , denoted by $m_{u\to v}$. If only the source or destination is specified, the notation is simplified as $m_{u\to}$ or $m_{\to v}$. According to the assumption above, The size of one message slot is $r_g T_{EC}$.

2.6. Percolation of Active Nodes

According to Assumption 1, a giant cluster always exists a.s. as the size of the network goes to infinity. With external constraints, for each time slot, we consider the percolation phenomenon among active nodes.

Since the states of nodes are i.i.d., the distribution of active nodes in $CN_\infty(\lambda, p)$ is according to a Poisson Point Process with constant point density $p\lambda$.Therefore, there exists a critical value for $p_c(\lambda) = \frac{\lambda_c}{\lambda}$ such that:

1) If $p > p_c(\lambda)$, $CN_\infty(\lambda, p)$ is in the supercritical case, where there exists a unique infinite connected cluster of active nodes a.s. during each time slot.Let $C(CN_\infty(\lambda, p))$ denote the infinite connected cluster of active nodes (active giant cluster) at the present time slot.In this case, $C(CN_\infty(\lambda, p)) \subseteq CN_\infty(\lambda, p)$, i.e. the active giant cluster is part of the giant cluster.

2) If $p < p_c(\lambda)$, $CN_\infty(\lambda, p)$ is in the subcritical case, where there does not exist a unique connected cluster of active nodes a.s. during each time slot.

Let $\theta(\lambda, p|active)$ denote the probability that an arbitrary active node belongs to the active giant cluster in an arbitrary time slot, then we have

$$\theta(\lambda, p|active) \begin{cases} > 0, p > p_c(\lambda) \\ = 0, p < p_c(\lambda) \end{cases}$$

3. Lower Bound for Buffer Size in Static Networks with Base Stations

In this section, we study the achievable lower bound for buffer size of connected nodes in static networks with BSs.

If the source node u and the destination node v is in the same cell, packets are transmitted by multi-hop simply. The probability for this case to happen is $\frac{1}{m}$. which decreases with the increasing number of BSs. Thus ignoring this case will not impact the conclusion.

If they are in two different cells, three steps are taken to finish the transmission, as shown in Figure 3.

Step 1: The source node u sends the packet to the local BS(A) by multi-hop.

Step 2: Once the packet reaches the BS(A), it is transmitted to another BS(B) in the cell which includes the destination node.

Step 3: The BS(B) sends the packet to the destination node v by multi-hop.

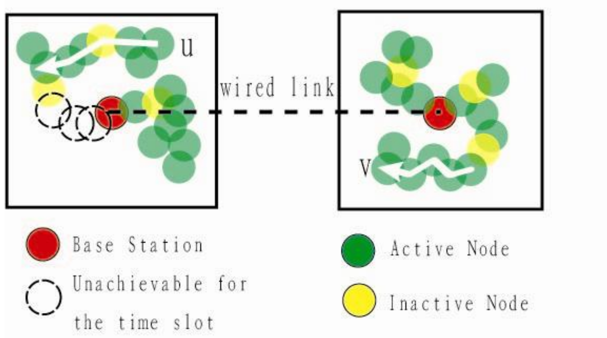

Figure 3. Transmission in Static Network with Base Stations.

The main result is that the expected value of minimum buffer occupation of each connected node is related to both the number of nodes and that of BSs in the network in subcritical case, and is independent of the size of the network in supercritical case.

In Subsection 3.1, we analyze the subcritical case, and in Subsection 3.2, we analyze the supercritical case.

3.1. Subcritical Case

In subcritical case, there exists an achievable lower bound for average buffer occupation, as stated in Theorem 1.

Theorem 1: In the static network with BSs, the lower bound of the average buffer occupation satisfies

$$\lim_{L\to\infty} \frac{S_L}{\frac{L}{\sqrt{m}}} \geq b_1 r_g T_{EC} \tag{1}$$

And it is achievable with some routing scheme that

$$\lim_{L\to\infty} \frac{S_L}{\frac{L}{\sqrt{m}}} \leq c_1 r_g T_{EC} \tag{2}$$

b_1,c_1 are finite positive constants irrelevant to L.

3.1.1 The Lower Bound of the Average Buffer Occupation

In this subcritical intermittently connected network, the following lemma on the minimum message existing time (delay) can be proved [11][12].

Lemma 1: In subcritical case, the minimum latency of message slot $m_{u\to v}$, $T_{m_{u\to v}}$, satisfy

$$\lim_{||X_u - X_v|| \to \infty} \frac{T_{m_{u\to v}}}{||X_u - X_v||} = \gamma_1 > 0 \quad a.s.$$

Proof for this lemma is based on the Subadditive Ergodic Theorem [11], and is omitted in this paper.

First we consider the average minimum required buffer size, $S_L^{(1)}$,in Step 1. Since the BS is supposed to be in the center of the cell, the average distance between a arbitrary node in the cell to the BS is

$$||X_n - X_{BS}|| = \frac{1}{(\frac{L}{\sqrt{m}})^2 \lambda} \int_0^{\frac{L}{2\sqrt{m}}} r\lambda 2\pi r dr = \frac{\pi}{12} \frac{L}{\sqrt{m}}$$

Therefore, the average minimum latency for a message slot is $\Theta(\sqrt{\frac{n}{m}})$. Since messages are generated continuously, by Little's Law, the average number of message slots generated by one node existing in the cell in a given time slot is $\Theta(\sqrt{\frac{n}{m}})$. There are $\Theta(\frac{n}{m})$ connected nodes,thus the number of all message slots existing in a given time slot is $\Theta(\frac{n}{m}\sqrt{\frac{n}{m}})$. Hence, the minimum average number of message slots one connected node should buffer is $\Theta(\sqrt{\frac{n}{m}})$. Since $S_L^{(1)} = T_{m_{u\to BS}} r_g$, we have

$$\lim_{L\to\infty} \frac{S_L^{(1)}}{\frac{L}{\sqrt{m}}} \geq b_1 r_g T_{EC}$$

where b_1 is positive constant independent with L.

The result for Step 3 is almost same with that for Step 1, so we omitted the proof of it. Thus we have the result

$$\lim_{L\to\infty} \frac{S_L}{\frac{L}{\sqrt{m}}} \geq b_1 r_g T_{EC}$$

3.1.2 A Constructive Upper Bound of the Average Buffer Occupation

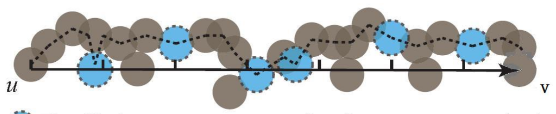

Figure 4. The Source Extending Path from u to v.

In this section, we will present a scheme in which a $\Theta(\sqrt{\frac{n}{m}})$ buffer size requirement can be achieved in the subcritical case. To achieve this, we designate *Source Extending Path (SEP)*[10] for each source destination pair where the number of hops is asymptotically linear to the distance between them.

*Source Extending Path (SEP):*We draw a segment between the source node u and the destination node v. Then divide the segment into a string of smaller segments of constant length. The nearest connected node to each segment endpoints is a*flag node*, and we connect every two neighboring flag nodes by the shortest path. It has been proved that the number of nodes in each segment is finite (see [2]). Then we let $P_{u\to v}$ denotes the path of this kind between u and v. One example is shown in Figure 4.

This scheme assures that the number of hops in $P_{u\to v}$, $N(P_{u\to v})$, is asymptotically linear to the $||X_u - X_v||$ (see [2]). Thus we have

$$\lim_{L\to\infty} \frac{N(P_{u\to v})}{||X_u - X_v||} \leq \gamma_2 < \infty$$

γ_2 is a positive constant.

We consider the occupied buffer in Step 1 first.

Consider a ring centered at BS with radius r and width Δr, which is shown in Figure 5. Then the number of nodes outside the ring in the cell is $\lambda(L^2 - \pi r^2)$. The number of nodes on the ring is $2\pi r \Delta r \lambda$. Since the message rate r_g is constant, it is obvious that the buffer occupied by the u on $P_{u\to BS}$ is also constant($\Theta(1)$)[10].

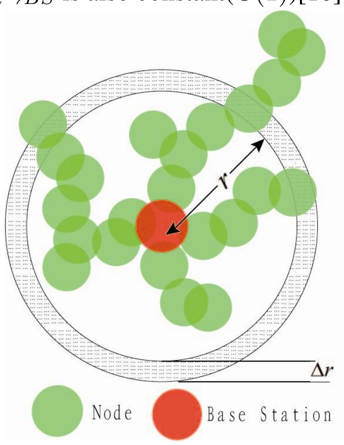

Figure 5. The Method of Integration.

Then the average buffer occupation in Step 1,$S_L^{(1)}$, satisfies

$$\begin{aligned} S_L^{(1)} &= \frac{1}{(\frac{L}{\sqrt{m}})^2} \int_0^{\frac{L}{2\sqrt{m}}} 2\pi r\lambda \times N(P_{u\to v}) \times \Theta(1) dr \\ &\leq \frac{1}{(\frac{L}{\sqrt{m}})^2} \int_0^{\frac{L}{2\sqrt{m}}} 2\pi r\lambda \times \gamma_2 ||X_u - X_v|| \times \Theta(1) dr \\ &= c_1 \frac{L}{\sqrt{m}} \end{aligned} \quad (3)$$

c_1 is a positive constant which is independent of L.

The result for Step 3 is the same as that for Step 1, so we draw the conclusion that

$$\lim_{L\to\infty} \frac{S_L}{\frac{L}{\sqrt{m}}} \leq c_1 r_g T_{EC}$$

Hence, Theorem 1 is proved, and it indicates that there is an achievable lower bound of the average buffer occupation in networks with BSs.

$$S_L = \Theta(\sqrt{\frac{n}{m}})$$

3.2. Supercritical Case

In supercritical case, the achievable lower bound for average buffer occupation is $\Theta(1)$. A lot of work has been done for this case by Yuanzhong XU in [10], so the analysis in this part is much simpler than the that of the previous case.

Lemma 2: In supercritical case, the achievable lower bound for occupied buffer size is $\Theta(1)$, which is independent of the size of the network.

Lemma 2 indicates that in supercritical case, the total buffer size occupied in the network is $\Theta(n)$. Since there are $\Theta(2n)$ transmission pairs in the network, the buffer in need by a source-destination pair is $\Theta(1)$, which is independent of the size of the network. That is to say, in a single cell, the buffer in need by the transmission between BS and a node is also $\Theta(1)$. Taking the fact into consideration that the overlying of transmission routes do not effects the average buffer size, and the average distance between any node and BS has been proved to be $\Theta(\frac{n}{m})$(That is to say the average length of transmission routes is not effected by the change that one end of a route is always BS), we can draw the conclusion that the achievable lower bound is still $\frac{\Theta(1)\times\Theta(2\frac{n}{m})}{\Theta(\frac{n}{m})} = \Theta(1)$, for both Step 1 and Step 2.

Then we come to the result that in supercritical case the achievable lower bound for average buffer occupation is $\Theta(1)$.

4. Buffer Size Occupation in Mobile Networks

In this section, we study the achievable lower bound for buffer size of nodes in mobile networks for subcritical case. In supercritical case, a transmission scheme is designed to reach the buffer size occupation $\Theta(1)$. The scheme of *Two-hop Transmission* is applied to this case.

Two-hop Transmission: In this kind of scheme, each packet goes through one relay node that temporarily buffers the packet until final delivery to the destination is possible. For a sourceCdestination pair, all the other nodes can serve as relay nodes. The goal is that in steady-state, the packets of every source node will be distributed across all the nodes in the network, hence ensuring that every node in the network will have packets buffered destined to every other node (except itself). This ensures that a scheduled sender-Creceiver pair always has a packet to send, in contrast to the case of direct transmission. We assume that redundancy is not allowed in this scheme, which means that one package is transmitted to only one relay node.

The transmission is divided into two steps (Figure 6).

Step 1: Each packet is transmitted by the source to a close-by relay node. If the close-by node happens to be the destination node, the transmission is finished in advance.

Step 2: The packet carried by the relay node is handed off to its destination if the relay node and the destination node is connected.

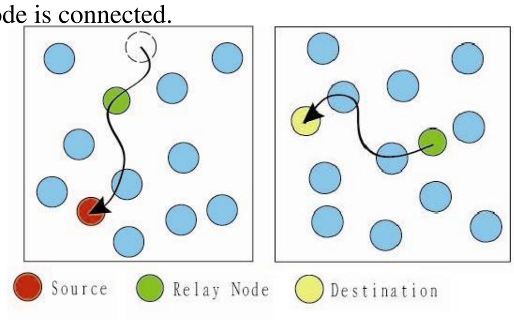

Figure 6. Transmission in Mobile Networks.

The main result is that the expected value of minimum buffer occupation in subcritical case is $\Theta(n)$, which means that the buffer size in need will increase if the number of users in the network grows. In supercritical case, the occupied buffer can be cut down to $\Theta(1)$, if a special transmission scheme is applied.

In Subsection 4.1, we analyze the subcritical case, and in Subsection 4.2, we analyze the supercritical case.

4.1. Subcritical Case

In subcritical case, there exists an achievable lower bound for average buffer occupation, as stated in Theorem 2 and Theorem 3.

Theorem 2: In the i.i.d. mobile network where two-hop scheme without redundancy is applied, the lower bound of the average buffer occupation satisfies

$$\lim_{L\to\infty} \mathbb{E}(\frac{S_L}{L^2}) \geq b_2 r_g T_{EC} \quad (4)$$

And it is achievable with some routing scheme that

$$\lim_{L\to\infty} \mathbb{E}(\frac{S_L}{L^2}) \leq c_2 r_g T_{EC} \quad (5)$$

b_2,c_2 are finite positive constants irrelevant to L.

4.1.1 The Lower Bound of the Average Buffer Occupation

In the work of Michael J. Neely [13], it has been proved that the delay time in i.i.d. networks with two-hop scheme is $\Theta(n)$, if redundancy is not allowed. We proof this result informally here.

Lemma 3: The delay time before one packet is handed off to the destination is $\frac{1}{ln(\frac{L^2}{L^2-\pi(2r)^2p^2})}$.

$Proof$: The possibility for the relay node to connect with the destination is $\frac{\pi(2r)^2p^2}{L^2}$, which is easy to learn considering the area of the network. According to the property of negative exponential distribution, the delay time is expected to be $\frac{1}{ln(\frac{L^2}{L^2-\pi(2r)^2p^2})}$.

Lemma 3 indicates that the delay time of one packet is $\Theta(n)$, because

$$\lim_{L\to\infty}\frac{\frac{1}{ln(\frac{L^2}{L^2-\pi(2r)^2p^2})}}{L^2}=\gamma_3$$

where γ_3 is a constant.

The probability for a relay node to connect with at least one source node in a time slot is

$$\begin{aligned} P_l &= 1-(1-\frac{\pi(2r)^2p^2}{L^2})^{n-2} \\ &\sim 1-e^{-(n-2)\frac{\pi(2r)^2p^2}{L^2}} \\ &\to 1-e^{-\pi(2r)^2p^2}, \quad n\to\infty \end{aligned} \tag{6}$$

Then the number of packets received by the relay node during the delay time is at least $\Theta(n)\times(1-e^{-\pi(2r)^2p^2})$.

Since the size of a single packet is r_gT_{EC}, the expected lower bound of buffer occupation for the i.i.d. mobile network with two-hop transmission is

$$\mathbb{E}(S_L)\geq\Theta(n)\times(1-e^{-\pi(2r)^2p^2})\times r_gT_{EC},\quad n\to\infty$$

So we have

$$\lim_{L\to\infty}\mathbb{E}(\frac{S_L}{L^2})\geq b_2r_gT_{EC}$$

Then the first part of Theorem 2 has been proved.

4.1.2 A Constructive Upper Bound of the Average Buffer Occupation

In this section, we will prove that the lower bound of $\Theta(n)$ is achievable. We designate that each relay node can receive more than one packets in one time slot, if it is connect to more than one source node in this time slot.

The expected number of source nodes that are connected to one relay node in one time slot is

$$\begin{aligned} N_u &= \sum_{i=1}^{n-1} i\times C_{n-1}^i\times(\frac{\pi(2r)^2}{L^2})^i(1-\frac{\pi(2r)^2}{L^2})^{n-1-i} \\ &\leq (1-\frac{\pi(2r)^2}{L^2})^{n-1}\frac{(n-1)(n-2)...(n-k)}{(n-\pi(2r)^2)^k}\times\frac{an-a-1}{n-1} \\ &\to a(1-e^{-\pi(2r)^2}),\quad n\to\infty \end{aligned} \tag{7}$$

where a and k are constants. The proof of this inequality in detail is included in the appendix of this paper.

According to Lemma 3, the delay time is still $\Theta(n)$, so we get the constructive upper bound of the average buffer occupation

$$\mathbb{E}(S_L)\leq\Theta(n)\times a(1-e^{-\pi(2r)^2p^2})\times r_gT_{EC},\quad n\to\infty$$

Thus the second part of Theorem 2 has been proved.

$$\lim_{L\to\infty}\mathbb{E}(\frac{S_L}{L^2})\leq c_2r_gT_{EC}$$

Hence, Theorem 2 is proved, and it indicates that there is an achievable lower bound of the average buffer occupation in i.i.d. mobile networks.

$$S_L=\Theta(n)$$

4.2. Supercritical Case

If the transmission scheme is not changed, the occupied buffer size cannot be improved even in the supercritical case, because the giant cluster is not taken advantage of. The buffer occupation is still $\Theta(n)$. However, it can be cut down if some special changes are made to the transmission scheme.

In the supercritical case, we assume that all relay nodes knows if they are in the giant cluster, and if and only if one relay node and the destination is in the giant cluster at the same time, the packet will be handed off to the destination through the giant cluster directly, and therefore no delay will be made in Step 2. One example is shown in Figure 7.

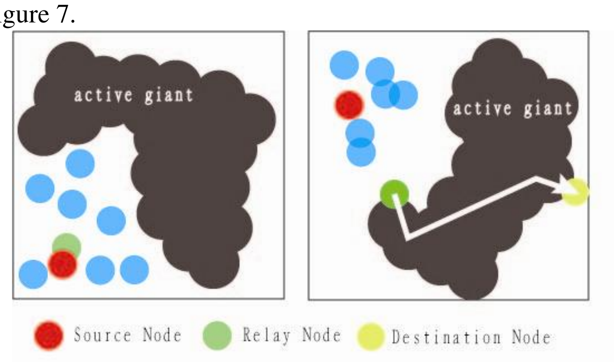

Figure 7. Transmission in Supercritical Case.

At this new scheme, buffer occupied will be $\Theta(1)$, which is greatly improved compared with the one in subcritical case.

Lemma 4: In supercritical case, the delay time before one packet is handed off to the destination is $\Theta(1)$.

$Proof$: According to the theory of percolation, the probability for one node to be in the giant cluster is $\theta(\lambda, p|active)$, which is a positive constant in supercritical case. Obviously, it is independent of the number of nodes in the network.

Thus at any time slot, the probability for a relay node and the destination node to be in the giant cluster at the same time is $(\theta(\lambda, p|active))^2$. According to the property of negative exponential distribution, the delay time is expected to be $\frac{1}{ln(\frac{1}{1-(\theta(\lambda,p|active))^2})}$, which is a constant independent of the number of nodes in the network.

It has been proved that the expected number of nodes that a relay node is connected to is $\Theta(1)$, then the expected average buffer occupation is

$$\mathbb{E}(S_L) = \frac{\frac{1}{ln(\frac{1}{1-(\theta(\lambda,p|active))^2})}\Theta(1)\Theta(1)\Theta(n)}{\Theta(n)} = \Theta(\frac{1}{ln(\frac{1}{1-(\theta(\lambda,p|active))^2})}) = \Theta(1) \quad (8)$$

Then it is proved that in supercritical case, the buffer size occupied is $\Theta(1)$, if a new transmission scheme is applied to make use of the giant cluster.

5. Buffer Size Occupation in Mobile Networks With Base Stations

In this section, we study the occupied buffer size of nodes in mobile networks with BSs. A new scheme is applied to this case, which is a combination of the scheme in Section 3 and 4.

The transmission is divided into three steps, as shown in Figure 8.

Step 1: The source node u sends one packet to BS(A) by two-hop transmission. It should be noted that the BS(A) is not necessarily to be the local BS. Since the relay node is able to move from cell to cell, the BS(A) is actually the first BS that the relay node is connected to. Therefore the value of p has no effect on Step 1.

Step 2: Once the packet reaches the BS(A), it is transmitted to another BS(B) in the cell which includes the destination node at the time slot.

Step 3: The BS(B) sends the packet to the destination node v by two-hop transmission through a relay node. It should be noted that both v and the relay node have opportunity to jump outside the cell. The number of packets that a BS sends to a connect node must be more than one to ensure the stability of the network.

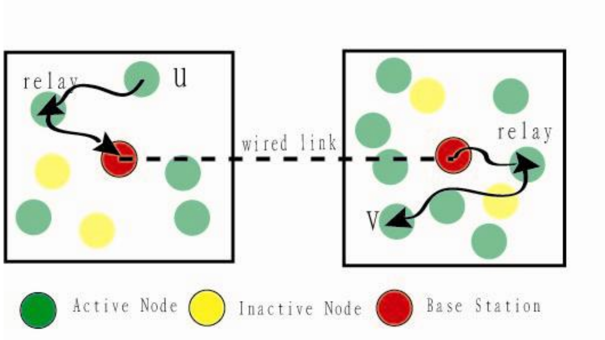

Figure 8. Transmission in Mobile Networks with Base Stations.

The average number of packages that a BS must send out at a time slot is $\Theta(\frac{r_g T_{EC} n}{m})$. Since the number of nodes that are connected to the BS at a time slot is $\Theta(1)$, which will be proved below, the BS has to send $\Theta(\frac{n}{m})$ packets to every connected relay node at one time slot.

The main result for this model is that the expected value of minimum buffer occupation in subcritical case is $\Theta(\frac{n}{m})$, if q is not 1. This result indicates that the buffer size occupied is related to both the number of nodes and the number of BSs in the network. In supercritical case, the occupied buffer can be cut down to $\Theta(1)$, if the existence of giant cluster is made use of.

In Subsection 5.1, we analyze the subcritical case, and in Subsection 5.2, we analyze the supercritical case.

5.1. Subcritical Case

Since many useful theorems have been proved in the previous sections, the analysis in this section is very simple.

Before analysis, we divide the packets into two classes:

Up Packets :The packets sent from source nodes to relay nodes. They are created during Step 1.

Down Packets :The packets sent from BSs to relay nodes. They are created during Step 2.

For Step 1, we analyze the buffer size caused by Up Packets, and for Step 3, buffer size needed by Down Packets is considered.

Since the probability for a relay node to connect to a BS is

$$((1-q)\times\frac{\pi(2r)^2}{\frac{n}{m}}+q\times\frac{m\pi(2r)^2}{n}=\frac{\pi(2r)^2}{\frac{n}{m}}$$

The delay time for Step 1 is

$$\frac{1}{ln(\frac{\frac{n}{m}}{\frac{n}{m}-\pi(2r)^2p^2})}\rightarrow\Theta(\frac{n}{m}),\quad \frac{n}{m}\rightarrow\infty$$

If $\frac{n}{m}$ is constant, the delay time is also constant without the condition $\frac{n}{m}\rightarrow\infty$, so the result is still in the right form even in this case.

Since it has been proved that the expected number of nodes that a relay node is connected to at any time slot is $\Theta(1)$, the average buffer size occupied in Step 1 is

$$S_L^{(UP)}=\frac{1}{ln(\frac{\frac{n}{m}}{\frac{n}{m}-\pi(2r)^2p^2})}\times\Theta(1)\rightarrow\Theta(\frac{n}{m}),\quad \frac{n}{m}\rightarrow\infty$$

The analysis for Step 3 is a little more complicated. The probability for the relay node and v to be in the home cell at the same time is $(1-q)^2$, and the probability that they are connected to each other successfully is $\frac{\pi(2r)^2}{\frac{n}{m}}$. The case that one of the relay node and v is outside the home cell is $2(1-q)q$, and for this case it is impossible for the transmission to be finished. If none of them are in the home cell (q^2), the probability that they are connected to each other successfully is $\frac{\pi(2r)^2}{n}$. Thus the probability that Step 3 is finished at any time slot is

$$\begin{aligned}&(1-q)^2\times\frac{\pi(2r)^2}{\frac{n}{m}}+2(1-q)q\times 0+q^2\times\frac{\pi(2r)^2}{n}\\&=\pi(2r)^2\frac{(1-q)^2m+q^2}{n}\end{aligned}\tag{9}$$

Thus the expected delay time is

$$\mathbb{E}(D_3)=\frac{1}{ln(\frac{n/m}{n/m-\pi(2r)^2(1-q)^2-\pi(2r)^2q^2/m})}$$

If $q\neq 1$, $\pi(2r)^2(1-q)^2$ is a positive constant, and $\pi(2r)^2q^2/m$ is supposed to be little enough to be ignored compared with $\pi(2r)^2(1-q)^2$, if m is large enough. Then we have

$$\lim_{\frac{n}{m}\rightarrow\infty,n\rightarrow\infty}\mathbb{E}(D_3)=\Theta(\frac{n}{m})$$

It should be noted that if $\frac{n}{m}$ is constant, D_3 is also a constant, thus the result $\Theta(\frac{n}{m})$ is still in the right form.

On the other hand, if $q=1$, which means that all the nodes are able to move in the whole network without any restriction, the delay time is much longer

$$\lim_{\frac{n}{m}\rightarrow\infty,n\rightarrow\infty}\mathbb{E}(D_3)=\lim_{n\rightarrow\infty}\frac{1}{ln(\frac{n}{n-\pi(2r)^2})}=\Theta(n)$$

Down Packets are created only when relay nodes are connected to BSs, and the opportunity for a relay node to be connected to a BS is always $\frac{m\pi(2r)^2}{n}$, which is independent of q.

Since the number of packets sent by BSs to a single relay node at one time slot is $\Theta(\frac{n}{m})$, the average buffer size occupied in Step 3 is

$$\lim_{\frac{n}{m}\rightarrow\infty}\mathbb{E}(S_L^{(DP)})=\begin{cases}\Theta(\frac{n}{m}\frac{m\pi(2r)^2}{n}\frac{n}{m})=\Theta(\frac{n}{m}),q<1\\ \Theta(n\frac{m\pi(2r)^2}{n}\frac{n}{m})=\Theta(n),q=1\end{cases}$$

Since the expect average buffer occupation is the sum of $S_L^{(UP)}$ and $S_L^{(DP)}$, we have

$$\lim_{\frac{n}{m}\rightarrow\infty,n\rightarrow\infty}\mathbb{E}(S_L)\begin{cases}=\Theta(\frac{n}{m}),q\neq 1\\ =\Theta(n),q=1\end{cases}$$

This result shows us that if the mobility of nodes are not limited, the buffer size occupied is still $\Theta(n)$ even if BSs are applied in the network. If the mobility of nodes are restricted with home cell, the average buffer size in need is cut to $\Theta(\frac{n}{m})$. The main reason accounts for this phenomenon is that since redundancy is not allowed in the network, the delay time is too long.

5.2. Supercritical Case

In this case the buffer size occupied is $\Theta(1)$ if the giant cluster is made use of.

In the mobile network without BSs, it has been proved that the average delay time for transmission is $\Theta(1)$, regardless of the size of the network. So this result can be used here directly. Since q has no effect on Step 1, The buffer in need for Up Packets is still $\Theta(1)$.

The buffer in need for Down Packets is also $\Theta(1)$. The proof is similar to that in the previous section. The probability for one relay node and a destination to be connected is

$$\begin{aligned}&(1-q)^2\times\ \theta^2(\lambda,p|active)+2q(1-q)\times\theta^2(\lambda,p|active)\\&+q^2\times\ \theta^2(\lambda,p|active)\\&=\theta(\lambda,p|active)^2\end{aligned}\tag{10}$$

which is a positive constant. So the expected delay time is also $\Theta(1)$. Thus the expected average occupied buffer size is $S_L=\Theta(1)$, regardless whether $q=1$.

6. Discussion

6.1. The Effect of the Radius of Base Stations

In the analysis for mobile networks with BSs, we assume that the radius of BSs is $R = r$. However in the real world the radius of BSs is not necessarily the same as that of normal nodes. For example, in modern communication network the radius of BSs is much larger than the terminals of users. In this section we discuss about the effect of the changing radius of BSs.

6.1.1 Subcritical Case

In the previous work we have ignored the fact that the number of available channels is always limited in the real world. That is reasonable in the case that $R = r$, because

$$\frac{n}{m} \times \frac{\pi(2r)^2 p}{(\sqrt{n/m})^2} = \Theta(1)$$

which indicates that the expected number of nodes that are connected to a BS at a single time slot will not change with the increasing number of users. However, this result is not practical in the case that R is changeable. The expected number of nodes that are able to be connected to the BS is

$$\frac{n}{m} \times \frac{\pi(R+r)^2 p}{(\sqrt{n/m})^2} = \pi(R+r)^2 p$$

which would increase if R become larger. In this case we assume that the maximum number of channels provided by one BS is k. Then each node that is in the range of BS has a probability $\frac{k}{\pi(R+r)^2 p}$ to be connected successfully, or it has to wait for the next chance to send packets to the BS. Then the probability for a relay node to finish Step 1 in a single time slot is

$$\frac{\pi(R+r)^2 p}{\frac{n}{m}} \times \frac{k}{\pi(R+r)^2 p} = \frac{k}{n/m}$$

It is interesting that the factor R disappeared during the calculation.

Much similar to the previous analysis, the time it takes to send a packet to a BS in Step 1 is $\frac{1}{ln(\frac{n/m}{n/m-k})}$. Then we have the average buffer occupation for Up Packets

$$\begin{aligned}\mathbb{E}(S_L^{(UP)}) &= \frac{1}{ln(\frac{n/m}{n/m-k})} \times \Theta(1) \\ &\times (1 - (p^2 \frac{n/m - \pi(R+r)^2 - \pi(2r)^2}{n/m - \pi(R+r)^2})^{\frac{n}{m}-1}) \\ &= \Theta(\frac{1}{ln(\frac{n/m}{n/m-k})})\end{aligned} \quad (11)$$

The average number of packages that a BS must send out at a time slot is $\Theta(\frac{r_g T_{EC} n}{m})$. Since the average number of nodes that are connected to the BS at a time slot is limited to k, the BS has to send $\Theta(\frac{n}{mk})$ Down Packets to every connected relay node at one time slot.

In Step 3, since the number of nodes that are in the range of a BS is $\pi(R+r)^2 p$, the probability that a relay node is connected to a BS successfully is

$$\frac{m\pi(R+r)^2 p}{L^2} \times \frac{k}{\pi(R+r)^2 p} = \Theta(\frac{km}{n})$$

If the destination node is in the range of the BS already, it can be connected to BS directly without the help of relay node. Then in the case $q \neq 1$, the probability for a destination node to be connected to a BS in its home cell is

$$\frac{p\pi(R+r)^2}{n/m} \times \frac{k(1-q)}{\pi(R+r)^2 p} = \Theta(\frac{km}{n})$$

Since the delay time for Step 3 is still $\frac{n}{m}$, the expected occupied buffer size is

$$\begin{aligned}\mathbb{E}(S_L^{(DP)}) &= \Theta(\frac{km}{n} \times \frac{n}{m} \times \frac{n}{mk} \times (1 - \frac{km}{n})) \\ &= \Theta(\frac{n}{m} - k)\end{aligned} \quad (12)$$

According to the result, when k is in the constant order, then $\mathbb{E}(S_L^{(UP)}) = \Theta(\frac{n}{m})$, and $\mathbb{E}(S_L^{(DP)}) = \Theta(\frac{n}{m})$, so the average buffer occupation in the whole network is still

$$\mathbb{E}(S_L) = \Theta(\frac{n}{m})$$

. which is independent of R.

If $k = \Theta(R^2)$, the result is

$$\mathbb{E}(S_L^{(UP)}) = \Theta(\frac{1}{ln(\frac{n/m}{n/m-(R+r)^2})})$$

$$\mathbb{E}(S_L^{(DP)}) = \Theta(\frac{n}{m} - R^2)$$

When $R = \Theta(\sqrt{\frac{n}{m}})$, $\mathbb{E}(S_L^{(1)}) = \Theta(1)$, but $\mathbb{E}(S_L^{(3)}) = \Theta(\frac{n}{m})$. Then the average buffer occupation in the whole network is still

$$\mathbb{E}(S_L) = \Theta(\frac{n}{m})$$

.

When $R = \Theta(1)$, the result is still $\Theta(\frac{n}{m})$.

In the case where $q = 1$, the delay for Step 3 is $\Theta(n)$, and the probability for a destination node to be connected to a BS in its home cell becomes

$$\frac{p\pi(R+r)^2}{n} \times \frac{k}{\pi(R+r)^2 p} = \Theta(\frac{k}{n})$$

The following calculation does not change, so we give the result here directly

$$\mathbb{E}(S_L^{(DP)}) = \Theta(n-k)$$

Since $k \leq \Theta(\frac{n}{m})$, the buffer size occupied is always $\Theta(n)$, regardless of R or k.

The analysis show us that even if the radius of R is changeable and loose channel limitation is applied, the average buffer occupation cannot be improved because the buffer in need by Down Packets still increases with the increasing $\frac{n}{m}$. However, this method indeed cut down the buffer in need because the result becomes $\Theta(\frac{n}{m}-k)$ instead of $\Theta(\frac{n}{m})$.

6.1.2 Supercritical Case

In this case, the probability for a node to be able to connect a BS is $(1-(1-\theta)^k)\theta$, where θ denotes $\theta(\lambda, p|active)$. So the probability for a node to connected with the BS in its home cell is independent of q.

The average buffer occupied in Step 1 is

$$\mathbb{E}(S_L^{(UP)}) = \Theta(\frac{1}{ln(\frac{1}{1-\theta(1-(1-\theta)^k)})})$$

Proof for this equation is similar to that in subcritical case, so it is omitted in this paper.

The expected number of nodes that are able to connect a BS at a single time slot this $n\theta(1-(1-\theta)^k)$. In the same way of proof for subcritical case, it can be found that

$$\begin{aligned}\mathbb{E}(S_L^{(DP)}) &= \theta(1-(1-\theta)^k) \times \frac{mk}{n\theta(1-(1-\theta)^k)}\frac{n}{mk} \\ &\times (1-\theta(1-(1-\theta)^k)) \\ &= 1-\theta(1-(1-\theta)^k)\end{aligned} \tag{13}$$

Then the expected average occupied buffer in supercritical case is

$$\mathbb{E}(S_L) = \Theta(\frac{1}{ln(\frac{1}{1-\theta(1-(1-\theta)^k)})} + 1 - \theta(1-(1-\theta)^k))$$

which is independent of q.

6.2. The Security for Network Coding

In this section we analysis the impact of increasing number of users on security in the network where Network Coding is applied.

Network Coding Model: In mobile networks with BSs, once a source node u intends to send a message to a destination node v, it encode the message by creating $(1+\beta)\alpha$ packets, and transmits these packets to v. Once v receives α packets, it is able to decode them and get the message.

We assume that there exist malicious relay nodes in the network, which will decode the message from any other sources once they get enough packets. If the average number of packets from a single source buffered in a relay node is larger than the order of α, the network is not security.

Consider subcritical case first. According to the previous analysis, the average buffer size in static network with BSs is $\Theta(\sqrt{\frac{n}{m}})$, which means that averagely every node is carrying $\Theta(\sqrt{\frac{1}{nm}})$ packets from another node. Then if $\alpha > \Theta(\sqrt{\frac{1}{nm}})$, the network is supposed to be security, because the need for α decreases with the increasing number of users.

In the mobile network without BSs, that would be $\alpha > \Theta(1)$, which indicates that a constant α is not necessarily large enough to ensure the security of the network.

In the mobile network with BSs ($q \neq 1$), since the distribution of mobile nodes are limited, the possible location of sources are also restricted. If $q = 0$, all the nodes move in their home cells only, and the requirement for α is $\alpha > \Theta(1)$. If q is larger but still $q < 1$, the result is about to be $\alpha > \Theta(\frac{1}{m})$. This result show us that the need for α is independent of the number of users in the network, and the increasing number of BSs would help to improve the security of the network. If $q = 1$, which means that all the node can move around the network without limitation, the result is $\alpha > \Theta(1)$ again.

In supercritical case, almost all the buffer occupation is $\Theta(1)$, so the minimum requirement for α is $\Theta(\frac{1}{n})$. Particularly, for mobile networks with BSs, the result becomes $\alpha > \Theta(\frac{m}{n})$ when $q = 0$, which show us that too many BSs in mobile networks may even harm the security of network coding.

Then it occurs to us that static network with BSs plays best among the three kind of networks for the security of network coding.

Table 1. Main Achievements

Network	Subcritical Case	Supercritical Case
Staticn Wireless Networks with BSs	$\Theta(\sqrt{\frac{n}{m}})$	$\Theta(1)$
Mobile Wireless Networks	$\Theta(n)$	$\Theta(1)$
Mobile Wireless Networks with BSs	$\Theta(\frac{n}{m})(q<1)$ $\Theta(n)(q=1)$	$\Theta(1)$
Mobile Wireless Networks with BSs(changeable radius)(channel limit: k)	$\Theta(\frac{1}{ln(\frac{n/m}{n/m-k})} + \frac{n}{m} - k)(q<1)$ $\Theta(\frac{1}{ln(\frac{n/m}{n/m-k})} + n - k)(q=1)$	$\Theta(\frac{1}{ln(\frac{1}{1-\theta(1-(1-\theta)^k)})} + 1-\theta(1-(1-\theta)^k))$

7. Conclusion

In this paper, we have studied buffer size occupation in intermittently connected static networks with base stations, mobile networks and mobile network with base stations. Fundamental lower bound on node buffer size in the first two networks are also studied. In conclusion, all the achievements are shown in Table 1.

Moreover, the impact of changeable radius of BSs in both subcritical case and supercritical case is analyzed. Another interesting study about the impact of user number on the security of network coding is also operated, and we find that static network with BSs plays best among the three kind of networks for the security of network coding.

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