Project Report—Throughput and Delay with Network Coding in Hybrid Mobile Ad-Hoc Network: A Global Respective

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***Abstract*—In this paper, we study the network coding under different mobility models for 2-hop and multi-hop schemes with n nodes and k original packets for each pair of source-destination. We propose two ad-hoc network mobility models- hybrid random walk mobility models and discrete random direction mobility models with the constraint of parameters β and α, respectively. And we find that under 2-hop relay scheme with network coding ,there is only a $logn$ gain on delay when the mobility model is random walk mobility model, while there will be a delay gain $logn$ whatever the mobility model is when multi-hop scheme is used. At last,we propose the network model with network coding and infrastructure mode. And in this mode, we obtain the result of throughput and delay, which are all illustrated as well.**

I. Introduction

In this paper, we study the delay and capacity trade-offs in mobile ad hoc network with network coding. The research on wireless network transmission capacity, delay and their tradeoffs has been carried out for many years. Researchers started considering this issue because it is able to offer us an outline of estimated network capability under different network models, which is a guide for future developing directions for wireless networks. Gupta and Kumar were the first ones to come up with the notation of "capacity" for wireless ad hoc networks. In this paper [1], they proposed a protocol interference unicast mobility mode and they found that the capacity of a random wireless network with static nodes scales as $\Theta(\frac{1}{\sqrt{n\log n}})$ bits per second, where n is the number of nodes. After the unicast is studied by many researchers, they began to consider multicast transmission because it is more representative in industry applications. In [2] Özkasap presented a classification of epidemic-based approaches utilized in the MANETs with a focus on reliable multicast protocols. Later, Li et al. [3] studied the multicast capacity of a static random wireless network where each node sends packets to $k-1$ destinations. They showed that per-node multicast capacity is $\Theta(\sqrt{\frac{1}{n\log n}}\cdot\frac{1}{\sqrt{k}})$ when $k = O(\frac{n}{\log n})$, and $\Theta(\frac{1}{n})$ when $k = \Omega(\frac{n}{\log n})$. In order to increase the capacity of wireless networks, Grossglauser and Tse [4] allowed the nodes to move in a 2-hop relaying algorithm which achieved a throughput of $\Theta(1)$ per node. Note that the price of improvement in capacity is the increase in delay. It has been shown in [5]that the 2-hop relay algorithm yielded a tremendous average delay of $\Omega(n)$. In most networking applications, a throughput obtained with an unreasonable delay may not be of practical use. Therefore, the key point is to achieve the optimal capacity and delay tradeoff. Neely et al. [5] applied redundancy with 2-hop and multi-hop i.i.d. mobility schemes to achieve a tradeoff of $delay/capacity \geq O(n)$. With a less restrictive network setting in Lin et al. [6], the capacity delay tradeoff was shown to be $delay/capacity \geq O(\sqrt[3]{n})$.

During the research, more details about how to operate the network model in a more advanced and realistic way are considered. Ying et al.[7] achieved the same tradeoff in [6] by employing a joint coding-scheduling algorithm. Gamal et al.[8] considered a mobile random walk and proposed that adjusting squarelet size and forwarding packets by multi-hop or mobility are fundamental schemes to achieve a tradeoff. The optimal unicast capacity and delay tradeoff has been discussed in detail while little research has focused on such tradeoff in the context of multicast. Hu et al. [9] first studied multicast for ad-hoc network through nodes' mobility, defined as MotionCast in their paper, where nodes move according to an i.i.d. pattern and each packet has k distinctive destinations. They found that the per-node capacity and delay for 2-hop algorithm without redundancy (For each time-slot, if more than one nodes are performing as relays for a packet, they defined there is redundancy in the network.) are $\Theta(1/k)$ and $\Theta(n\log k)$, respectively; and for 2-hop algorithm with redundancy they are $\Omega(1/k\sqrt{n\log k})$ and $\Theta(\sqrt{n\log k})$, respectively. Besides above, they first pointed out a tradeoff in multicast.

After a few works on pure wireless networks with nodes mobility and various transmission algorithms, researchers began to think about more approaches to improve the transmission capacity and delay performances. One of the most famous approaches is the network coding. In above network models, the relay nodes simply receive the packets and then forward them to the next nodes. Ahlswede et al. [10] proposed a concept of network coding that allowed not only information replicating but also information mixing at the intermediate nodes. Later, Li et al. [11] proposed a linear network coding algorithm which proved to achieve the optimal max-flow. Tracey et al. [12] presented a distributed random linear network coding approach for multicast. Ghaderi et al. [13]analyzed the network coding performance against automatic repeat request scheme for reliable multicast transmission.

However, network coding in static networks doesn't have the ability to improve capacity and delay tradeoffs efficiently. Liu et al.[15],[16] demonstrated that there was no order change of the capacity but a constant amplitude gain. In mobile networks, Zhang et al. [17] first showed that there was a $\log n$ capacity gain in delay tolerant networks. Based on the previous works which are mostly depended on $i.i.d.$ mobility model, some challenging questions naturally raised:

- How representative is the $i.i.d.$ mobility model in the study and in the industry application?
- Can the capacity-delay relationship be significantly different under some other reasonable mobility models, such as random walk mobility model, random way-point mobility model and Brownian motion model?
- Can the mobility time scales-fast mobility and slow mobility-have a big influence on the capacity-delay relationship?

Gaurav[18] studied the delay and capacity trade-offs in mobile ad hoc networks in four mobility models-i.i.d mobility model, random walk mobility model, random way-point mobility model and Brownian motion model-without Network coding. The result is thought-provoking because they pointed that under a given mobility model in order to achieve a per-node capacity of ,the minimum average delay(they call it critical delay)is inversely proportional to the characteristic path length, which is the distance nodes travel without changing direction.

Inspired by their works, we may analysis the capacity-delay relationship in other mobility models like random walk mobility model. Gaurav[18] assumed that the information the source wanted to deliver can only be replicated in the source node and the relay nodes only play a role of holding a packet. If we use the Network coding in analysis, the information-we call a packet-must be coded by the source node in use of RLC, and when the relay nodes get the packet, it also has to code the packet in use of RLC in order to create redundancy. The destination node finally decodes the packets after getting enough packets to achieve the rank of decoding matrix. In this situation, we may deliver the same amount of information in a shorter time because the Network coding allows us to transmit more than one packet in each time slot due to the butterfly model. But the problem rises at the same time, with the use of Network coding, the destination node must achieve enough packets to decode the message, which produce a large amount of delay. In our work, we set two mobility models to give a global perspective on the network with network coding. And we show that compared to the previous work, there is a capacity gain even though no gain is achieved when network coding is utilized.

The paper is organized as follows. In section II, we describe the network model, network coding(RLC) scheme and introduce some critical definitions and notations. In section III and IV, the capacity and delay in hybrid random walk mobility model and discrete random direction mobility model with network coding are introduced respectively. In section V, we give a detailed analysis about the capacity and delay with RLC under hybrid random walk mobility model. And the analysis for discrete random direction mobility model is presented in section VI. A discussion on the results are in section VII. Finally we conclude in section VIII.

II. Network Model with Coding Scheme

We consider an ad hoc network consisting of n mobile nodes, distributed uniformly on a unit square S. The square is assumed to be a $torus^2$,$i.e.$, the top and bottom edges are assumed to touch each other and similarly the left and right edges also are assumed to touch other. We consider a homogeneous scenario in which each node generates traffic at the same rate. We take the 2-hop relay scheme and multi-hop relay scheme into consideration. The communication between any source-destination pair can possibly be carried out via multiple other nodes, acting as relays. That is , a source node can, if possible send a packet directly to its destination node; or, the source node can forward the packet to one or more relay nodes; and finally ,a relay node or the source itself can deliver the packet to its destination node.

A. Network Models

Hybrid Random Walk Mobility Models: These models are parametrized by a single parameter β, that takes values between 0 and $\frac{1}{2}$. The unit square is divided into $n^{2\beta}$ squares of area $1/n^{2\beta}$ each which is called a cell, resulting in a discrete torus of size $n^\beta \times n^\beta$. Each cell is then further divided into $n^{1-2\beta}$ square subcells of area $1/n$ each, as shown in Fig.1. Time is divided into slots of equal duration. At each time slot a node is assumed to be in one of the subcells inside a cell. Initially, each node is equally likely to be in any of the n subcells, independently of the other nodes. At the beginning of a slot, a node jumps from its current subcell to one of the subcells in an adjacent cell, chosen in an uniformly random fashion. Here we have an explanation about the adjacent cells: Let a node be in cell(i,j), where $i, j = 0, 1, 2, ..., n^\beta - 1$ at timeslot t, then, at time slot $t+1$, the node is equally likely to be in the same cell (i,j) or any of the four adjacent cells (i-1,j),(i+1,j),(i,j-1),(i,j+1), where the addition and subtraction operations are performed modulo n^β. Note that for $\beta = 0$, the above mobility model is essentially the $i.i.d.$ mobility model and for $\beta = 1/2$, it is the random walk mobility model.

Discrete Random Direction Mobility Models: These models are parametrized by a single parameter α, that takes values between 0 and 1/2. The unit square S is divided into $n^{2\alpha}$ squares of area $1/n^{2\alpha}$ each (henceforth referred to as cells), resulting in a discrete torus of size $n^\alpha \times n^\alpha$. The initial position of each node is assumed to be uniformly distributed within S. The motion of each node under these models is independent and identical to the other nodes. The motion of a node is divided into multiple trips. At the begin of a trip, the node chooses a direction θ uniformly between $[0, 2\pi]$, and moves a distance of $n^{-\alpha}$ in that direction, with a speed of v_n, and the process repeats itself. Here we choose $v_n = \Theta(\frac{1}{\sqrt{n}})$ because we keep the network area fixed and let the number of nodes increase to infinity, which means that the average neighborhood size scales as $\Theta(\frac{1}{\sqrt{n}})$. Time is divided into slots of equal duration. At the beginning of a slot, each node jumps from its current cell to an adjacent cells, chosen uniformly from within the set of adjacent cells. The motion of a node during the slot is as

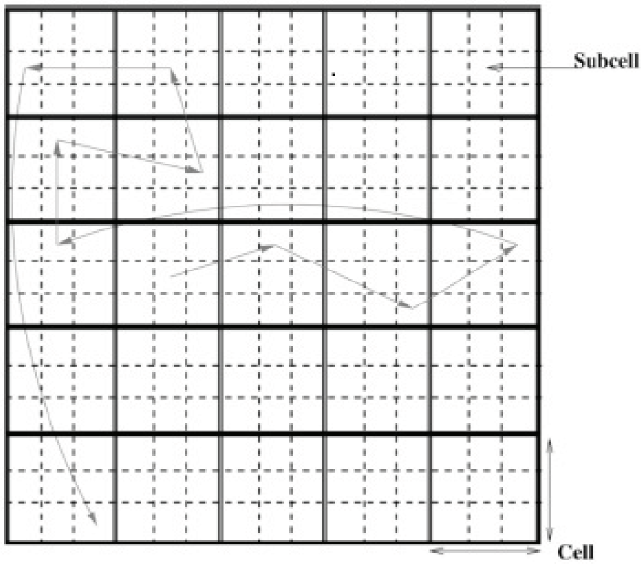

Fig. 1. The division of unit square into cells and subcells; and the motion of a node under a hybrid random walk model

follows: The node chooses start point and end point uniformly within the current cell. During the slot, the node moves from the start point to the end point. In our model, the duration of a slot should be $\Theta(n^{1/2-\alpha})$.

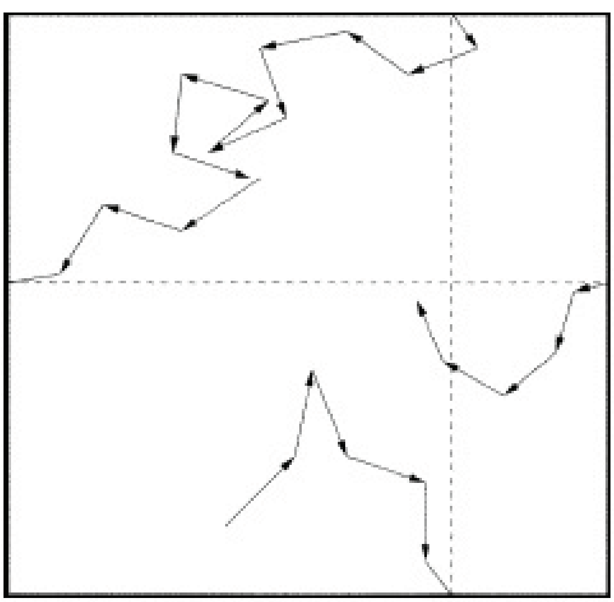

Fig. 2. An example motion path of a node under the discrete random direction model

Model for successful transmission: For simplicity, we assume that the success or failure of a transmission between a pair of nodes is governed by the protocol model of [1].Let W be the bandwidth of the system in bits per second. Let X_t^i denote the position of node i, for i=1...n, at time t. Under the protocol model, node i can communicate directly with node j at a rate of W bits per second at time t, if and only if, the following interference constraint is satisfied:

$$d(X_t^k, X_t^j) \geq (1+\delta)d(X_t^i, X_t^j)$$

for every other node $k \neq i, j$ that is simultaneously transmitting. Here δ is some positive number; and $d(x, y)$ is the distance between points $x = (x_1, x_2)$,$y = (y_1, y_2)$∈ S, defined as follows:

$$d(x, y) = min_{x^i \in [x], y^j \in [y]} \mid\mid x^i - y^j \mid\mid,$$

where [x] is set of points $(x_1, x_2), (x_1 - 1, x_2)$,$(x_1 + 1, x_2)$,$(x_1, x_2 - 1)$,$(x_1, x_2 + 1)$, and [y] is defined similarity.

Definition of throughput: A throughput $\lambda > 0$ is said to be feasible/achievable if every node can send at a rate of at least λ bits per second to its chosen destination. We denote by $T(n)$, the maximum feasible throughput $w.h.p.$. Given a scheme Π, let $M_\Pi(i, t)$ be the number of packets from source node i that destination node d(i) receives in t timeslots under scheme Π, for $1 \leq i \leq n$. Note that this could be a random quantity for a given realization of the network. Define the long term throughput of S-D pair i, denoted by $\lambda_\Pi^i(n)$, to be

$$\lambda_\Pi^i(n) = \lim_{t\to\infty} inf \frac{1}{t} M_\Pi(i, t)$$

Scheme Π is said to have throughput $T_\Pi(n)$ if

$$\lim_{n\to\infty} P(\lambda_\Pi^i(n) \geq T_\Pi(n) \quad for \quad all \quad i) = 1.$$

Note that when network coding is utilized in scheme Π, $M_\Pi(i, t)$ is the number of successfully decoded packets received by the destination d(i) of S-D pair i in t timeslots under scheme Π.

Definition of delay: The delay of a packet is the time it takes the packet to reach the destination after it leaves the source. We do not take queueing delay at the source into consideration, since our interest is in the network delay. Let $D_\Pi^i(j)$ denote the delay of packet j of S-D pair i under scheme Π, then the sample mean of delay for S-D pair i is

$$\overline{D_\Pi^i} = \lim_{k\to\infty} sup \frac{1}{k} \sum_{j=1}^{k} D_\Pi^i(j).$$

The average delay over all S-D pairs for a particular realization of the random network is then $\overline{D_\Pi} = \frac{1}{n}\sum_{i=1}^{n} \overline{D_\Pi^i}$. The delay for a scheme Π is the expectation of the average delay over all S-D pairs and all random network configurations,$i.e.$,

$$D_\Pi(n) = E[\overline{D_\Pi}] = \frac{1}{n}\sum_{i=1}^{n} E[\overline{D_\Pi^i}].$$

When network coding is utilized, we consider the delay of getting original packets. When an original packet m_i belongs to the generation M, the delay of m_i under scheme Π is the time from the first packet belonging to M leaves the source to the original packet m_i has been decoded in the destination.

B. Network Coding Operation

Random linear coding (RLC for short)[12] is applied toa finite set of k original messages, $M = \{m_1, m_2, \cdots, m_k\}$, which is called a generation. We assume that all the k packets in M are linearly independent. In the RLC protocol, destination nodes start collecting several linear combinations of the messages in M. Once the destination nodes have k independent linear combinations of the messages, they can recover all the messages successfully. Let f_l denote one of the encoded packets. Then f_l has the form $f_l = \sum_{i=1}^{b} a_i \cdot m_i$ where a_i is the RLC vector known to the destinations. Note that sources hold "messages" at first, and each source encodes k messages of its own into an encoded packet. The transmitted units in the network are "packets", but not "messages".

For decoding purposes, the transmitting nodes also send the random coding vectors as overhead within each packet. Each node v collects the coding vectors for the packets it receives in a decoding matrix G_v. A received packet is said to be innovative if its coding vector increases the rank of the matrix G_v.

Random linear network coding scheme within a cell partitioned model can be illustrated in Figure 3. Note that the source originally has a packet with k messages. In the same cell, if the source meets with the relays, it will send encoded packets to the relays. The relays will take the packet moving around until they encounter new relay or destination nodes within the same cell. Then the relays will encode the packet and transmit the new random linear coding messages, i.e., new massages, to relays or destinations. Note that when relays do "encoding" again, they treat the received "packets" (from the same source) as "messages" and encode then into a new "packet". The reason why the relays encode the messages again is to introduce the redundancy for decoding purpose. The destinations finally decode the packets after receiving enough packets to achieve the rank of decoding matrix.

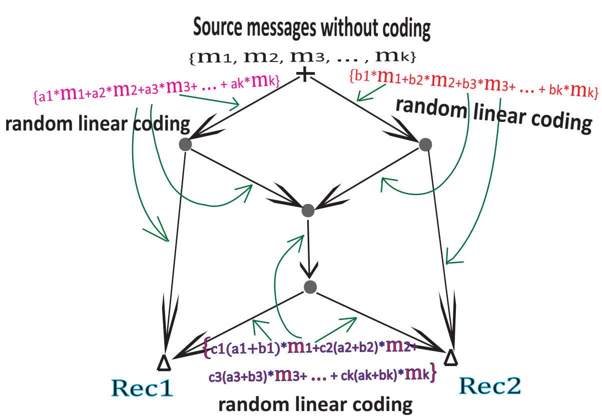

Fig. 3. Network coding scheme

C. RLC-Based Relay Schemes

Schemes 1: 2-hop Relay with RLC: (1)k original packets in each source node will be grouped into one generation. Each source will send $m = (1 + \epsilon)k$ coded packets for each generation, where ϵ is a constant. (2)When the relay nodes have received the coded packet, it will store it in the buffer which has infinite capacity and wait encoding and transmission. After all the nodes have received the coded packet from the source, it will be deleted from the buffer. (3)In each time slot, only one of $n^{1-2\beta}$ be active. And in the active cell, transmission is always between nodes within the same cell. (4)For an active cell with at least two nodes, randomly assign a node as sender and independently choose another node in the cell as receiver. With equal probability of 1/2, the transmission is scheduled to operate in either "Source-to-Relay" or "Relay-to-Destination" mode, describing as follows:

- Source to Relay transmission: If the sender has a new encoded packet with random linear combination of messages that has never been transmitted before, send the packet to the receiver and delete it from the buffer. As for the receiver which is a relay node, it collects packets from the sender and store them in a buffer of infinite capacity in order to await encoding and transmission. If one transmission fails due to the lossy and erasure channel, retransmit the packet. Otherwise, stay idle.
- Relay to Destination transmission: If the sender has a new encoded packet with random linear combination of messages from other nodes destined for the receiver, transmit it. Before a relay transmit a packet to its destined destination, it has to undergo RLC encoding again, on all the packets in its buffer for the same destination. The receiver that is the destination node must receive enough innovative packets to achieve the rank k of the decoding matrix G_v so that it can recover the original packet. If one transmission fails due to the lossy and erasure channel, retransmit the packet. According to the successful transmission acknowledgements, if all the destinations have received the source packet, that packet will be dropped from the buffer in the sender. Otherwise, stay idle.

Schemes 2 Multi-hop Relay with RlC: The transmission way for the source and relay node is similar to the 2-hop relay scheme with network coding. The only difference is that the number of relay nodes can be more than one.

III. Throughput-Delay Tradeoffs Without Networking Coding Under Hybrid Random Walk Models: Results

In this section, we give a brief overview of the schemes in [18] and present the main result about throughput and delay under hybrid random walk models without network coding.

We first describe some definitions and assumptions in [18]. **Critical Delay:** Let ς be the class of scheduling and relaying schemes under consideration. For $c \in \varsigma$, let D_c, λ_c be the average delay and per-node throughput, respectively, under scheme c. The critical delay for the class of schemes ς, denoted by D_ς, is the minimum average delay that must be tolerated under a given mobility model in order to achieve a per-node capacity of $\omega(1/\sqrt{n})$, that is

$$D_\varsigma = \inf_{c \in \varsigma : \lambda_c = \omega(1/\sqrt{n})} D_c.$$

Assumption: Only the source node can initiate a replication; i.e.,the relay nodes holding a packet do not initiate a replication.

Now, we analyze the performance of the schemes described above. We give the main results about the throughput-delay under hybrid random walk models without network coding presented in [18].

Theorem 1: Under the critical-delay scheme, the delay under the hybrid random walk models is $\Theta(n^{2\beta} \log n)$, and the per-node throughput is $\omega(1/\sqrt{n})$.

Theorem 2: Under the 2-hop delay scheme, the delay under hybrid random walk models is $\Theta(n)$ when $\beta < 1/2$ and $\Theta(n \log n)$ when $\beta = 1/2$. And the per-node throughput is still $\omega(1/\sqrt{n})$.

IV. Throughput-Delay Tradeoffs Without Network Coding Under Discrete Random Direction Models: Results

In this section, we present the main results about throughput and delay under discrete random direction models discussed in [18].

Theorem 3: Under the critical-delay scheme, the delay under the discrete random direction models is $\Theta(n^{1/2+\alpha} \log n)$, and the per-node throughput is $\omega(1/\sqrt{n})$.

Theorem 4: Under the 2-hop delay scheme, the delay under discrete random direction models is $\Theta(n)$ when $0 \le \alpha < 1/2$ and $\Theta(n \log n)$ when $\alpha = 1/2$.

V. Throughput-Delay Tradeoffs With Network Coding Under Hybrid Random Walk Models:Results

In this section, we give the main results about throughput-delay tradeoffs with RLC under hybrid random walk mobility models and make a comparison with the former works with RLC.

A. Throughput-Delay Tradeoffs With RLC Under 2-hop Relay Scheme

Theorem 5: When 2-hop relay with RLC scheme is used and $k = \Theta(n^{2\beta})$, we have $T(n) = \Theta(1)$ and $D(n) = \Theta(n^{2\beta})$ for hybrid random walk mobility models,here $0 < \beta \le 1/2$. When $\beta = 0$, it is the i.i.d mobility model, and we have $T(n) = \Theta(1)$, and $D(n) = \Theta(n)$

B. Throughput-Delay Tradeoffs With RLC Under Multi-hop Relay Scheme

Theorem 6: When multi-hop relay with RLC scheme is used, under hybrid random walk mobility model with $k = \theta(n^{2\beta})$, we have $T(n) = \theta(1)$ and $D(n) = \theta(n^{2\beta})$.

VI. Throughput-Delay Tradeoffs With Network Coding Under Discrete Random Direction Models:Results

A. Throughput-Delay Tradeoffs With RLC Under 2-hop Relay Scheme

Theorem 7: Under the discrete random direction models, 2-hop relay with RLC scheme is adopted and $k = \Theta(n^{2\alpha})$, then we get $T(n) = \Theta(n^{\alpha-1/2})$ and $D(n) = \Theta(n^{\alpha+1/2})$, here $0 \ge \alpha < 1/2$. And when $\alpha = 1/2$, $D(n) = \Theta(n)$.

B. Throughput-Delay Tradeoffs With RLC Under Multi-hop Relay Scheme

Theorem 8: When multi-hop relay with RLC scheme is used, $k = \Theta(n^\alpha)$, the throughput and delay under discrete random direction models are $T(n) = \Theta(n^{\alpha-1/2})$, $D(n) = \Theta(n^{1/2})$ respectively.

VII. Throughput-Delay Tradeoffs With Network Coding Under Hybrid Random Walk Models:Analysis

In this section, we present details about the proofs for the results on RLC-based scheme under hybrid random walk mobility models which are discussed in the previous section.

A. Preliminaries

To make the analysis more apparent and easily, we need first define some notation for the hybrid random walk mobility models: inter-meet delay, minimal flooding delay and 2-hop whole-meet delay. Here inter-meet time represent all schemes while the 2-hop relay represent any schemes in which the number of hops for each packet is 2.

Considering the hybrid random walk mobility models, the definition for inter-meet delay is that after the nodes are distributed uniformly in the unit square, the time it costs the source node to encounter one of any other nodes. If we go further: when the source node encounter $n^{2\beta}$ nodes, the corresponding time is called as 2-hop whole-meet delay. From the definition we can see, these two time reflect the intrinsic properties of how mobility will facilitate information propagation. And these two factors are independent of any schemes.

Lemma 7.1: Letτ be the random variable representing the inter-meeting time for two nodes in the same subcell of a random walk mobility model on a 2-d torus of size $n^\beta \times n^\beta$, we have

$$E[\tau] = \Theta(n^{2\beta})$$

Proof: El Gamal et al. once proved that $E[\tau] = \Theta(n)$ on a 2-d torus of size $\sqrt{n} \times \sqrt{n}$. Under the hybrid random walk mobility model, there are $n^{2\beta}$ cells and every timeslot, the node will be in any one of the $n^{2\beta}$ cells instead of the n cells in random walk mobility model. Therefore it is easy to come to the conclusion that $E[\tau] = \Theta(n^{2\beta})$ under the hybrid random walk mobility model. ■

Lemma 7.2: The minimal flooding delay under hybrid random walk mobility model is $\Theta(n^{2\beta})$

Proof: We cite the following important result about rumor spreading on torus: Theorem 3 in [44] states that following the flooding rule mentioned above, at timeslot t, there exists a sub-torus of size $\sqrt{t} \times \sqrt{t}$, where for each cell in this sub-torus, there exists at least one red node. Therefore, in $\Theta(n^{2\beta})$ timeslots, we can cover the whole torus of size $n^{\beta} \times n^{\beta}$ $w.h.p.$ ■

Lemma 7.3: The 2-hop whole-meet time under hybrid random walk mobility models is $\Theta(n^{\beta})$.

Proof: Let N be the number of distinct nodes the source node has met in $n^{2\beta}$ timeslots. From *Lemma 1*, we can obtain that $E[N] = (1-\epsilon)n^{2\beta}$, where $0 < \epsilon < 1$ is a constant, and $\sigma_N = O(n^{2\beta}\log n)$. By Chebyshev inequality, for any $0 < \varsigma < 1$,

$$P(N \leq (1-\varsigma)E[N]) \leq \frac{\sigma_N}{\varsigma^2 E[N]^2} = O\left(\frac{\log n}{n^{2\beta}}\right) \to 0,$$

which means that $N = \theta(n^{2\beta})w.h.p.$ That is to say the 2-hop whole-meet time under hybrid random walk mobility models is $\Theta(n^{2\beta}\log n)$. ■

B. Proof for Main Results

Proof for 2-hop relay with RLC Under Hybrid Random Walk Mobility Models(Theorem 5)

Proof: As already having been proved in *Lemma 2*that, after $N_1 = \Theta(n^{2\beta})$ timeslots, the source node has already delivered coded packets to $m_1 = \Theta(n^{2\beta})$ different nodes. From [45], we can infer that the mixing time of a simple random walk on a $n^{\beta} \times n^{\beta}$ torus is also $\Theta(n^{2\beta})$. Therefore, there exists a constant ϵ such that after $N_2 = \epsilon n^{2\beta}$ timeslots, these m_1 nodes with coded packets are uniformly distributed in the torus $w.h.p.$ which means that each node in the network has coded packets with a constant probability. Then after $N_1 + N_2$ timeslots, the destination node begins to collect coded packets. It can be proved that after $N_3 = \Theta(n^{2\beta})$ timeslots, the destination will collect $\Theta(n^{2\beta})$ coded packets. Therefore, the whole delay $N = N_1 + N_2 + N_3 = \Theta(n^{2\beta})$ $w.h.p.$.

In the hybrid random walk mobility models, each source node sends $m = \Theta(n^{2\beta})$ coded packets for a "big generation"[change it later], and each big generation has $\Theta(n^{2\beta})$ original packets, then each coded packet contains $\Theta(1)$ information of original packets. Therefore we can get $T(n) = \Theta(1)$. ■

Proof for Multi-hop relay with RLC Under Hybrid Random Walk Mobility Models (Theorem 6)

Proof: There we adopt the same analysis method in the 2-hop circumstance. The key problem is that how many timeslots it costs the destination node to get $\theta(k)$ coded packets. Suppose that after N timeslots, the destination nodes gets $\theta(k)$ coded packets, then based on the Proposition the destination has enough coded packets to recover k original packets $w.h.p.$. Then the delay is upper bound by N. From Zhang. we know that $E[N] \leq \theta(k)$ therefore, under the hybrid random walk mobility model, we replace k with $\theta(n^{2\beta})$. That is the delay under hybrid random walk mobility model with RLC is $\theta(n^{2\beta})$, where $k = \theta(n^{2\beta})$. Since we get k original packets in $\theta(k)$ timeslots, the throughput for the network model is $\theta(1)$. ■

Contrast to the result in Gaurav.S[], we can see that with the use of network coding, there is capacity gain $\omega(\sqrt{n})$ and no gain is found in the delay.

VIII. Throughput-Delay Tradeoffs With Network Coding Under Discrete Random Direction Models:Analysis

In this section, we present details for the proof for the results on RLC-based scheme under discrete random direction models which are discussed in the previous section.

Lemma 8.1: The minimal flooding delay under discrete random direction model is $\Theta(n^{1/2+\alpha})$.

Proof: As proved in lemma 5.2, if the timeslot under the discrete random direction model is the same under the hybrid random walk model, then the minimal flooding delay will still be $\Theta(n^{2\alpha})$. However, from the definition of discrete random direction model we can see the timeslot is $\Theta(n^{1/2-\alpha})$, therefore the minimal flooding delay under discrete random direction model will be $\Theta(n^{1/2-\alpha}) \times \Theta(n^{2\alpha})$, that is $\Theta(n^{1/2+\alpha})$. ■

B. Proof for Main Results

Proof for 2-hop relay with RLC Under Discrete Random Direction Models(Theorem 7)

Proof: From the definition of discrete random direction models and hybrid random walk models, we can see that the timeslot is different, which are $\Theta(n^{1/2-\alpha})$ and $\Theta(1)$ respectively. And under the 2-hop relay scheme, we adopt the same RLC scheme under discrete random direction models and hybrid random walk models, the only difference is the duration of one timeslot. Therefore we can easily calculate the delay under discrete random direction models. That is

$$D(n) = \Theta(n^{1/2-\alpha}) \times \Theta(n^{2\alpha}) = \Theta(n^{1/2+\alpha})$$

■

Proof for Multi-hop relay with RLC Under Discrete Random Direction Mobility Models (Theorem 8)

Proof: Here we use the same analysis method with the multi-hop scheme under hybrid random walk models. The key problem is that how many timeslots do we need in order to receive at least $\Theta(k)$ coded packets at the destination. Similarly, we denote it as N. From Theorem 2, we have $E[N] < \Theta(k)$, and under discrete random direction model we replace k with $\Theta(n^{2\alpha})$. And under discrete random direction model model the timeslot lasts $\Theta(n^{1/2-\alpha})$. Therefore the delay under discrete random direction model with RLC is $\Theta(n^{1/2+\alpha})$. Since we achieve k original coded packets at the destination, then the destination can decode all the information, then $T(n) = \Theta(n^{1/2-\alpha})$. ■

IX. Discussion For The Throughput-delay With Network Coding Scheme Under Hybrid Random Walk Models And Discrete Random Direction Models

The results obtained in the previous sections are summarized in Fig.4

And we summarize the results of network coding under hybrid random walk models and discrete random direction models with 2-hop and multi-hop schemes and make a comparison with the results in [18], which are illustrated in Table I and Table II respectively.

TABLE I
COMPARISON AMONG CAPACITY, DELAY IN 2-HOP RELAY WITH NETWORK CODING ALGORITHMS

Scheme	Condition	Capacity	Delay
Hybrid Random Walk Models w.o. NC. for 2-hop	$\beta < 1/2$	$\omega(1/\sqrt{n})$	$\Theta(n)$
Hybrid Random Walk Models w.o. NC. for 2-hop	$\beta = 1/2$	$\omega(1/\sqrt{n})$	$\Theta(n \log n)$
Hybrid Random Walk Models w. NC. for 2-hop	$\beta = 0$	$\Theta(1)$	$\Theta(n)$
Hybrid Random Walk Models w. NC. for 2-hop	$k = \Theta(n^{2\beta})$ $0 < \beta \leq 1/2$	$\Theta(1)$	$\Theta(n^{2\beta})$
Discrete Random Direction Models w.o. NC. for 2-hop	$0 \leq \alpha < 1/2$	$\omega(1/\sqrt{n})$	$\Theta(n)$
Discrete Random Direction Models w.o. NC. for 2-hop	$\alpha = 1/2$	$\omega(1/\sqrt{n})$	$\Theta(n \log n)$
Discrete Random Direction Models w. NC. for 2-hop	$k = \Theta(n^{2\alpha})$ $0 \leq \alpha < 1/2$	$\Theta(n^{\alpha-1/2})$	$\Theta(n^{\alpha+1/2})$
Discrete Random Direction Models w. NC. for 2-hop	$\alpha = 1/2$	$\Theta(n^{\alpha-1/2})$	$\Theta(n)$

TABLE II
COMPARISON AMONG CAPACITY, DELAY IN MULTI-HOP RELAY WITH NETWORK CODING ALGORITHMS

Scheme	Condition	Capacity	Delay
Hybrid Random Walk Models w.o. NC. for multi-hop	$0 \leq \beta \leq 1/2$	$\omega(1/\sqrt{n})$	$\Theta(n^{2\beta} \log n)$
Hybrid Random Walk Models w. NC. for multi-hop	$k = \Theta(n^{2\beta})$	$\Theta(1)$	$\Theta(n^{2\beta})$
Discrete Random Direction Models w.o. NC. for multi-hop	$0 \leq \alpha \leq 1/2$	$\omega(1/\sqrt{n})$	$\Theta(n^{\alpha+1/2} \log n)$
Discrete Random Direction Models w. NC. for multi-hop	$k = \Theta(n^{2\alpha})$	$\Theta(n^{\alpha-1/2})$	$\Theta(n^{\alpha+1/2})$

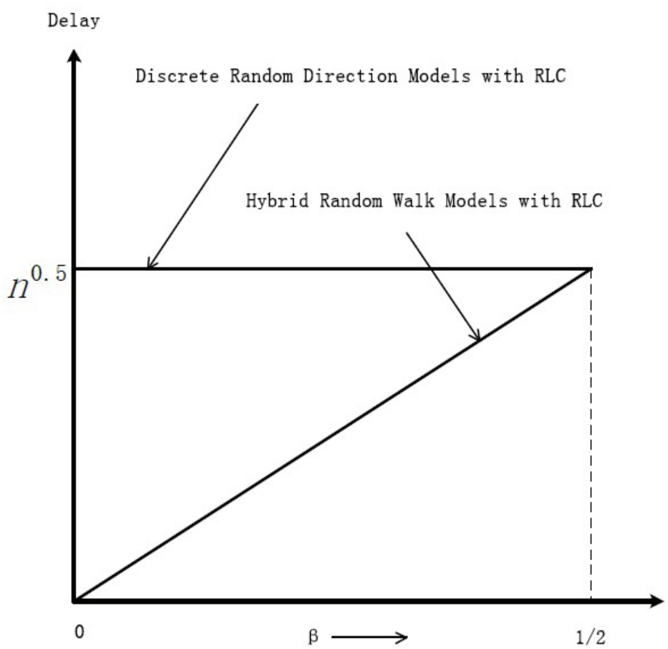

Fig. 4. The delay in case of hybrid random walk and discrete random direction models with RLC

X. NETWORK MODEL WITH BASE STATION IN RLC

Form this section, we will adopt a new scheme to analyze the throughput and delay for the ad-hoc network. We regular allocate the base stations in the network and propose the 2-hop relay algorithm to deduce the throughput and delay.

A. Network Model with Base Station

Cell Partitioned Network Model With Base Station:The w-hole network is cell partitioned as previous hybrid random wall models. Then $m = n^b (0 \leq b \leq 1)$base stations are regularly distributed in $n^{2\beta}$ cells.All the base stations are connected by wires so that they can communicate in $\Theta(1)$ delay. Every cell contain one base station at the center position, and the cell includes $n^{1-2\beta}$ subcells. A base station can communicate with all the nodes in the same cell at the same time, while a node can only deliver packets to the base station when it is in the same subcell as the destined base station. In other words, base station has enough transmission power to cover the whole cell. What's more, we assume uplink and downlink use different frequency to avoid interference. This means when base station is transmitting packets,all the other transmissions between two nodes or one node and a base station can still go on without any problems. The nodes transmission schemes are the same as previous hybrid random walk models. To make the analysis clear, here we assume the k packets information has the order of $k = n^d$ where $0 \leq d \leq 1$. The source and destination relationship will never change while nodes move.

B. 2-hop Relay Algorithm Under Hybrid Random Walk Models With Base Stations

When we introduce the base station into the network, there are two conditions in a subcell now: it has a base station or not. When a node jumps into a new subcell in a timeslot, if the subcell doesn't have a base station, the node acts in the same with as described in the previous section. If the cell has a base station, then the node will act as follows:

- Source to base station transmission: If the nodes in the subcell have packets to transmit, randomly choose such a node as the source, and send the packet to the base station within the subcell. If no such node exists, stay idle.
- Base station to base station transmission: As defined above, in $\Theta(1)$ time, the base station will broadcast the received packet to all the other base station via wires.
- Base station to destinations transmission: In this last step, the base stations send the packets to all the destination nodes within their corresponding cell. The transmission ends.

XI. Throughput And Delay Under 2-hop Relay Algorithm With RLC In The Hybrid Random Walk Network With Base Stations: Results

In this section, we will present the main results under 2-hop relay algorithm with RLC when the base stations are added to the network. The network model is illustrated in the previous section.

Theorem 9: In the network with base stations, under 2-hop relay algorithm with network coding, when $k = \Omega(m)$, that is $0 \leq b \leq d \leq 1$, we can achieve the throughput $\Theta(1)$ and the delay $\Theta(n^{2\beta+b-1} + k)$, where $k = n^d$ and $m = n^b$.

Theorem 10: In the network with base stations, under 2-hop relay algorithm with network coding ,when $k = o(m)$, that is $0 \leq d \leq b \leq 1$, we can achieve the throughput $\Theta(1)$ and the delay $\Theta(n^{2\beta+d-1} + k)$, where $k = n^d$ and $m = n^b$.

XII. Throughput And Delay Under 2-hop Relay Algorithm With RLC In The Hybrid Random Walk Network With Base Stations: Analysis

In this section, we will give details about the proof of the results under 2-hop relay algorithm with RLC in the network with base stations in the previous section.

Lemma 12.1: When $k = \Omega(m)$, the input rate of each queue in the base stations is $\Theta(\frac{1}{m})$; and the input rate of each queue in the base stations is $\Theta(\frac{1}{k})$ when $k = o(m)$.

Proof: The lemma is proved in [20]. ■

Here, we recall the definition of first hitting time and first return time in [18].

Definition of First Hitting Time: The first hitting time for the set of states $A \subset S_X$ is given by $\tau_H{}^A = inf t \geq 0 : X(t) \in A$ with $X(0)$ being distributed according to Π_X.

Definition of First Return Time: The first return time for the set of states $A \subset S_X$ is given by $\tau_H{}^A = inf t > 0 : X(t) \in A$ with $X(0) \in A$.

Then we from the result of first hitting time and first return time in the case of 2-D torus of size $\sqrt{n} \times \sqrt{n}$, we can derive the result for the 2-D torus of size $\frac{n^\beta}{\sqrt{m}} \times \frac{n^\beta}{\sqrt{m}}$.

Lemma 12.2: Let H denote the first hitting time for a single state on a 2-D torus of size $\frac{n^\beta}{\sqrt{m}} \times \frac{n^\beta}{\sqrt{m}}$, then $E\{H\} = \Theta(\frac{n^{2\beta}}{m} \log n)$.

Lemma 12.3: Let H denote the first return time for a single state on a 2-D torus of size $\frac{n^\beta}{\sqrt{m}} \times \frac{n^\beta}{\sqrt{m}}$, then $E\{H\} = \Theta(\frac{n^{2\beta}}{m})$.

In the hybrid random walk models, the motion of nodes on a 2-D torus of size $n^\beta \times n^\beta$ with m base stations regularly distributed is equivalent of motion on a 2-D torus of size $\frac{n^\beta}{\sqrt{m}} \times \frac{n^\beta}{\sqrt{m}}$ with a single base station.

Lemma 12.4: When $k = \Omega(m)$, we have the delay $D = \Theta(n^{2\beta})$ and throughput $\lambda = \Theta(\frac{1}{n})$.

Proof: The successful transmitting of packet from the nodes to a base station needs two procedures: The source node is to transmit the packet to the base station and the base station is ready to receive the packet. The probability that the source node is scheduled to transmit the packet to the base station is $\frac{1}{mq}$, where q is the density of nodes in the network. According to the Lemma 6 and Lemma 7, the first hitting time of a cell with a base station is $\Theta(\frac{n^{2\beta}}{m} \log n)$, and the inter meeting time of a cell with a base station is $\Theta(\frac{n^{2\beta}}{m})$. Thus the delay is $D = \Theta(\frac{n^{2\beta}}{m} \log n) + \Theta(\frac{n^{2\beta}}{m})(\frac{1}{mq})^{-1} = \Theta(n^{2\beta})$.

From Lemma 5, we know the input rate of each queue in base stations is $\Theta(\frac{1}{m})$, during the time interval [0,T], the total number of packets sent to base stations is $\Theta(\frac{1}{m}) \times Tm$. To guarantee a stable network, the throughput of whole network cannot exceed the packets that base stations are able to serve in time interval [0,T]. Thus we have $\lambda Tm \leq \Theta(\frac{1}{m}) \times Tm$, that is $\lambda \leq \Theta(\frac{1}{n})$. Therefore, the capacity in 2-hop relay algorithm with base stations is $\Theta(\frac{1}{n})$. ■

Lemma 12.5: When $k = o(m)$, we have the delay $D = \Theta(n^{2\beta} + d - b)$ and throughput $\lambda = \Theta(n^{b-d-1})$.

Proof: When $k = o(m)$,the probability that the source node is scheduled to transmit the packet to the base stations is $\frac{1}{kq}$. The first hitting time of a cell with a base station is $\Theta(\frac{n^{2\beta}}{m} \log n)$, and the inter meeting time of a cell with a base station is $\Theta(\frac{n^{2\beta}}{m})$. Thus the delay is $D = \Theta(\frac{n^{2\beta}}{m} \log n) + \Theta(\frac{n^{2\beta}}{m})(\frac{1}{kq})^{-1} = \Theta(n^{2\beta+d-b})$.

Since the input rate of each queue in base stations is $\Theta(\frac{1}{k})$, during the time interval [0,T], the total number of packets sent to base stations is $\Theta(\frac{1}{k}) \times Tm$. To guarantee a stable network, the throughput of whole network cannot exceed the packets that base stations are able to serve in time interval [0,T]. Thus we have $\lambda Tm \leq \Theta(\frac{1}{k}) \times Tm$, that is $\lambda \leq \Theta(n^{b-d-1})$. Therefore, the capacity in 2-hop relay algorithm with base stations is $\Theta(n^{b-d-1})$. ■

Proof for Main Results

Proof for 2-hop relay with RLC Under Hybrid Random Walk Mobility Models with base stations(Theorem 9) Under the hybrid random walk models with base stations, the probability that a node meets a base station is $P_1 = \frac{m}{n^{2\beta}} \frac{1}{n^{1-2\beta}} = \frac{m}{n} = \Theta(n^{b-1})$. From Theorem 5 and lemma 8, we have

$$\lambda = P_1\lambda_1 + P_2\lambda_2 = \Theta(n^{b-1})\Theta(\frac{1}{n}) + \Theta(1-n^{b-1})\Theta(1) = \Theta(1)$$

Where λ_1 is the throughput when only base stations are adopted in the hybrid random walk models while λ_2 is the throughput when only network coding is used in the hybrid random walk models.

The delay is

$$D = P_1D_1 + P_2D_2 = \Theta(n^{b-1})\Theta(2\beta) + \Theta(1 - n^{b-1})\Theta(k) = \Theta(n^{2\beta+b-1} + k)$$

Where D_1 is the delay for hybrid random walk models with only base stations and D_2 is the delay for hybrid random walk when network coding is used.

Proof for 2-hop relay with RLC Under Hybrid Random Walk Mobility Models with base stations(Theorem 10)

$$\lambda = P_1\lambda_1 + P_2\lambda_2 = \Theta(n^{b-1})\Theta(n^{b-d-1}) + \Theta(1-n^{b-1})\Theta(1) = \Theta(1)$$

The delay is

$$D = P_1D_1 + P_2D_2 = \Theta(n^{b-1})\Theta(2\beta + d - b) + \Theta(1-n^{b-1})\Theta(k)$$

$$= \Theta(n^{2\beta+d-1} + k)$$

XIII. Conclusion

We propose two techniques to improve the network performance including capacity, delay in this paper. We design two mobility models-hybrid random walk models and discrete random direction models and adopted two-hop relay scheme and multi-hop relay scheme respectively. In such network coding models, we conclude that there is a $\log n$ gain on delay under 2-hop relay schemes when the mobility model is random walk mobility models. And there is $\log n$ gain on delay in hybrid random walk mobility models under multi-hop schemes whatever the mobility model is. As we suppose that at the beginning of a generation, there are already k original packets for the source to deliver, then the definition of capacity in my work is different from the normal one, then we ignore the comparison of capacity between models with network coding and without network coding.

Furthermore, we study the infrastructure mode in which there are m base stations regularly distributed in the network model. The results show that the capacity and delay have an order gain after we apply the infrastructure mode in the network, but the function depends on the relationship between the number of original packets k and the number of base stations m.

At last, we combine the network coding technique and the infrastructure mode technique together in one network model, and we calculate the capacity and delay performance in such network. We find that under 2-hop relay algorithm with network coding, when $k = \Omega(m)$, the throughput is $\Theta(1)$ and the delay is $\Theta(n^{2\beta+d-1} + k)$. And when $k = o(m)$, we achieve that the throughput $\Theta(1)$ and the delay $\Theta(n^{2\beta+d-1} + k)$, here $k = n^d$ and $m = n^b$

References

[1] P. Gupta, and P.R. Kumar, "The Capacity of Wireless Network," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388-404, March 2000.

[2] Ö. Özkasap, Z. Genc, and E. Atsan, "Epidemic-based approaches for reliable multicast in mobile ad hoc networks," *ACM SIGOPS Operating Systems Review*, vol. 40, no. 3, pp. 73-79, July 2006.

[3] X. Li, S. Tang and O. Frieder, "Multicast capacity for large scale wireless ad hoc networks," *in Proc. of ACM MobiCom 2007*, pp. 266-277, Montreal, Canada, September 9-14 2007.

[4] M. Grossglauser and D. Tse, "Mobility Increases the Capacity of Ad-hoc Wireless Networks," *in Proc. IEEE INFOCOM*, pp. 1360-1369, Anchorage, Alaska, USA., April 2001.

[5] M. Neely, and E. Modiano, "Capacity and delay tradeoffs for ad hoc mobile networks," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 1917-1937, June 2005.

[6] X. Lin, G Sharma, R. Mazumdar, and N. Shroff, "Degenerate Delay-Capacity Trade-offs in Ad Hoc Networks with Brownian Mobility," *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2777-2784, June 2006.

[7] L. Ying, S. Yang, and R. Srikant, "Optimal Delay-Throughput Tradeoffs in Mobile Ad Hoc Networks," *IEEE Transactions on Information Theory*, vol. 54, no. 9, pp. 4119-4143, September 2008.

[8] A. E. Gammal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay trade-off in wireless networks," *in Proc. IEEE INFOCOM*, vol. 1, Hong kong, China, March 2004.

[9] C. Hu, X Wang, and F Wu, "MotionCast: on the capacity and delay tradeoffs," *in Proc. of ACM MobiHoc 2009*, pp. 289-298, New Orleans, Louisiana, USA., May 2009.

[10] R. Ahlswede, N Cai, SYR Li, and RW Yeung, "Network Information Flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204-1216, July 2000.

[11] S. Li, R. Yeung, and N. Cai, "Linear Network Coding," *IEEE Transactions on Information Theory*, vol. 49, no. 2, pp. 371-381, February 2003.

[12] T. Ho, M. Medard, R. Koetter, D. Karger, M. Effros, J. Shi, and B.Leong, "A Random Linear Network Coding Approach to Multicast," *IEEE Transactions on Information Theory*, vol. 52, no. 10, pp. 4413-4430, October 2006.

[13] M. Ghaderi, D. Towsley, and J. Kurose, "Network Coding Performance for Reliable Multicast" *in Proc. IEEE MILCOM 2007*, pp. 1-7, Orlando, Florida, USA., October 2007.

[14] Z. Li, B. Li, and L. Lau, "A Constant Bound on Throughput Improvement of Multicast Network Coding in Undirected Networks," *IEEE Transactions on Information Theory*, vol. 55, no. 3, pp. 1016-1026, March 2009.

[15] J. Liu, D. Goeckel and D. Towsley, "Throughput Order of Ad Hoc Networks Employing Network Coding and Broadcasting," *in Proc. of IEEE MILCOM 2006*, Washington DC, Alaska, USA., October 2006.

[16] J. Liu, D. Goeckel and D. Towsley, "Bounds on the Gain of Network Coding and Broadcasting in Wireless Networks," *in Proc. IEEE INFOCOM*, pp. 724-732, Anchorage, Alaska, USA., May 2007.

[17] C. Zhang, Y. Fang and X. Zhu, "Throughput-Delay Tradeoffs in Large-Scale MANETs with Network Coding," *in Proc. IEEE INFOCOM*, pp. 199-207, Rio de Janeiro, Brazil, April 2009.