# Project 2: An Optimal Pricing Mechanism in Cognitive Radio Network With Congestion Control

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***Abstract*—In this paper, I design a novel pricing mechanism for primary spectrum operator(PSO) in cognitive radio network to cope with the heterogeneous types of secondary users(SU) and their strategic behavior in CDMA sense. In cognitive radio networks, how to pricing a spectrum usage, particularly transmission power in the CDMA channel in my work, for secondary users is an essential problem to solve. However, all the previous works on this topic only consider the linear pricing mechanism and thus hamper the possibility of further extraction of SU's value, or generate profits. My work now extends the pricing function to non-linear pricing mechanism and form a quasi-Stackelberg Game between SU and PSO. I indeed notice that there are many works that utilize Stackelberg Game as an approach towards the pricing mechanism. However, my works is significantly different from the previous ones because they all fall into the traditional follower model and use the backward induction to derive the equilibrium solution of this game. My work, however, gives the PSO a positive position in pricing by using the non-linear pricing and help PSO achieve the desired, but rational, profits in the presence of selfish and strategic SUs. What's more, my work also considers the heterogonous QoS requirement for users and the impact of congestion in the whole network. My work give the result of PSO's optimal profits in this game and the pricing mechanism he should use to achieve it despite of the selfishness of SUs. Simulation result shows that the revenue generated by my mechanism is 40% larger than the previous Stackelberg Game approaches.**

## I. Summary and Recent Work Report

This report summarize a result of my work in the last semester. I think this work is worth developing because it is connected to the pricing mechanism of transmission power in cognitive radio network. And the performance of my mechanism is shown to be with significant improvement towards the original works. I believe that this work will be appreciated by the cognitive service provider because I give them a new mechanism to make more money

As I illustrated in the last section of this report, complete information is a too strong assumptions and the simulation is not extensive. Therefore, I know it can only be a project-level report for our class.

My recent work is not this, however, because I find it is hard to extent to incomplete or asymmetry information networks. My recent work are:(1) Consider the effect of the time-varying QoS of SU in the pricing and auction mechanism. All the previous works assume SU's QoS of the service is static with time. However, the QoS may change due to the mobility of SU, the presence PU and some other factors. I want to incorporate this time-varying property of QoS in the modeling of auction or pricing game. I believe this will be a mile-stone level work in CR if I can finish it. I am now working on a proper mathematical model to define the time-varying behavior.(2)The k-connectivity of the cognitive radio network is a fundamental question of the CR and I am also very interested in this.

## II. Introduction

**NOTE: This is a brief report for the this problem. Though the motivation and mean technical parts are illustrated, some trivial and time consuming proofs are omitted and all citation works are also omitted.**

With the ongoing growth in wireless communication services, the demand for radio spectrum increases dramatically. However, the spectrum resource is limited and most of them has already been licensed to existing operators. Former studies show that the actual licensed spectrum remains unoccupied for large periods of time []. Thus, cognitive radio (CR) networks were proposed [] in order to efficiently exploit these spectrum holes and distribute the spectrum resource among the secondary users (SUs). In cognitive radio networks, how to pricing a spectrum usage, particularly transmission power in my work, for secondary users is an essential problem to solve. CITATIONS OMITTED.

However, all the previous works on this topic only consider the linear pricing mechanism and thus hamper the possibility of further extraction of SU's value, or generate profits. My work now extends the pricing function to non-linear pricing mechanism and form a quasi-Stackelberg Game between SU and PSO. I indeed notice that there are many works that utilize Stackelberg Game as an approach towards the pricing mechanism. CITATIONS OMITTED However, my works is significantly different from the previous ones because they all fall into the traditional follower model and use the backward induction to derive the equilibrium solution of this game. My work, however, gives the PSO a positive position in pricing by using the non-linear pricing and help PSO achieve the desired, but rational, profits in the presence of selfish and strategic SUs. What's more, my work also considers the heterogonous QoS requirement for users and the impact of congestion in the whole network. My work give the result of PSO's optimal profits in this game and the pricing mechanism he should use to achieve it despite of the selfishness of SUs. Simulation result shows that the revenue generated by my mechanism is 40% larger than the previous Stackelberg Game approaches.

My work makes the following contributions:

- This mechanism utilize the non-linear pricing into the transmission power pricing mechanism of cognitive radio network under CDMA network settings and thus use a more realistic mechanism for the PSO to lease spectrum. In our work, the PSO can firstly derive his optimal revenue by only knowing the user type of secondary users and then design a non-linear pricing mechanism to achieve this desired profit. The previous work all assume the iterative actions of PSO and SU and this assumption is unrealistic in real network scenarios.

- This mechanism consider the different type of SUs' QoS requirement under this network settings and also takes into account the congestion effect of secondary users network. And this assumption render this work more technical complexity and more practical value to be implemented in the real network.
- I prove that the optimal solution of the revenue the 40% higher than the traditional Stackelberg Game approach. What's more, I also notice that due to the indifference effect of the users choice, the optimal solution can not be fully achieved but can only be approximate with any arbitrary approximation rate.

## III. System Model and Problem Formulation

### A. Network Setting, SUs' Utility Function and PSO's Revenue

We consider a cognitive radio network under CDMA sense and incorporate the concept of primary spectrum operator(PSO) into our model. The primary spectrum operator in our network setting acts as an agent of the primary users. He collects all the spectrum opportunity exist in the network and lease them to the secondary users. This network setting of the CR is almost a widely accepted framework for the real implementation of the CRN. CITATION OMITTED. We assume that the whole system works in an CDMA manner. We assume the total power capacity of the cognitive radio network is $P$ and the PSO will divide the power capacity to N SUs denoted as $SU = \{SU_1, SU_2, \cdot, SU_N\}$. To illustrate the problem more clearly, we need to define two concept: SU's utility and the PSO's revenue.

**Definition 1.** The utility of $SU_i$ is defined as follow:

$$U_i(p_i, p_{-i}, c_i) = T_i \log(1 + p_i) - \frac{1}{P - p_i - p_{-i}} - c_i(p_i) \quad (1)$$

where the $T_i$ indicates the type of SUs, $p_i$ is the transmission power allocated to the $SU_i$, $p_{-i} = \sum_{j=1}^{N} p_j - p_i$ and $c_i$ is the price function of $SU_i$ indicating how much he needs to pay in order to get the service.

**Remarks:** In definition 1, the utility is consist of 3 part. The first term use log arithmetic utility function, which is a widely used utility function, to indicate the raw utility of the service received by $SU_i$ and the multiplied factor $T_i$ illustrate the heterogenous property of SUs. The second term is called congestion control factor. In this network model, we assume that the whole power gap could not be fully utilized because this will cause severe interference to the PU when they get back to the network and the network cannot cater to time-varying usage requirement. Tough hold this factor is rational in piratical view, this model introduce inherent coupling effect in the system and thus introduce more technical complexity. The third term is cost of the usage of $SU_i$ and this function is not fixed for all users but varies by PSO pricing design mechanism. Additionally, I indeed notice that this model is actually rather deviated from the physical layer but I will still use this model because it is a good abstraction of the actually utility function. And because this is only a project.

**Definition 2:** The revenue of the PSO is defined as $\sum_{i=1}^{N} c_i(p_i)$.

**Remarks:** Though the definition of the PSO's profits is simple, it is critical to notice that this $c_i$ is a pricing function of $p_i$. What's more, different from the previous pricing mechanism, where $c_i = \alpha p_i$ is a linear function, my definition give the pricing function an important flexibility to be non-linear function of $p_i$ and thus have many surprising effects.

### B. Pricing Design Game—The Quasi-Stackelberg Game

In this project report, I will firstly use the complete information assumption that all the SU's $T_i$ is known to the PSO. Though it is a strong assumption, but it is easy to convey my mean idea and the extension to incomplete information or even asymmetry incomplete information is not hard due to the existing extension framework from complete information game to incomplete information game(The stochastic distribution of $T_i$ is known to PSO).

In this network, while each SU tries to maximize his utility by choosing how much transmission power he wants to buy($b_i$) from PSO, the PSO's task is different from the previous work where the PSO acts negatively by using backward induction to figure out the optimal pricing. In my mechanism, PSO can act actively to *design* a set of pricing function for every kind of users and maximizing his own revenue. In another word, the PSO tries to induce the SUs to buy the amount of goods that can maxmizing his own revenue. However, there are many coupling effects in it and I will firstly give the formal definition of this game.

**Definition 3.** The mentioned quasi-Stackelberg Game between SUs and PSO is defined as follow. The player set of this game is $N$ SU and the single PSO. The out come of this game is denoted as $R = \{(p_1^r, c_1^r), (p_2^r, c_2^r), ..., (p_N^r, c_N^r)\}$, where the tuple $(p_i^r, c_i^r)$ means that for $SU_i$, he buys $p_i^r$ transmission power from the PSO and pays $c_i^r$.

The PSO's objective is to maximizing his own revenue of this game by achieving the outcome fits the following constraint optimization problem:

$$\{(p_i^r, c_i^r)\}_{i=1}^{N} = \arg \max_{p_i \geq 0, \sum_i p_i < B, c_i \geq 0} \sum_{j=1}^{N} c_j \quad (2)$$

$$s.t U_i(p_i, p_{-i}, c_i) \geq U_i(0, p_{-i}, c_i) \quad (3)$$

Assume that PSO can find this solution and the next step for him to do is to give different SU a different pricing policy so that he can a achieve this desired profit. Considering the SUs are all selfish and only wants to maximizing their own utility so that the pricing mechanism must fit the following equations to achieve the optimal solution $R$.

$$\arg \max_{0\le p_i \le P-p^r_{-i}} U_i(p_i, p_{-i}, c_i) = p^r_i, \quad (4)$$

$$c_i(p^r_i) = c^r_i, \quad (5)$$

$$c_i(0) = 0. \quad (6)$$

## IV. PSO Profit Optimization with a Asymptotic Solution

In this section, we will examine the possible optimal profit can be achieved by the PSO in this network settings. Note that we can denote the power threshold of the whole CDMA channel as $P = Np_c$. The rational behind this assumption is that the provider will increase the total spectrum opportunity linearly as the total user grow. For analytical clearance, we will here set the $p_c$ to be 1 and conduct the analysis in the specific situation. Therefore we have $P = N$ for all the equations above.

### A. Decomposition of the Optimization Problems

**Theorem 1.** The optimization problem illustrated in (2) and (3) is equivalent to the following two steps: First, we can get the optimal power by solving:

$$\{p^r_i\}^N_{i=1} = \arg \max_{p_i\ge 0, \sum_{j=1}^N p_i \le N} S(p_1, p_2, ..., p_n), \quad (7)$$

where $S(p_1, p_2, ..., p_n) = \sum_{j=1}^N \hat{F}(p_i, p_{-i}, T_i)$, and $\hat{F}(p_i, p_{-i}, T_i) = T_i \log(1 + p_i) - \frac{1}{N-p_i-p_{-i}} + \frac{1}{N-p_{-i}}$. Then the optimal pricing can be obtained from

$$c^r_i = \hat{F}(p^r_i, p^r_{-i}, T_i) \quad (8)$$

*Proof:* OMITTED.

### B. Solution of the Asymptotic Optimal Transmission Power Set

In this part, we assume $N$ to be sufficiently large and the rationality behind this assumption is that:(1) As we mentioned that the provider will provide the spectrum opportunity proportional to the SUs;(2)For any arbitrary N there can not guarantee and explicit solution for this.

Firstly, we assume there is a solution to (7) and thus the derivative of $\hat{F}$ must fit:

$$\hat{F}_{p_i} = \frac{T_i}{1 + p_i} - \frac{N}{(N - \bar{x})^2} + \sum_{j\ne i} \frac{1}{(N - p_{-j})^2} = 0 \quad (9)$$

where $\bar{p} = \sum_{j=1}^N p_i$.Note that the $\bar{p}$ is a function of $N$. And suppose there is two constant $(\alpha, \beta)$, we have

$$\lim_{n\to\infty} \frac{N - p(\bar{N})}{N^\alpha} = \beta \quad (10)$$

Then we have the asymptotic solution with (9) and (10). Then, we denote $T_{av}(N) = \sum_{j=1} NT_i/N$ and assume that $\sum_{j=1} NT^2_i/N$converge to a limiters as $N \rightharpoonup \infty$.

I will now omitted the proof that the (10) hold only when $0 < \alpha \le \frac{1}{3}$ and $\beta > 0$. We now give another assumption that $\lim_{N\to\infty} \frac{p_i(N)}{N-p(N)=0}$, this seems to be an irrational assumption but I will show that the solution actually satisfies this assumption. Then we have: $\lim_{N\to\infty}(N - p(\bar{N}))^{-2} = \lim_{N\to\infty}(N - p(\bar{N}))^{-2}(1 + \frac{p_i}{N-\bar{p_i}}) = 0$ Thereore, from (9) we have: $\frac{T_i}{1+p_i(N)} \sim \frac{T_j}{1+p_j(N)} \sim \frac{nT_{av}}{N+\hat{p}(N)}$ and we got:

$$p_i(n) \sim \frac{T_i}{T_{av}}(1 + \frac{\bar{p}}{N}) - 1, i \in N \quad (11)$$

Combing (10),(11) and(9), we use the Taylor Series expansion and have $\beta^{-2}N^{1-2\alpha} - T_{av}/2 \sim \sum_j(\beta N^\alpha + \frac{2T_i}{T_{av}-} - 1)^{-2} = \beta^{-2}N^{1-2\alpha} - 2\beta^{-3}N^{1-3\alpha} + o(N^{1-3\alpha})$ . Therefore we got $\alpha = \frac{1}{3}$ and$\beta = 4^{\frac{1}{3}}T_{av}^{-\frac{1}{3}}$. That is to say the optimal solution is as follow:

$$p(\bar{N}) \sim N - 4^{\frac{1}{3}}T_{av}^{-\frac{1}{3}}N^{\frac{1}{3}} \quad (12)$$

$$p_i(N) \sim \frac{2T_i}{T_{av}} - 1 - 4^{\frac{1}{3}}T_{av}^{-\frac{4}{3}}T_iN^{\frac{-2}{3}} \quad (13)$$

if and only if:

$$T_i > \frac{T_{av}}{2} \quad (14)$$

Then the solution satisfies the assumption that $\lim_{N\to\infty} \frac{p_i(N)}{N-p(N)=0}$.

**Theorem 2** The solution from (12)-(14) is the optimal solution of the revenue generation problem of PSO.

*Proof:* I can prove that this unique positive solution fits first-order condition is the global optimal solution. However, the proof will be time-consuming and will be *omitted*.

### C. Pricing Policy Design to Achieve the Optimal Revenue

Now for the profit maximizing problem depicted in the beginning, we have solve the first phase that is to calculate the optimal power usage of SUs to get the optimal profits for the PSO. Now we will move to the second phase of this optimization problem to get the actual optimal revenue of the PSO. However, this is rather simple because we can directly use equation (8) to get the revenue.

$$c^r_i(N) \sim T_i \log(2T_i/T_{av}) - (4^{\frac{1}{3}}T_iT_{av}^{-\frac{1}{3}} + 4^{-\frac{2}{3}}T_{av}^{\frac{2}{3}})N^{-\frac{2}{3}} \quad (15)$$

Then the total profit can be evaluated as

=$\sum_{i=1}^N r_i$(16)Using the Taylor expasion we have

$$\hat{c}^r \sim \sum_{j=1}^N T_j \log(2T_i) - 3(4^{-\frac{2}{3}}T_{av}^{\frac{2}{3}})N^{\frac{1}{3}} + T_{av}^{-1}\sum_j T^2_i/N - 3T_{av}/4 \quad (17)$$

Considering the $\sum_{j=1}^N 2T_j/T_{av} = 2N$, we have

$$\hat{c} \sim \sum_j T_j \log(\frac{2T_j}{T_{av}}) \ge T_{av}(\log 2)N \quad (18)$$

where the quality holds iff $T_i = T_av$.

Now, we already have the optimal revenue the PSO can achieve for the amount of N transmission power. However, the above discussion only consider the optimal solution itself but not mention how to achieve the solution. However, whether this revenue is achievable or not is a critical question to solve.

Because all the users are strategic and will not follow any compel or order, the PSO must design a proper pricing policy for different type of users and thus can induce the users to converge to the optimal revenue solution-$R$.

By (6),(5)and (8) we have that

$$U_i(0, p^r_{-i}, c(0)) = U_i(p^r_i, p^r_{-i}, c(p^r_i)) \tag{19}$$

Therefore, SUs will be indifferent to these two choices and there is a possibility that SU will not participate in this game. No participant means no profits for PSO. As a result, the optimal solution cannot be guaranteed. Fortunately, I discover that if the PSO can make a slight compromise on the profit such that substituting $(p^r_i, c^r_i)$ with $(p^r_i, c^r_i - \delta)$ where$\delta$ is an arbitrary small positive number. Then the $(p^r_i, c^r_i - \delta)$ become the unique utility maximizing point for $SU_i$ and thus he will stick to this point. Therefore, the optimal profit of PSO can be achieved with arbitrary approximation.

Now I will give an specific example of $c_i$. Note that  is a strictly concave function, there is no linear function $c_i$ such that $c_i$ goes through $(0,0)$ and $(p^r_i, c^r_i)$. This means that the all the previous works concerning the Stackelberg Game using the linear pricing cannot give a solution that is optimal solution in my paper. Considering the fact that if $p^\star_i$ fits $\hat{F}_{p_i}(p^\star_i, p^r_{-i}, T_i) = 0$ than $\hat{F}$ will reach the global maximum. We can let $c^r_i$take the value of $\hat{F}(p^r_i)$ everywhere except $c_i(0) = 0$ and $c_i(p^r_i) = c^r_i$ and this pricing will induce the SUs to choose the optimal solution for PSO.

## V. Evaluation of the Proposed Mechanism

With the optimal linear pricing, the PSO can achieve a very close approximation of the optimal revenue. Now I will compare the total profit of my mechanism with that of the traditional linear pricing mechanism. The optimal revenue of the tractional Stackelberg Game with linear pricing can be written as follow:

$$\hat{c}^l \sim T_{av}N - \frac{1}{2}w^2 N \tag{20}$$

where $w = \sum_j T_j^{\frac{1}{2}}/N$

It can be proved that $\bar{c}^r > \bar{c}^l$ and when the $T_i = T_{av}$ for all $i$, the improvement of our non-linear pricing mechanism towards the original linear optimal one is 38.2%. This proof is also omitted because of the time being. And we anticipate that in the presence of the heterogenous SUs the improvement can be larger because the non-linear pricing reflect the heterogeneity much better than the linear one. However, numerical solution is not done yet.

## VI. Extension to non-complete information game(Not solved totally)

In the previous sections, we talked about the how to apply the non-linear incentive based scheme to a incomplete game. It is not likely that the PU knows complete information about the SU network because the SU network is highly volatile with time and space. We assume the PU can only know the distribution of the SUs' type but not the exact information about the SUs network. We now assume that we can classify SU into m discrete types. And SU $i$ is of type $T^l_i$ for $' \in 1, 2, 3...., m$ with probability of $q_l > 0$. And $\sum_{l=1}^m q_l = 1$. We thus deal here with a discrete distribution. Let us assume, without loss of any generality, that $T^1 > T^2 > ..... > T^m > 0$. With incomplete information, the service provider only knows this distribution, but not the users' true types. Furthermore, we assume that each users' true type is private information to himself such that the other users only knows the distribution too.

### A. Incomplete Information Game Problem Formulation

With incomplete information, the PSO's objective is to maximize the expected total profit. Also, he cannot have price discrimination fro different users according to their true types. Thus, he should have the same pricing policy, , for any users, which consist of m optimal flow-charge pairs, one pair for each possible user type:

$$\{(x^{lt})\}_{l=1}^m = \max_{0 \le x^l, r^l \ge 0} {}_{l=1}^m n \sum_{l=1}^m q_l r^l \tag{21}$$

However, this problem is very hard to solve and not be solved yet.

## VII. Conclusion and Future Work

This paper give a pricing mechanism for the PSO as an active participant in the pricing game. In our mechanism, the PSO can firstly calculate the optimal revenue he can achieve in this network and than design a non-linear pricing policy to induce SUs to choose the optimal solution. This mechanism has significant improvement to the previous pricing mechanisms.

However, the complete information assumption is a too strong assumption of this paper. In practical view, it is impossible for PSO to know all the SU's QoS requirement. Therefore, if the mechanism can extend to the incomplete information scenario, it will be much more valuable.