Analysis of Reliable Distributed Data Fusion Structure in Collaborative Spectrum Sensing in Cognitive Radio Networks: A Network Formation Game Approach

Haiming Jin (5080309195)
Dept. of Electronic Engineering
Shanghai Jiao Tong University, China
Email: jinhaiming@sjtu.edu.cn

***Abstract*—By virtue of cooperative sensing of PUs' channels in a cognitive radio network, SUs are capable to enhance their sensing performance significantly. This is because cooperative sensing can yield desirable results in dealing with the inherent trade-off between spectrum sensing and spectrum access. One specific scheme for cooperative spectrum sensing in CRNs is the coalitional game based cooperative sensing in which SUs form coalitions before they sense the PUs' channels cooperatively. Usually, such a network structure involves the fusion of sensing bits within a coalition on a common control channel. However, in the scenario of cognitive Ad hoc networks, it is unjustifiable to assume the existence of such a common control channel. As a result, for the purpose of sensing result fusion, cognitive Ad hoc SUs have to set up communication links with other SUs within the coalition. In this report, we calculate the most reliable data fusion structure ie. one that minimizes reporting error of sensing bits, within a coalition. Then, by formulating the problem as a network formation game, we try to ferret out the result fusion structure that cognitive Ad hoc SUs will seek to form distributedly and voluntarily while maximizing their individual reliability of sensing bits collection. Moreover, according to the analysis provided in this report, the two kinds of network structures are both star-like structures in which one SU acts as the SU and the other SUs are in the peripheral positions.**

I. Introduction

The recent burgeoning growth of the number of wireless devices has yielded huge requirements for radio spectrums. Consequently, the problem of spectrum scarcity arises under the current spectrum allocation framework where spectrums are assigned statically to licensed users or Primary Users (PUs). According to the measurement conducted by FCC [1], only 5% to 15% of these spectrums have been efficiently utilized. Thus, in order to solve the spectrum scarcity for increasing numbers of Secondary Users (SUs), the notion of cognitive radio has been proposed [2]. In cognitive radio networks, a SU is designed to be sensitively aware of its surrounding spectrum environment so as to enable the opportunistic access of PUs' idle spectrums. Yet, the implementation of cognitive radio networks faces several challenges [3], one of which is effective spectrum sensing. That is SUs have to consistently sense PU spectrums in order to detect spectrum holes and dynamically access PUs' channels without causing severe interference to PUs' transmission. Therefore, it is imperative to establish a sound and robust strategy to conduct the spectrum sensing process in cognitive radio networks.

To enable the process of spectrum sensing, various detection methods have been proposed in [4] including match filter detection, energy detection, and feature detection. Coherent detection non-coherent detection can be respectively performed by match filter detection and energy efficient detection. Moreover, feature detection exploits the periodicity inherent in the received signal to detect primary signals with a specific modulation pattern.

Of all the aforementioned detection methods, limited by current technology of radio devices, it is impossible for cognitive radios to conduct the process of spectrum sensing and spectrum access at the same time. Thus, the time-slotted periodic sensing and transmission model has been widely discussed. In such frameworks, an inherent trade-off between sensing time and access time should be considered. An increase of the former will enhance the SU's knowledge of PU channels and reduce the potential interference between PUs and SUs. And apparently, a longer transmission time will contribute to a larger throughput for SUs. Early works of collaborative spectrum sensing [5] [6] and [7] provide some Detection, Location estimation, and transmit-power estimation for smallscale primary users. insight in solving this trade-off problem, because by sharing sensing knowledge, SUs in the CR network can significantly reduce their sensing time but still acquire relatively exhaustive knowledge about the conditions of PUs' channels. Therefore, SUs will be able to improve their transmission time without incurring serious interference for the PUs. Additionally, degradation of SUs' sensing performance can be alleviated to a great extent by collaborative sensing [7]. However, enlightened by [8] and [9], the presence of a central controller in [5] [6] and [7] results in undesirable consequences for the CR network, e.g. large sensing overhead. What's more, as illustrated in [10], sensing results such as channel side information (CSI) can also help determine SUs' choices of transmission power in the transmission period of any arbitrary time slot.

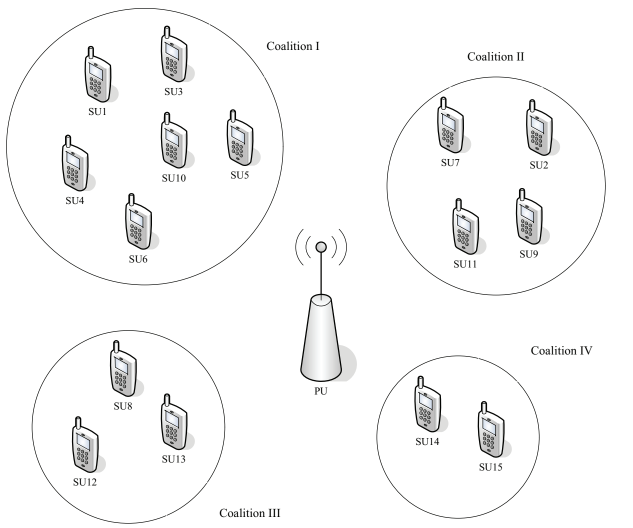

Fig. 1. **Illustration of Coaltional Network Structure:** SUs form 4 coalitions i.e. {1,3,4,5,6,10},{2,7,9,11},{8,12,13} and {14,15} to cooperatively sense the PU's channels

One specific scheme for cooperative sensing in cognitive radio networks is the implementation of coalition structures proposed by Walid Saad in [8] and [9]. However, the implementation of such structures involves the process of local result fusion over a common control channel. Such assumption of the existence of a common control channel is unjustifiable in the circumstance of cognitive Ad hoc networks where SUs have to establish communication links directly between each other to share the sensing bits. Hence motivated by this problem, we analyze the reliable data fusion structure and try to characterize the network structure that SUs will try to seek to form distributedly and voluntarily. The main contributions of this report are as follows:

- The unique efficient network structure is the symmetric star where the hub SU invests an equal amount in all links with peripheral SUs.
- A strongly pairwise stable network in our model must be a star in which all players are connected.

The rest of this report is organized as follows: in section II, we discuss the related works on the topic of cooperative sensing in cognitive radio networks. In section III, we introduce the system model and the problem formulation. In section IV, we present the major results based on our study utilizing the approach of network formation game and demonstrate related proofs. In section V, hopefully we will be able to present of simulations in the work of the subsequent weeks.

II. Related Work

Extensive work has been done to delve into the problem of spectrum sensing in cognitive radio networks to deal with a wide range of problems in the cooperative or non-cooperative sensing process.

Authors in [11], utilizes spectrum network coding as a spectrum shaper in a CR network to enhance spectrum discovery for non-cooperative sensing, where adaptive channel sensing is carried out by dynamically updating the list of the PU channels that are predicted by the SU to be idle. In [5], the authors analyze channel probing in wireless networks based on the recursive Bellmen equation to maximize the throughput of a wireless sensor. And in the authors' subsequent work [10], they take into the consideration the effect of the parameter of transmission power on the overall throughput of the wireless sensor and establish correlations between the detected channel CSI with the transmission power. However, the optimal solution based on convex-maximization cannot be solved within polynomial time. Therefore, the authors also propose a near optimal on-line strategy which is has the computational complexity of $\Theta(N)$. Although [5] and [10] are not carried out under the setting of CR networks, the gist of the strategy of channel CSI probing and power allocation can be utilized in SUs' cooperative sensing of PUs' channels in CR networks. In [12], the authors propose an energy-efficient scheme for distributed spectrum sensing (DSS) in CR networks. In this work, a cluster-and-forward based DSS scheme is proposed by developing a two-tier hierarchical CR network. Moreover, the authors of [12] propose in [13] a DSS scheme to deal with two security issues, the Incumbent Emulation (IE) attacks and Spectrum Sensing Data Falsification (SSDF) attacks in CR networks. Furthermore, authors of this paper [14] offer some insight into the sensing framework in CR networks that can carry out.

In [8] and [9], the authors unprecedentedly incorporate the method of coalition game into solving the problem of cooperative sensing in CR networks. Specifically, in [9] the authors model the problem as a coalition formation game in partition form to deal with the trade-off between sensing time and access time of SUs in cooperative sensing. In this framework, a SU can take individual distributed decisions to join or leave a coalition maximizing its utility that accounts for the average sensing time and the resulted capacity while accessing the detected spectrum. In [8], the authors utilize a non-transferrable coalition game to deal with the trade-off between increasing the detection probability with reducing the false alarm probability. These aforementioned two works offer valuable insights in modeling cooperative sensing in CR networks as a coalitional game approach.

III. System Model and Problem Formulation

In the scenario of cognitive Ad hoc networks, the local result fusion structure proposed by [8] for utilizing a common control channel within a coalition is impractical, because Ad hoc SUs need to establish communication links directedly among each other to transmit sensing bits. Moreover, limited by the resources and conditions of report channels between any arbitrary two SUs in this cognitive Ad hoc network, SUs will selectively choose to build communication links with other SUs to carry out local sensing result fusion within a coalition. Thus, in this report we assume that all the SUs in the current cognitive radio network seek to cooperatively sense the PUs' channels in a coalitional structure similar to that in [8] demonstrated in Figure 1 in which SUs form coalitions to cooperatively sense the PUs' channels. Our focus is the structure of the most reliable network structure for sensing result fusion within a coalition.

In this report, we formulate the local communication link construction procedure as a network formation game in which SUs are players that seek to maximize their own reliability of sensing bits collection. We assume that in any arbitrary coalition $\mathcal{S}$, there are N cogitive Ad hoc SUs constituting a SU set $\mathcal{N} = \{1, \cdots, N\}$. Then in definition 1, we formally define the network formation game with heterogeneous link strength.

Definition 1: A network formation game with heterogeneous link strength can be defined as a tuple $\mathcal{G}(\mathcal{P}, \mathcal{S}, \mathcal{U})$, where $\mathcal{P}$ denotes the set of players in this game, $\mathcal{S}$ denotes the strategy space of all the players in which $S_i \in \mathcal{S}$ is the strategy space of a particular player i and $\mathcal{U}$ is the set of utility functions for the players where $u_i \in \mathcal{U}$ is the utility for player i. In this game, any arbitrary player $i \in \mathcal{P}$ selectively determine its limited investment I_i based on a common criterion γ. The outcome of the game is a weighted graph, $G(\mathcal{V}, \mathcal{E}, \mathcal{W})$ in which the vertex set $\mathcal{V}$ is the player set $\mathcal{P}$, the edge set $\mathcal{E}$ is the set of the communication links formed between any two players in this game and $\mathcal{W}$ is the set of weights for each edge that has been formed, which is in direct relationship with the investment players invest in the links they form.

In this report, it is obvious that the player set $\mathcal{P}$ is actually the SU set $\mathcal{N}$ i.e. $\mathcal{P} = \mathcal{N}$. For simplification of analysis, we assume that every SU has fixed and homogeneous power budget P used to establish the communication links with other SUs and report the sensing bits among them in order to share the sensing results. Although it is intuitive to denote the power budget of any arbitrary SU as its investment in the establishment of its communication links with other SUs, we seek to model the investment in an alternative manner which is related to the correct reporting probability between any two arbitrary SUs.

Definition 2: Given the circumstance with Rayleigh fading environments and BPSK modulation of reporting sensing bits among SUs, the probability of correct reporting from SU i to SU j can be defined as [15]:

$$\begin{aligned} I_i^j = P_{c,i,j}(P_{i,j}) &= \frac{1}{2}\left(1 + \sqrt{\frac{\bar{\gamma}_{i,j}}{1+\bar{\gamma}_{i,j}}}\right) \\ &= \frac{1}{2}\left(1 + \sqrt{\frac{P_{i,j}h_{i,j}}{\sigma^2 + P_{i,j}h_{i,j}}}\right) \end{aligned} \tag{1}$$

where $\bar{\gamma}_{i,j} = \frac{P_{i,j}h_{i,j}}{\sigma^2}$ is the SNR in the report channel from SU i to SU j, $P_{i,j}$ is the transmission power that is utilized by SU i to report sensing bits to SU j and $h_{i,j}$ is the path loss between SU i and SU j.

Notice that in definition 2, for simplification of analysis, we assume that the correct reporting probability only depends on the reporting power $P_{i,j}$ from SU i to SU j, i.e. other parameters such as path loss $h_{i,j}$ and average noise power σ^2 are the same between any arbitrary two SUs. Then in definition 3, definition 4 and definition 5, we define the investment of SU i on the link it establishes to SU j, contribution function that maps the investment of SU i on the link it establishes from SU i to SU j and the reliability of a bidirectional reporting channel between SU i and SU j respectively.

Definition 3: The investment I_i^j of SU i on the link it establishes to SU j is defined as the correct reporting probability $P_{c,i,j}(P_{i,j})$ defined in definition 2, i.e. $I_i^j = P_{c,i,j}(P_{i,j})$, which is positively correlated with the reporting power $P_{i,j}$. As a result, the investment vector of SU i on all the links it establishes can be denoted as $\mathbf{I}_i$ and the overall investment vector consists of all the investment vector of all the SUs in the current coalition is denoted as $\mathbf{I}$. Moreover, we assume that every SU has the same total investment budget $\mathcal{I}$.

Definition 4: The contribution function mapping the investment $P_{c,i,j}$ of SU i in the communication link between SU i and SU j to its contribution on the reliability of this link is:

$$\varphi(P_{c,i,j}) = P_{c,i,j} - \frac{1}{2} \tag{2}$$

Definition 5: The reliability of a bidirectional reporting channel between SU i and SU j can be defined as:

$$r_{ij} = r_{ji} = \varphi(P_{c,i,j}) + \varphi(P_{c,j,i}) \tag{3}$$

Since only bidirectional communication links are meaningful in the cognitive Ad hoc network that we consider, a link is established is established between SU i and SU j if and only if $\varphi(P_{c,i,j}) > 0$ and $\varphi(P_{c,j,i}) > 0$ hold simultaneously.

In that the investment $P_{c,i,j}$ defined in definition 3 is positively correlated with the reporting power $P_{i,j}$, the reliability of a bidirectional link defined in definition 5 is also positively correlated with the reporting power $P_{i,j}$. Hence, SUs can increase the link reliability by increasing the power allocation in the establishment and maintaining of the particular link. Moreover, definition 4 and definition 5 together ensure several properties that are desirable in this framework including the linearity of φ and the fact that $r_{ij} \in (0, 1)$. Then in definition 6, we define connectedness between two SUs and the related path connect these two SUs.

Definition 6: A path between SU i and SU j is a sequence of $M(M \leq N)$ SUs $p(i,j) = \{i, i_1, \cdots, i_{M-2}, j\}$ satisfying that between any two adjacent SUs along the path bidirectional communication links have been established.

Between any two connected SUs, the benefit that SUs can get from each other depends on the reliability with which SUs can access each others' information, that is the reliability of the entire path. Then in definition 7, definition 8 and definition 9, we define the reliability of a path $p(i,j)$ and the optimal reliability that SU i and SU j can be guaranteed from connectedness between these two SUs and the value of the entire resulted graph G.

Definition 7: The reliability of a path $p(i,j) = \{i, i_1, \cdots, i_{M-2}, j\}$ is defined as equation (4):

$$r(p(i,j)) = r_{ii_1} r_{i_1 i_2} \cdots r_{i_{M-2}j} \tag{4}$$

Definition 8: We denote $P(i,j)$ as the set of paths between SU i and SU j, the optimal reliability of path $p(i,j)$ such that $p(i,j) \in P(i,j)$ is defined as equation (5):

$$R(i,j) = r^*(p(i,j)) = \max_{p(i,j)\in P(i,j)} r(p(i,j)) \tag{5}$$

Definition 9: The utility of any arbitrary SU i given a resulted graph G and the value of this particular graph can be denoted respectively in equation (6) and equation (7):

$$U_i(G) = \sum_{j\neq i} R(i,j) \tag{6}$$

$$V(G) = \sum_i U_i(G) = \sum_i \sum_{j\neq i} R(i,j) \tag{7}$$

In that we formulate the problem as a network formation game in this report, it is necessary to introduce the definition of network efficiency and definitions equilibrium network structure incorporated in this report.In definition 10, definition 11 and definition 12, we provide several related definitions.

Definition 10: A resulted network structure, representing by the graph $G(\mathcal{V},\mathcal{E},\mathcal{W})$ is efficient if for any arbitrary graph $G^{'}(\mathcal{V},\mathcal{E}',\mathcal{W}')$ the condition $V(G) > V(G')$ holds.

Definition 11: Given any overall investment vector $\mathbf{I}$, the graph $G(\mathbf{I})$ is Nash stable if there exists no SU i and investment vector $\mathbf{I}_i^{'}$ such that $U_i(G(\mathbf{I}_{-i},\mathbf{I}_i^{'})) > U_i(G(\mathbf{I}))$

Definition 12: A graph $G(\mathbf{I})$ is pairwise stable if it is Nash Stable and there is no pair of SUs (i, j) and joint deviation $(\mathbf{I}_i^{'},\mathbf{I}_j^{'})$, such that $U_k(G(\mathbf{I}_{-i,-j},\mathbf{I}_i^{'},\mathbf{I}_j^{'}) > U_k(G(\mathbf{I}))$ where $k = i,j$.

As defined in definition 12, a Nash stable graph is strongly pairwise stable if no pair of SUs can both be strictly better off by changing their investment strategy. Consequently given those aforementioned definitions, we can formally describe the problem formulation in this report.

O1: We seek to calculate the network structure that maximizes the reliability of sensing bits reporting within the network. That is $G^*(\mathcal{V},\mathcal{E},\mathcal{W})$ satisfying:

$$G^* = \arg\max_G V(G) \tag{8}$$

O2: We seek to ferret out the equilibrium i.e. Nash-stable network structure $G^{*^{'}}(\mathcal{V},\mathcal{E},\mathcal{W})$ given that SUs strategically allocate their investment budgets in the reporting channels they establish with other SUs. If such network structure exists and it is unique, we can then arrive at the conclusion that SUs will distributedly and voluntarily form this kind of network structure.

IV. Results of Efficient and Reliable Network Structure

A. *Efficient Network Structure*

In this subsection, we introduce our results about the characteristics of the efficient the network within a coalition to carry out local result fusion. Before we present the major results in this subsection, we introduce several lemma 1 and lemma 2.

Lemma 1: If the contribution function φ is convex, then unique efficient network is a star where one SU, namely the hub, is connected to all other players and peripheral SUs are only connected with the hub.

Proof: Firstly, we relax our condition and assume that a link is successfully constructed between any arbitrary two SUs, SU i and SU j, as long as $\varphi(P_{c,i,j}) \geq 0$ or $\varphi(P_{c,j,i}) \geq 0$. Later, in our proof, we will see that actually the result of the efficient network structure is a structure where both $\varphi(P_{c,i,j}) > 0$ and $\varphi(P_{c,j,i}) > 0$ hold. We consider a connected component H of G of size M. Thus in H, the total amount of investment is $M\mathcal{I}$. The proof of lemma 1 follows the following two stage.

- **Stage I**
  We demonstrate that it is able to construct a star S with higher overall utility than H, whenever H is not a star.
- **Stage II**
  We demonstrate that it is always possible to construct a single connected star with higher overall utility than the graph G when G is constituted by several disconnected stars.

For proof of **stage I**, we assume that in component H, the ordered sequence of the reliability of all its links is $\{r_1,\cdots,r_M\}$, where $r_1 < \cdots < r_M$. Then we construct a star S by choosing an arbitrary SU within H as the hub. Without loss of generality, we assume the hub to be SU m and the reliability of all the $m-1$ links are $r_1, r_2, \cdots, \sum_{k=m-1}^{M} r_k$ and the investment of all the SUs in star S can be defined as:

$$\begin{aligned} &I_i^m = \min\{\varphi^{-1}(r_i),\ \mathcal{I}\} \\ &I_m^i = \varphi^{-1}(r_i) - I_i^m \\ &\forall i = 1,\cdots,m-2 \end{aligned} \tag{9}$$

$$\begin{aligned} I_{m-1}^m &= \min\{\varphi^{-1}(\sum_{k=m-1}^{M} r_k),\ \mathcal{I}\} \\ I_m^{m-1} &= \varphi^{-1}(\sum_{k=m-1}^{M} r_k) - I_{m-1}^m \end{aligned} \tag{10}$$

If we use $s_i,\ i = 1,\cdots,m-1$ to denote the reliability of the $m-1$ links in the constructed star S. Then, obviously for $i = 1,\cdots,m-2$, the condition $s_i = r_i$ holds and for $i = m-1$, the condition $s_{m-1} \geq r_{m-1}$ is true. In star S direct reliability of communication links are exactly the same with those of the component H. And the overall indirect reliability of communication links are given by $V'(S) = 2\sum_{i\neq j\ i,j=1}^{m-1} s_i s_j$. Then we denote D as the set pairs of nodes that are not directly connected in H. Then we consider the case in which H is a tree and the case in which H is not a tree.

Suppose that H is a tree but not a star. For a pair of SUs $(i,j) \in D$, we use r_{t_i} and r_{t_j} to denote the reliability of the two terminal links in the path $p^*(i,j)$. Then, obviously for every pair $(i,j) \in D$, we have $r(p^*(i,j)) \leq r_{t_i} r_{t_j}$.Each pair of nodes in D is associated with a unique pair of terminal links and the number of terminal nodes that one can construct is C_{m-1}^2. Thus the indirect reliability in graph H, $V'(H)$ satisfies $V'(H) < 2\sum_{i\neq j\ i,j=1}^{m-1} r_i r_j = V'(G)$.

In the case that H is not a tree, we have the fact that the cardinality of D is strictly less than C_{m-1}^2. We can again associate a unique pair of terminal links to any pair of SUs such that $(i,j) \in D$. Then apparently, the condition that $V'(H) < 2\sum_{i\neq j\ i,j=1}^{m-1} r_i r_j \leq V'(G)$

The star constructed above might not be a feasible star because the resulted investment of the hub of the star is entirely possible to be larger than the investment budget of that SU,

$\mathcal{I}$. Hence, we need to construct a feasible star in which the investment of every SU is no larger than the predetermined investment budget $\mathcal{I}$.

Lemma 2: For the star S constructed in lemma 1, if the condition $I_i^m = \mathcal{I}$ holds, then $\sum_{i=1}^{m-1} I_m^i \leq \mathcal{I}$ i.e. the star S is feasible.

Proof: In a star constructed using the aforementioned method, if all the all peripheral players invest $\mathcal{I}$ on the link with the hub, SU m. Then, we have:

$$\begin{aligned} \sum_{i=1}^{m-1} I_m^i &= \sum_{i=1}^{m-2} \varphi^{-1}(r_i) + \varphi^{-1}(\sum_{k=m-1}^{M} r_k) - (m-1) \\ &\leq \sum_{i=1}^{M} \varphi^{-1}(r_i) - (m-1)\mathcal{I} \leq \mathcal{I} \end{aligned} \quad (11)$$

consequently, the star is feasible and we finish the proof of lemma 2.

If the star S constructed above is not feasible in that the hub SU invest more than $\mathcal{I}$ in establishing links with peripheral SUs. Then, we construct s star $\bar{S}$ such that $\bar{I}_k^m = \min\{\mathcal{I}, I_k^m + I_m^i\}$ and $\bar{I}_m^i = I_m^i - (\bar{I}_k^m - I_k^m)$. And let $\epsilon = \bar{s}_k - s_k$ and $\delta = s_i - \bar{s}_i$. Consequently, from the convexity of φ, we have $\epsilon \geq \delta$. Then we consider the difference between the indirect reliability in star $\bar{S}$ and star S:

$$\begin{aligned} & V'(\bar{S}) - V(S) \\ &= 2((\epsilon - \delta)\sum_{j \neq i,k} s_j + (s_k + \epsilon)(s_i - \delta) - s_k s_i) \\ &\geq 2(\epsilon s_i - \delta s_k - \epsilon\delta) \geq 0 \end{aligned} \quad (12)$$

So the overall indirect reliability of $\bar{S}$ is at least as high as that of S. If $\bar{S}$ is not feasible, then we can continue to transfer resources from the hub to some peripheral node to construct a feasible star. Now, we have finished the proof of stage I.

For proof of **stage II**, we consider two feasible stars S_1 and S_2 with size M_1 and M_2. Next, we will demonstrate that it is invariably possible to constuct a new star S^* with size $M_1 + M_2$ centered around the hub of S_2, say SU m_2 with the condition that $I_i^{m_2 *} = \mathcal{I}, \forall i \neq m_2$ and $I_{m_2}^{i\ *} = I_{m_2}^i$. By construction, we can see that the direct reliability of the communication links inside the star S_2 has not been changed. Then, we need to consider the change of indirect reliability of communication links due to the changes of the star S_1:

$$\begin{aligned} \Delta V'(S_1) = 2(s_1 - 1)\varphi(\mathbf{I})^2 - \\ \sum_{i,j \in S_1 \setminus m_1} \varphi(\mathbf{I})(\varphi(I_{m_1}^i) + \varphi(I_{m_1}^j)) - \\ \sum_{i,j \in S_1 \setminus m_1} \varphi(x_{m_1}^i)\varphi(x_{m_1}^j) \end{aligned} \quad (13)$$

and from the convexity of function φ, we can arrive at the conclusion that $\Delta V'(S_1) > 0$. Hence, the proof of lemma one has been finished.

Theorem 1: The unique efficient network structure is the symmetric star where the hub SU invests an equal amount in all links with peripheral SUs.

Proof: In the proof of lemma 1, we did not restrict the function φ to be strictly convex. So, we can conclude that lemma 1 is also valid when φ is a linear function. According to definition 4, φ is linear. As a result, the resulted efficient network structure is also a star.

Then we characterize the investment of the SU in the hub position. Suppose S^* is a star with hub at SU n in which all the communication links have a reliability of $\mathcal{I} + \frac{\mathcal{I}}{n-1}$ and S is another star with hub at SU n where the reliability might not be the same among different communication links, but also satisfies the condition $\sum_{i=1}^{n-1}(I_i^n + I_n^I) = n\mathcal{I}$. The direct reliability of communication links are also the same in star S^* and star S. Next we demonstrate that the stat S^* has a larger indirect reliability than the star S. Let s_{1n} and s_{2n} to be the weakest and strongest links in star S. We consider, then, the effect of increasing the investment on s_{in} by ϵ and decreasing the investment on s_{2n} by ϵ. Then the difference in the value of the star is $\Delta V(S) = 2(\epsilon(s_{2n} - s_{1n}) - \epsilon^2)$. For sufficiently small ϵ the value $\Delta V(S) > 0$ holds. This implies that the symmetric star in which the hub SU invests equally to all the links it established between peripheral SUs is the unique efficient network structure.

Notice that at the beginning of lemma 1, we assume that a link is established between two SUs as long as one of the contribution function is not less than 0. Up to now, we have been able to prove that the unique efficient network structure is a network where both players invest in the links between them, i.e. SUs establish meaningful bidirectional communication links.

B. Strongly Pairwise Stable Network Structure

In the previous subsection, we have demonstrated that the unique efficient network structure is a symmetric star in which the hub SU invests equally on all the links it established between peripheral SUs. In this subsection, we characterize the network structure of the strongly pairwise stable network structure and demonstrate that actually, such structures are also stars. Also, lemma 3 and lemma 4 need to be introduced before we introduce the major results in this subsection.

Lemma 3: Given that φ is linear and a graph G is strongly pairwise stable, if there exists a SU i that invests in multiple links, then all of the neighbors of SU i invest completely their investment budget to the link to SU i.

Proof: Suppose a SU i invests on two links to SU j and SU k. Then two cases exists for us to consider:

- **Case I**
  Both SU j and SU k invest on two or more links, so that $I_j^i < \mathcal{I}$ and $I_k^i < \mathcal{I}$.
- **Case II** Only one of the two SUs, SU j and SU k invests on two or more links. Without loss of generality, we assume that $I_j^i = \mathcal{I}$ and $I_k^i < \mathcal{I}$

For **case I**, without loss of generality we assume the reliability of the link between SU i and SU j and the link between SU i and SU k satisfies $r_{ij} \leq r_{ik}$. We then demonstrated that we are able to construct a joint allocation of resources for SU i and SU j that makes both SUs better off. Moreover, we use the notation $B_j^{i,k}$ to denote the equilibrium marginal value to SU j of the connection to SU k through SU i. Suppose SU i shifts ϵ investment from the link to k to the link to j.

Then, we consider the utility change to SU j resulted from the reallocation of investment:

$$\begin{aligned}\Delta U_i =&\epsilon(1+\sum_{m\neq j,k} r_{im}B_j^{i,m} + B_j^{i,k}(r_{ik}-r_{ij})) \\ &-\epsilon^2 B_j^{i,k}\end{aligned} \tag{14}$$

Notice that for ϵ that is close to zero, the change in utility is positive as $r_{ij} < r_{ik}$. Next, we consider another SU l such that $I_j^l > 0$ and a reallocation where SU j shifts δ resources from the link to SU l to the link to SU i. One can find $\delta(\epsilon)$ such that the total effect of the reallocation of resources (δ, ϵ) on the utility of j is strictly positive. So, the condition that $B_i^j = 1 + r_{jl}B_i^{j,l} + \sum_{m\neq i,l} B_i^{j,m} \leq B_i^{j,l}$. Then consider the change in utility of SU i resulted from the reallocation of investment:

$$\begin{aligned}\Delta U_i =&\delta(1+\sum_{m\neq i,l} r_{jm}B_i^{j,m} + B_i^{j,l}(r_{jl}-r_{ij})) \\ &-\delta^2 B_i^{j,l}\end{aligned} \tag{15}$$

Then, we have $\Delta U_i \geq \delta(1-r_{ij})B_i^{j,l} - \delta^2 B_i^{j,l}$. ΔU_i is strictly positive for δ close to zero. Consequently, we have constructed a joint allocation of investments making both SU i and SU j strictly better off.

For **case II**, in that we have proved that given $r_{ij} \geq r_{ik}$, SU i and SU j can jointly plan a profitable deviation, we assume that $r_{ij} < r_{ik}$. For some other SU l, suppose that $I_k^l > 0$. For the reason that i invests on k, then $B_k^i \geq B_k^j$. Suppose SU k transfers some investments from the link to l to the link to i. Then the total utility of SU i increases since he gets the additional direct benefit and there are no loss in indirect benefit since $B_i^k \geq B_i^l$. Moreover, since $B_k^i = B_k^i$. For exactly the same reason, a transfer of investment allocation by SU i from the link to SU j to the link to SU k makes SU k better off and leaves SU i indifferent. Consequently, SU i and SU k have a profitable joint deviation. Up to now, we have finished the proof of lemma 3.

Lemma 4: Given that graph G is a connected network which is not a tree, if φ is linear, then G is not strongly pairwise stable.

Proof: Given that the network graph G is not a tree and it is connected, we can assume that G contains a cycle. Without loss of generality, we assume that the cycle contains the SU set $\mathcal{N}' = \{1, \cdots, n\}$ such that $\mathcal{N}' \subset \mathcal{N}$. Moreover the cycle can be represented by the notation $\{(1,2),(2,3),\cdots,(n-1,n),(n,1)\}$

Firstly, we seek to prove that if φ is linear and G is strongly pairwise stable, all SUs in the cycle invest only on one link. Suppose a SU i in the cycle invests on multiple links. From lemma 3, both SU i−1 and SU i+1 invest fully on the link with SU i. If k is the smallest integer such that i−k invests both on i−k+1 and i−k−1. Then, lemma 3 requires that i−k+1 invests fully on the link with i−k. But, by assumption i−k+1 invests fully on the link with i−k+2.

Suppose L_i and $L_{i+1}$is the set of nodes contained in $\mathcal{N}\backslash\mathcal{N}'$ which can be accessed though nodes $i \in \mathcal{N}'$ and $i+1 \in \mathcal{N}'$. Now take any arbitrary player j in L_i who is connected to SU i and SU k, such that $k \in L_{i+1}$ which is connected to SU i+1. Lemma 3 implies that j invests fully on the link with SU i and SU k invests fully on the link with i+1. Repeated application of lemma 3 implies that all SUs in L_i and L_{i+1} in fact invest only on one link.

The rest of the proof of lemma 4 follows a similar pattern of the proof of theorem 2 in [16].

After introducing lemma 3 and lemma 4, we present the major results in this subsection in theorem 2.

Theorem 2: A strongly pairwise stable network in our must be a star in which all players are connected.

Proof: Firstly notice that in definition 4, the function φ is justifiably defined to be a linear function with respect to the investment of any arbitrary SUs. Then the results of lemma 3 and lemma 4 in the scenario that φ is linear is entirely applicable to the proof of theorem 2.

Since, φ in our framework is linear, the only pairwise stable network structure can only be a tree. Thus we consider a tree of diameter greater than or equal to 3. Let SU i and SU j be two terminal nodes at distance that is no less than 3 and let SU k and SU l be their predecessors in the tree. If SU i chooses to invest its investment on the link it establishes to SU l instead of SU k, then $B_i^k \geq B_i^l = B_j^l + r_{lj} - r_{li}$,, where B_j^l denotes the marginal equilibrium value the SU j can obtain though the direct link it established to SU l. Moreover, we have $B_k^l \geq B_i^k + r_{ki} - r_{kj}$. By summing uo these inequalities, we can get: $r_{li}+r_{kj} \geq r_{ki}+r_{lj}$. Thus we get a contradiction since $r_{li} = r_{ki}r_{kl}$ and $r_{kj} = r_{lj}r_{kl}$ with $r_{kl} < 1$. This demonstrates that since the function φ is linear, the network structure cannot be strongly pairwise stable if the result network graph G is not a collection of stars.

Then we suppose that the network contains two stars S_1 and S_2 such that $|S_1| = N_1$, $|S_2| = N_2$ and $N_1 \geq N_2$. Consider a peripheral SU that possesses the least reliable communication links with other SUs in the network, say SU i. Then, without loss of generality, suppose that it belongs to star S_2 and denote the hub of star S_2 to be i_2^* and the hub of star S_1 to be i_1^*. Consider the following joint deviation carried out by SU i and i_1^*: SU i connects fully to SU i_1^*, i.e. $I_i^{i_1^*} = \mathcal{I}$ and SU i_1^* shifts away investment $x_{i_1^*}^j$ from some peripheral SU j in star S_1 and invests this amount in the link to SU i. After this deviation, SU the payoff of i_1^* has increased from $N_1\mathcal{I}$ to $(N_1+1)\mathcal{I}$ and the payoff of SU i also has increased. Then, based on the previous proof, a strongly pairwise network structure must be a connected star given the condition that φ is linear.

Remarks: Based on theorem 1, we can conclude that the unique efficient network structure is a symmetric star where the hub SU invests an equal amount in all links with peripheral SUs. Based on theorem 2, we can conclude that a strongly pairwise stable network in our must be a star in which all players are connected. Note that the equilibrium network structure is also a star-like network structure. However, the investment of the hub SU might not be homogeneous among all the links it establishes with peripheral SUs. The problem that whether the equilibrium network structure is also a star i.e. the equilibrium network structure converges to the efficient network structure has not been figured out up till now. Consequently, it is left for future work.

REFERENCES

[1] F.C.C.S.P.T. Force. Report of the spectrum efficiency working group. *Washington DC, noviembre de*, 2002.

[2] J. Mitola III and G.Q. Maguire Jr. Cognitive radio: making software radios more personal. *Personal Communications, IEEE*, 6(4):13–18, 1999.

[3] T.C. Clancy III. *Dynamic spectrum access in cognitive radio networks*. PhD thesis, Citeseer, 2006.

[4] D. Cabric, S.M. Mishra, and R.W. Brodersen. Implementation issues in spectrum sensing for cognitive radios. In *Signals, Systems and Computers, 2004. Conference Record of the Thirty-Eighth Asilomar Conference on*, volume 1, pages 772–776. Ieee, 2004.

[5] P. Chaporkar and A. Proutiere. Optimal joint probing and transmission strategy for maximizing throughput in wireless systems. *Selected Areas in Communications, IEEE Journal on*, 26(8):1546–1555, 2008.

[6] A. Ghasemi and E.S. Sousa. Collaborative spectrum sensing for opportunistic access in fading environments. In *New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on*, pages 131–136. IEEE, 2005.

[7] E. Visotsky, S. Kuffner, and R. Peterson. On collaborative detection of TV transmissions in support of dynamic spectrum sharing. In *New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on*, pages 338–345. IEEE, 2005.

[8] W. Saad, Z. Han, M. Debbah, A. Hjorungnes, and T. Basar. Coalitional games for distributed collaborative spectrum sensing in cognitive radio networks. In *INFOCOM 2009, IEEE*, pages 2114–2122. IEEE, 2009.

[9] W. Saad, Z. Han, R. Zheng, A. Hjorungnes, and T. Basar. Coalitional Games in Partition Form for Joint Spectrum Sensing and Access in Cognitive Radio Networks. *Coalitional Game Theory for Distributed Cooperation in Next Generation Wireless Networks*, page 215, 2010.

[10] P. Chaporkar, A. Proutiere, and H. Asnani. Learning to optimally exploit multi-channel diversity in wireless systems. In *INFOCOM, 2010 Proceedings IEEE*, pages 1–9. IEEE, 2010.

[11] Junshan Zhang Shanshan Wang, Yalin E. Sagduyu and Jason H. Li. Spectrum Shaping via Network Coding in Cognitive Radio Networks. In *INFOCOM 2011, IEEE*. IEEE, 2011.

[12] J. Wei and X. Zhang. Energy-efficient distributed spectrum sensing for wireless cognitive radio networks. In *INFOCOM IEEE Conference on Computer Communications Workshops, 2010*, pages 1–6. IEEE, 2010.

[13] Wei J. and X. Zhang. Two-Tier Optimal-Cooperation Based Secure Distributed Spectrum Sensing for Wireless Cognitive Radio Networks. In *INFOCOM IEEE Conference on Computer Communications Workshops, 2010*, pages 1–6. IEEE.

[14] A.W. Min, X. Zhang, and K.G. Shin. Detection of Small-Scale Primary Users in Cognitive Radio Networks. *Selected Areas in Communications, IEEE Journal on*, 29(2):349–361, 2011.

[15] B. Sklar. *Digital communications*. Prentice Hall, 2001.

[16] F. Bloch and B. Dutta. Communication networks with endogenous link strength. *Games and Economic Behavior*, 66(1):39–56, 2009.