Spectrum Trading in Cognitive Radio Network:a Two-Stage Market Based on Contract and Stackelberg Game

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Abstract

Cognitive radio is a promising paradigm to achieve efficient utilization of spectrum resource by allowing the unlicensed users to access the licensed spectrum. Designing mechanisms with proper economic incentives is essential for the success of dynamic spectrum sharing. In this project report, we study both the long-term market (i.e. contract-theoretic model) and short-term market (i.e. Stackelberg game model) between a single primary spectrum owner (PO) and multiple unlicensed secondary users (SUs) in a Market-driven secondary spectrum trading. In long-term market, we design optimal contracts, which are offered by PO . Then SUs choose whether to accept the contract based on both their demand and their types. We can show these optimal contracts maximize both PO and SUs profit. After long-term market, secondary spectrum trading enters into short-term market, where SUs can buy some amount of licensed spectrum at each time slot. We model and analyze the interactions between PO and SUs as a Stackelberg game. Finally, our simulation result demonstrates that this integrated design mechanism is effective to improve spectrum utilization and address profit maximization problem in both PO and SUs' side.

1 Introduction

In the past decades, the FCC (Federal Communications Commission) and its counterparts have used command and control model to assign spectrum to license wireless service provider, however, with the explosive development of wireless services and networks in recent years, the remaining available spectrum becomes exhausting and scarce. Dynamic spectrum sharing based on cognitive radio has emerged as a promising paradigm to increase spectrum efficiency and alleviate spectrum scarcity. Dynamic spectrum sharing allows unlicensed secondary users to access the spectrum of licensed primary users in an opportunistic way [1],[2]. To realize this, it is essential to design a spectrum sharing mechanism [3] which offers PO incentives to share or lease his spectrum to SUs and the SUs also have incentive to employ or buy the spectrum from the PO. Market-driven secondary spectrum trading [4] is considered as a promising paradigm.

Many researchers focused on design mechanism for spectrum management, in which tools such as contract theory [5],[6],[9], game theory[8],[10],[11] and auction theory[12],[13],[14] are used. [6] introduced the concept of contract in economics and consider the issue of quality discrimination for the spectrum trading with multiple consumer types. [5] focused on cooperative spectrum sharing under incomplete information based on contract theory. In [7],[8],[9], stackelberg game is used to model and analyze the interactions between the PO and SUs in the spectrum market. Among them, Duan proposed the optimal investment and pricing decisions in [7]. However, these mechanisms only considered one market in process of secondary spectrum trading, thus QoS differences among SUs cannot be satisfied and the spectrum utilization has much room for improvement. As we can see, a new mechanism which tackles secondary spectrum trading with the coexistence of two modeling approach markets is needed.

In this report, we design a secondary spectrum trading mechanism between a single PO and multiples SUs in a two-stage market. This two-stage market consists of a long-term and short-term spectrum trading market. In the long-term market, PO designs a set of contracts for different type SUs. SUs choose whether to accept the contracts. Once a SU accepts a contract, PO will allocate a unit spectrum band to this SU. After long-term market, PO will confirm the total bandwidth of idle spectrum needed for contracts. Then our two-stage market enters into the short-term market. At each time slot, PO firstly senses the unused ”spectrum holes” in the licensed spectrum without violating the usage rights of the primary users. Then PO can obtain bandwidth of the residual spectrum, which is the difference between the total sensing bandwidth and the guaranteed bandwidth for contracts. Finally PO sells this part of residual spectrum to SUs who fail to sign a contract with PO but still desperately need licensed spectrum to transmit its own packet. Figure 1 illustrates the two-stage market.

Contract has been widely adopted as a well-known market-driven mechanism to allocate spectrum in cognitive radio study due to its economical properties[15]. Contract can provide incentives to all of the members in the trading market and maximize profits for both sides. Therefore, we choose contract model in long-term spectrum trading market. On the other hand, existing studies on cognitive radio have used various kinds of game theory to analyze the behavior between PO (or primary users) and SUs in process of selling and buying idle spectrum. We choose Stackelberg game to model the interactions between PO and SUs in short-term market because the equilibrium of Stackelberg game can be obtained just through a round of operations and do not need an iterative algorithm which is commonly seen in some Auctions and Non-cooperative game. This characteristic makes Stackelberg game particularly well-suited to be adopted during each time slot.

This two-stage market has some significant advantages. On one hand, it is flexible in achieving QoS differentiations. Some SUs need a stable spectrum supply to transmit series of packets, then they would choose contract in long-term market. For other SUs, packets may come randomly and they do not have enough money to pay for the contract, then they can trade in short-term market. On the other hand, two-stage market can enhance the maximum profit for PO by selling remaining spectrum in second market. Meanwhile, two-stage market improves spectrum utilization and enables more efficiency.

The main contributions of this project are as follows:

- Traditional studies on cognitive radio only consider one-stage trading market where the process of selling and buying spectrum resource between PO and SUs happens in one approach. We extend the spectrum trading market into two-stage with the coexistence of long-term market and short-term market.

- In our two-stage market, PO first formulates contracts in long-term market and then obtains bandwidth for contracts through spectrum sensing in short-term market.This is different from existing studies on contract-based spectrum trading market where bandwidth for contracts is obtained through spectrum leasing.

- We propose an optimal quantization algorithm to quantize SUs into discrete consumer types based

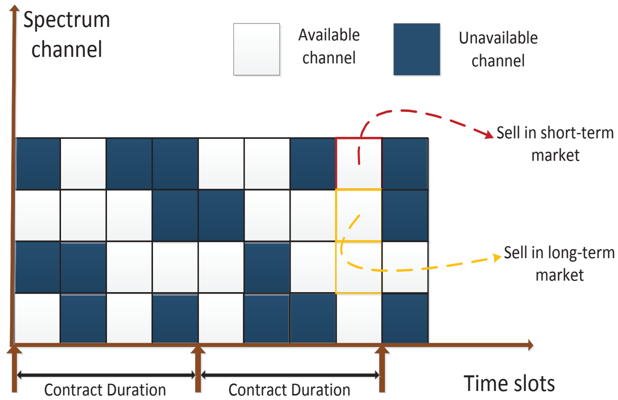

Figure 1: An illustration of the two-stage spectrum trading market

on the different distance between PO and SUs in the long-term market. This algorithm effectively alleviates PO's computation and guarantees that each SU will choose the contract designed for its type.

- The simulation result shows that two-stage market achieves more profit in both PO and SUs' sides. the utility of PO increases markedly(29% average) than one-stage market and the social utility for SUs also increase largely(33% average).

The rest of this report is organized as follows. In section II, we provide the system model. In section III and section IV, we propose the optimal contract formulation in long-term market and backward induction of Stackelberg game formed in short-term market. In section V, we present the simulation result. Finally, we conclude our work in section VI.

2 System Model

2.1 Wireless Network Model

We consider a cognitive radio network consisting of a single primary spectrum owner (PO) and multiple secondary users (SUs). The PO owns a set of licensed spectrums from subscribed primary users (PUs). The total transmission is divided into fixed-time intervals, called time slots. The spectrum possessed by the PO is under-utilized at each time slot. That means, there exists some idle spectrum bands not used by the primary users at a specified interval. Therefore, the PO is willing to lease this unused spectrum band to secondary users who desperately need spectrum to transmit their packets. In return, SUs pay for this portion of licensed spectrum and then PO improve profit.

In our model, the PO can employ spectrum sensing technique to access the idle spectrum from licensed spectrum and sell the idle spectrum to the SUs on a slot-by-slot basis in a secondary spectrum trading market. The total bandwidth of idle spectrum offered by the PO is varying with the time slot because the PUs' traffic is stochastic and the idle spectrum changes dynamically.

2.2 Long-term Market and Short-term Market

We consider secondary spectrum trading in a two-stage spectrum market consisting of long-term and short-term market.

1) $long\text{-}term\ spectrum\ trading\ market$

Long-term market is the first stage of the secondary spectrum trading market. A single PO and multiple SUs enter into this market to agree on the contracts. Once an SU accepts a contract from the PO, the PO need to deliver the unit bandwidth of idle spectrum to the SU in a given certain period of time slots T. If the PO fails to guarantee the contract, i.e. the PO does not have enough spectrum band to offer at one time slot, the penalty is needed to pay to the SU for compensating the loss. As the contracts are designed by the PO, PO dominates this trading market to some extent. Long-term spectrum trading market is modeled as a monopoly market, in which the PO is modeled as a monopolist and sets the particular contracts for different SUs and the SUs act as the consumers. The consumers decide which contract to choose and find out an optimal contract.

To provide an intuitive and meaningful expression, the contract in our model consists of three elements: (1)the maximum allowable transmission power p; (2)SU's payment P; (3)PO's penalty g when violating the contract. Then the contracts can be expressed as:

$$C \triangleq \{q, P, g\}$$

2) *short-term spectrum trading market*

After long-term, the PO and the SUs who fail to accept the contracts enter into this short-term spectrum trading market.

We model the interaction between the PO and the SUs in this short-term spectrum trading market as a Stackelberg game. The PO is the leader of game. First, the PO senses the idle spectrum from the licensed primary spectrum and realizes the total available bandwidth. The guaranteed bandwidth for contract is obtained in the previous long-term market, so the bandwidth of residual spectrum for this market is the difference between the total available bandwidth and the guaranteed bandwidth for contracts. Then the PO announce price π to the market based on the residual bandwidth which PO has owned. Finally the SUs in short-term spectrum trading market decide whether to buy and determine the demands for bandwidth from the PO.

We consider that the PO sell the idle spectrum to the SUs on a slot-by-slot basis. In this short-term market, SUs initiates a leasing request only when need. And the price charged by PO in this market is much cheaper than the payment in contract.

3 Contracts formulation in long-term market

Let **N**$= \{1, 2, ...N\}$ denotes the sets of SUs in the long-term spectrum trading market. Once an SU accepts a contract which is specially designed for its type, PO will allocate an unit bandwidth of spectrum to this SU. An unit bandwidth of spectrum is denoted as B_0. We assume $B_0 = 1$ in this report. The maximum allowable power through which SUs can transmit on the unit spectrum characterizes the quality of this unit spectrum q. In this way if an SU chooses a contract with quality q, he can transmit his own packets on this spectrum with power not more than q. We assume that each SU is rational and prefers higher transmission rate. Specifically, for a given maximum allowable transmission power q, the transmission rate for SU i can be written by Shannon-Hartley theorem.

$$\gamma(q) = B_0 \log(1 + q\frac{h_i}{n_0 B_0 + I_{PUs} + I_{PO}})$$

where h_i is the channel gain between the transmitter and receiver of SU i, I_{PUs} and I_{PO} represent the interference coming from PUs and PO respectively, n_0 is the background noise power. In this report, we focus on the interaction between SUs and PO so I_{PUs} and n_0 are assumed to be identical for all SUs.

SUs can be classified into different types and the expression $\frac{h_i}{n_0 B_0 + I_{PUs} + I_{PO}}$ is used to denote the type of SUs. Specifically, we define an SU i as a type-α SU if $\frac{h_i}{n_0 B_0 + I_{PUs} + I_{PO}} = \alpha$. Furthermore, we focus on the term I_{PO}, I_{PO} can be written as $I_{PO} = p_o h_o$ where p_o and h_o denotes the transmission power and channel gain between transmitter of PO and receiver of SU i. h_o exponentially decreases with the distance d_i between PO and SU i. Usually, the relationship between h_o and d_i can be modeled as expression $h_o = \frac{\lambda}{{d_i}^2}$, where λ is a constant. Therefore, the type-α is a distance-specified function which can be expressed as Eq.(1)

$$\alpha(d_i) = \frac{h_i}{n_0 + I_{PUs} + \frac{p_o \lambda}{{d_i}^2}} \tag{1}$$

3.1 Optimal Quantization

We define the SUs' type into a discrete-type model, that is, there is a set of discrete no-negative rational numbers denoting different types of SUs. In our model, the type of an SU is related to the distance d_i between SU i and PO. That means, one SU belongs to a type independent with others unless they share the same distance. In practice, however, a cognitive radio network consists of a single PO and a great amount of SUs such as 1000 mobile phones. PO does not have enough computational capability and power to formulate 1000 kinds of contracts for each SU types. It is essential and significant to quantize all types for overall SUs into a set of finite discrete number sequence.

We partition all SUs into K clusters depended on different distance between PO and SUs. The boundaries of these clusters are denoted as vector $\mathbf{A} = \{A_1, A_2, ..., A_{K-1}\}$, and the quantized types are denoted as vector $\mathbf{L} = \{L_1, L_2, ..., L_K\}$. We need to design an optimal quantization algorithm so that for a given distribution of SU's location in a real network situation, the distortion D, which describes the performance of this lossy compression system is minimized. The distortion D is defined as:

$$D = E[(\alpha(d_i) - correspondingL_i)^2] \tag{2}$$

Since the distortion D is related to both vector $\mathbf{A} = \{A_1, A_2, ..., A_{K-1}\}$, $\mathbf{L} = \{L_1, L_2, ..., L_K\}$, we rewrite D as Eq.(3):

$$D(\mathbf{A}, \mathbf{L}) = \sum_{i=1}^{K} \sum\nolimits_{d_i \in (A_{i-1}, A_i)} (\alpha(d_i) - L_i)^2 \tag{3}$$

Therefore, the optimal quantization can be expressed as:

$$best(\mathbf{A}, \mathbf{L}) = \arg\min D(\mathbf{A}, \mathbf{L}) \tag{4}$$

By setting the partial derivatives of $D(\mathbf{A}, \mathbf{L})$ with respect to $A_i, (i = 1, 2, ..K-1)$ and $L_i, (i = 1, 2, ..K)$ equal to zero, we arrive a series of equations which can be solved simultaneously to obtain the values of A_i and L_i.

$$\begin{cases} L_i = E[\alpha(d_i) | A_{i-1} < d_i < A_1] = \frac{\sum_{d_i \in (A_{i-1}, A_i)} \alpha(d_i)}{N_{d_i \in (A_{i-1}, A_i)}} \\ \\ A_i = \alpha^{-1}[\frac{1}{2}(L_{i-1} + L_i)] \end{cases} \tag{5}$$

where $N_{d_i \in (A_{i-1}, A_i)}$ is the total number of SUs locating at interval $[A_{i-1}, A_i]$

Specifically, we design an iterative algorithm to address the optimal quantization problem. According to Eq.(1), type function is a quasi-concave function with respect to d_i, which means there exists a maximum type value α_max when d_i approaches infinity. the detailed procedure is shown in Algorithm 1.

Algorithm 1 Optimal quantization algorithm.

1: PO determines K and obtain α_{max}.
2: PO divides $[0, \alpha_{max}]$ into K intervals, each boundary value $\mathrm{A}_i = \alpha^{-1}(\frac{i\alpha_{\max}}{K})$.
3: PO senses the existence of SUs and obtain $N_{d_i \in (A_{i-1}, A_i)}$ and $\sum_{d_i \in (A_{i-1}, A_i)} \alpha(d_i)$.
4: PO calculates type vector $\mathbf{L}$.
5: PO updates type vector $\mathbf{A}$ using Eq.(5)
6: if (SU switches its type)
7: go to step (3)
8: end if

3.2 Optimal contracts for both PO and SUs

For a given maximum allowable transmission power q, the value of the unit spectrum for a type-α SU can be measured with his transmission rate, i.e.$V(\alpha, q) = \log(1 + q\alpha)$. We define SU's payment for the unit spectrum is P, then the utility $U(\alpha, q, P)$ of this SU can be obtained as Eq.(6)

$$U(\alpha, q, P) = \log(1 + q\alpha) - P \tag{6}$$

In the long-term market, PO will formulate some contract for different type SUs and each SU choose the contract which always maximize his utility, thus the optimal contract for a type-α SU can be written as:

$$best(q, P) = \arg\max U(\alpha, q, P)\{(q, P) \in all\ contracts\} \tag{7}$$

At PO's side, PO focuses on the problem how he can maximize his profit by formulating contracts in an optimal approach. We assume that there are K SU's types in this market. Without loss of generality, we order types in the ascending order $\alpha_1 < \alpha_2 < ... < \alpha_K$. For each type, there are N_{α_i} SUs, that is, $N = \sum_{i=1}^{K} N_{\alpha_i}$. We rewrite q and P as $q(\alpha_i)$ and $P(\alpha_i)$ because contracts differ with SU's types. Therefore, the revenue of PO in long-term market can be expressed as Eq.(8)

$$R = \sum_{i=1}^{K} P(\alpha_i) N_{\alpha_i} \tag{8}$$

the cost of PO consists of three parts, the first part is the expense for sensing idle spectrum from licensed PUs. Let B_s denotes sensing amount, that is, the total bandwidth of spectrum which PO sense. C_s denotes the unit sensing cost.The second part is performance degradation of PUs induced by the interference of SUs. this cost is quality-specified and increase with q, moreover, it grows more rapidly in high quality than in low quality. We use expression $\lambda_1 q^2$ to describe the second part, where λ_1 is pre-defined parameter. The final part of cost is PO's penalty for violating the contracts. Let N_g denotes the number of SUs who sign contracts with PO but fail to obtain the unit spectrum. Thus the cost of PO can be written as Eq.(9)

$$\Omega = B_s C_s + \sum_{i=1}^{K} \lambda_1 q(\alpha_i)^2 N_{\alpha_i} + N_g g \tag{9}$$

Thus, the utility of PO can be expressed as Eq.(10)

$$U = R - \Omega = \sum_{i=1}^{K} [P(\alpha_i) - \lambda_1 q(\alpha_i)^2] N_{\alpha_i} - B_s C_s - N_g g \tag{10}$$

We define that $U_I(q,P)$ satisfies expression $U = U_I(q,P) - B_s C_s - N_g g$, that is,

$$U_I(q,P) = \sum_{i=1}^{K} [P(\alpha_i) - \lambda_1 q(\alpha_i)^2] N_{\alpha_i} \tag{11}$$

In this section we focus on how to maximize $U_I(q,P)$, and the residual part of U will be discussed in next section.Therefore, the optimal contracts for PO can be written as:

$$\begin{aligned} &best(q,P) = \arg\max U_I(q,P) \\ &subject\ to\ (q,p)\ satisfies\ Eq.(7) \end{aligned} \tag{12}$$

$U_I(q,P)$ maximized problem can be solved by citing $Lemma4$ and $Lemma5$ in [6], which is not the focus of this project report.

$$\begin{aligned} E[U^*] &= \int_0^{\frac{B_n}{B_s}} [U_I^{\max} - B_s C_s - (B_n - B_s\eta)g]dx \\ &+ \int_{\frac{B_n}{B_s}}^{1} [U_I^{\max} + (B_s\eta - B_n)\log(\tfrac{B}{B_s\eta - B_n}) - B_s\eta - B_n - B_s C_s]d\eta \\ &= U_I^{\max} - B_s(C_s + \tfrac{1}{2}) - B_n + \tfrac{B_n^2(3+g)}{2B_s} + \tfrac{(B_s - B_n)^2}{4B}(1 - 2\log\tfrac{B_s - B_n}{B}) \end{aligned} \tag{22}$$

$$\begin{aligned} E[U^*] &= \int_0^{\frac{B_n}{B_s}} [U_I^{\max} - B_s C_s - (B_n - B_s\eta)g]dx \\ &+ \int_{\frac{B_n}{B_s}}^{\frac{B_n + Be^{-2}}{B_s}} [U_I^{\max} + (B_s\eta - B_n)\log(\tfrac{B}{B_s\eta - B_n}) - B_s\eta - B_n - B_s C_s]d\eta \\ &+ \int_{\frac{B_n + Be^{-2}}{B_s}}^{1} [U_I^{\max} + Be^{-2} - B_s C_s]d\eta \\ &= U_I^{\max} - \tfrac{B_n(B_n g + B_n + 2Be^{-2})}{2B_s} + \tfrac{5Be^{-4}}{4} - B_s C_s - \tfrac{(B_n + Be^{-2})^2}{2B_s} + Be^{-2}(1 - \tfrac{B_n + Be^{-2}}{B_s}) \end{aligned} \tag{23}$$

4 Stackelberg game and backward induction in short-term market

4.1 Stackelberg game

We use a Stackelberg game to model the interaction between PO and SUs in the short-term spectrum trading market.

1. PO is the leader of this game. It first decides the sensing amount B_s according to the guaranteed bandwidth for contracts, which is denoted as B_n. Sense factor η is used to reflect the relationship between total available bandwidth and B_s. In this case, if PO decides to sense B_s spectrum at a time slot, only a portion of spectrum, i.e.$B_s\eta$ is unused by PUs. $\eta \in [0, 1]$, is a stochastic number and depends on PUs' activities. PO cannot tell the exact value of η at a time slot before sensing, but the distribution of η can be obtained by previous experience or the radio schedule of PUs. In this report, we assume η follows a uniform distribution for simplicity.

2. Then PO determines the price π to SUs given sensing amount B_s

3. Finally, SUs choose whether to buy spectrum and their demands of bandwidth denoted as ω_i to maximize their individual profit.

4.2 Backward Induction

The stackelberg game in this situation can fall into a set of dynamic game[8] and the Subgame Perfect Equilibrium is considered as the common solution, The backward induction can be considered as a general technique for obtaining equilibrium. That means we first calculate SUs demand bandwidth ω_i for a given price π. Then we analyze PO's price decision given fixed B_s. Finally,we derive the optimal sensing decision to achieve maximized profit for PO.

Similarly with long-term market, we use SU's achievable transmission rate as their valuation of spectrum. The difference is that we assume SUs use $OFDM$ to access the spectrum and interference from PO and others can be avoided.

$$V(\omega_i) = \omega_i \log(1 + \frac{p_i h_i}{n_o \omega_i}) \tag{13}$$

where ω_i is the bandwidth of spectrum allocated to SU i. p_i can be denoted as the maximum allowable transmission power through the entire allocated bandwidth because each SU is rational, h_i is the channel gain and n_o is the background noise power. Therefore SU i's utility function can be written as:

$$U(\omega_i) = V(\omega_i) - \pi\omega_i = \omega_i \log(1 + \frac{p_i h_i}{n_o \omega_i}) - \pi\omega_i \tag{14}$$

We also define types for SUs in short-term market, that is, $\beta_i = \frac{p_i h_i}{n_o \omega_i}$.the optimal bandwidth demand for a type-β_i SU is:

$$best(\omega_i) = \arg\max \omega_i \log(1 + \frac{\beta_i}{\omega_i}) - \pi\omega_i \tag{15}$$

$$\frac{\partial U}{\partial \omega_i} = \log(\frac{\omega_i + \beta_i}{\omega_i}) - \frac{\beta_i}{\omega_i + \beta_i} - \pi \tag{16}$$

So the optimal ω_i^* satisfies Eq.(16)$= 0$, to simplify analyze process in the further, we use $\beta_i e^{-1-\pi}$ to approximate ω_i^*. Therefore, PO obtains the total bandwidth demands in this short-term market as $\sum_{i=1}^{K'} \beta_i e^{-1-\pi} = \mathrm{B}e^{-1-\pi}$, where K' is the number of SUs' types in short-term market.

Next,we consider how PO chooses its price based on the total bandwidth demands and a fixed B_s to achieve PO's maximum profit.

If $B_s\eta < B_n$, available idle spectrum cannot satisfy the demand required by contracts. In this case, PO choose to give up short-term market, that means all bandwidth will be allocated to SUs who sign the contract, $\pi = Na$. Otherwise, the residual bandwidth is $B_s\eta - B_n$. PO's total profit is:

$$U = U_I^{\max} + \min(\pi B e^{-1-\pi}, \pi(B_s\eta - B_n)) - B_s C_s \tag{17}$$

where $U_I^{\max}$ is the maximized value of Eq.(11)

For a fixed $B_s\eta - B_n$andB_s, the optimal price decision is

$$best(\pi) = \arg\max \min(\pi B e^{-1-\pi}, \pi(B_s\eta - B_n)) \tag{18}$$

The result of Eq.(18) can be expressed as following:

- If $B_n \leq B_s\eta < Be^{-2} + B_n$, then optimal price decision is $\pi = \ln(\frac{B}{B_s\eta - B_n}) - 1$ and PO's maximized profit is $U^{\max} = U_I^{\max} + (B_s\eta - B_n)\ln(\frac{B}{B_s\eta - B_n}) - B_s\eta - B_n - B_sC_s$
- If $B_s\eta \geq Be^{-2} + B_n$, then optimal price decision is $\pi = 1$ and PO's maximized profit is $U^{\max} = U_I^{\max} + Be^{-2} - B_sC_s$

Now we enter into the final step where PO determines the sensing amount B_s to maximize his expected profit.Here we define $U^* = U^{\max}$ and the optimal sensing amount decision is :

$$best(B_s) = \arg\max U^* \tag{19}$$

According to the different value of B_s, Eq.(19) can divide into three cases.

- $B_s < B_n$, in this case,we always have $B_s\eta < B_n$, which means PO have to give up short-term market and pay the penalty to SUs who signed contracts but fail to utilize spectrum.

$$U^* = U_I^{\max} - B_sC_s - (B_n - B_s\eta)g \tag{20}$$

We derive Eq.(20)to obtain the expected profit,$E[U^*]$.

$$\begin{aligned} E[U^*] &= \int_0^1 f(\eta)[U_I^{\max} - B_sC_s - (B_n - B_s\eta)g]d\eta \\ &= U_I^{\max} - B_sC_s - B_ng + \frac{B_sg}{2} \end{aligned} \tag{21}$$

- $B_n < B_s < B_n + Be^{-2}$, In this case, we the expected profit as Eq.(22)
- $B_s > B_n + Be^{-2}$, in this case, we derive the expected profit as Eq.(23)

We can obtain the optimal sensing amount decision by deriving $\frac{\partial E[U^*]}{\partial B_s} = 0$ in Eq.(21),(22),(23)because $\frac{\partial^2 E[U^*]}{\partial^2 B_s} \leq 0$.

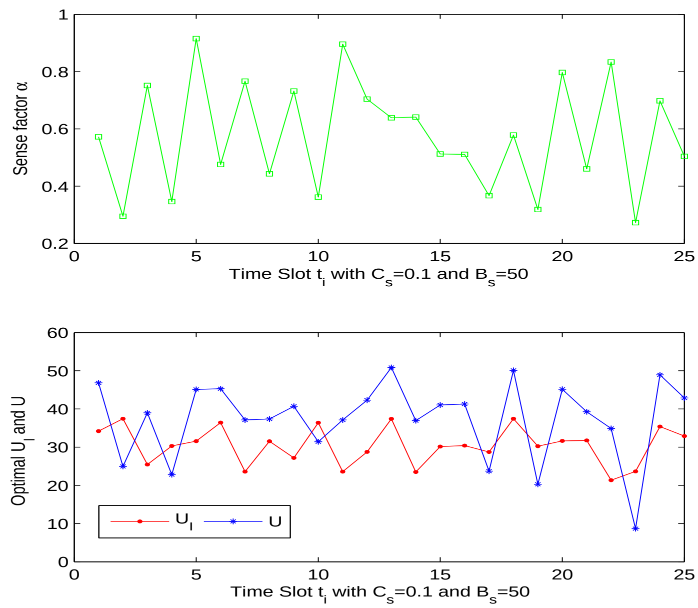

Figure 2: Maximized U_I and U over time with different sensing factor α

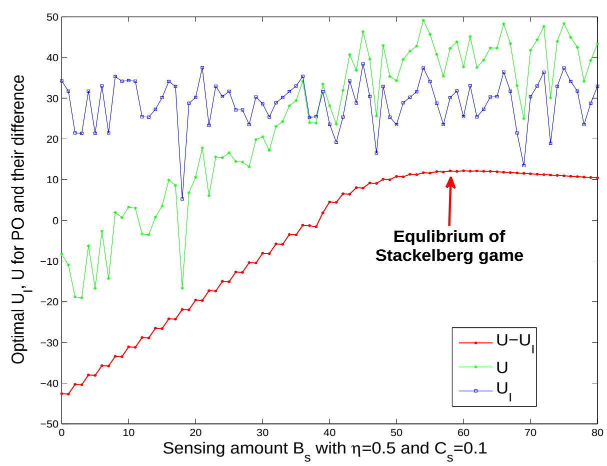

Figure 3: Maximized U_I, U and their difference over time with different sensing amount B_s

5 Simulation result

We study the optimal contracts in long-term market and focus on maximized utilities for both PO and SUs which can be obtained through formulating and choose optimal contracts. We use terms U_I or $profit\ in\ stageI$ to denote these maximized utilities in our simulation. Note that U_I does not include penalty $N_g g$. We also study stackelberg game in short-term market and focus on maximized utilities for SUs and expected

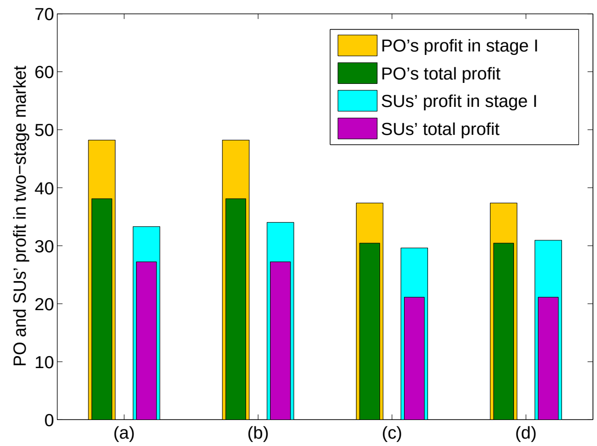

Figure 4: the utility of PO and social utility of SUs in two-stage market

utility for PO in this stage; combining with long-term market, we use term U or $total\ profit$ to denote utilities obtained through two-stage market. The different distribution of SUs in long-term market and short-term market can influence the optimal contracts and total profit, therefore we study the two-stage market in 4 scenarios. In case (a), there exist 25 SUs in long-term market and 10SUs in short-term market; in case (b), 25 SUs in long-term market and 15 SUs in short-term market; in case(c), 20 SUs in long-term market and 10 SUs in short-term market; in case (d), 20 SUs in long-term market an 15 SUs in short-term market. Without loss for generality, we assume K=5, and each type can be obtain by using Algorithm 1.

Figure 2 illustrates U_I and U for PO with different sensing factor α, we assume that case (c) holds in this scenarios with sensing amount $B_s = 50$ and sensing cost $C_s = 0.1$. From Figure 3,we find in some time slot U is smaller than U_I, this is because α is too low for PO to provide enough idle spectrum to SUs in long-term market, so the penalty is considered.

Figure 3 illustrates U_I, U for PO and their difference with different sensing amount B_s, we assume that case (c) holds in this scenarios with $\alpha = 0.5$ and $C_s = 0.1$. The difference between U and U_I denotes the profit achieved in short-term market. From Figure 4, we find that $U - U_I$ increase with B_S at small B_s and gradually decline at large B_s. this corresponds to expected profit for PO which we have analyzed in section IV. And the maximized $U - U_I$ point can be considered as the equilibrium of stackelberg game.

Figure 4 presents the U_I and U for both PO and all SUs in 4 cases. From Figure 4, we find that in our two-stage market model the utilities for both PO and SUs grow larger than traditional one-stage market. For PO, the utility increase 35.9% in case (a),(b) and 22.7% in case (c),(d). For SUs, the social utility increase 22.3%, 24.98%, 40.28% and 46.50% respectively.

6 conclusion

In this peoject, we study the issue of secondary spectrum trading between single PO and multiple SUs and design a two-stage spectrum trading market mechanism which consists of long-term market and short-term market. We use contract and Stackelberg game to model these two markets. We further analyze the optimal contracts and the equilibrium of Stackelberg game. We also discuss the maximized aggregate utility for both PO and SUs. The simulation result shows that our two-stage market model is efficient and significant to improve spectrum utilization and social utilities.

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