FINAL PROJECT REPORT

Connectivity of Mobile K-Hop Wireless Network with Correlated Mobility and Cluster Scalability

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***Abstract*—Since it was found that real mobility processes exhibit significant degree of correlation (*correlated mobility*) and nodes are often heterogeneously distributed in clustered networks (*cluster scalability*), there has been a great interest in studying their impact on network performance. However, works investigating their impact on the asymptotic connectivity are limited and the reason is threefold: (1) there is no available model to characterize the networks with correlated mobility and cluster scalability; (2) the ubiquity of correlated mobility and cluster scalability in reality makes the analysis hard to be tracked; and (3) the potential applications of correlated mobility and cluster scalability are not understood.**

In this paper, we study the effect of correlated mobility and cluster scalability on network connectivity and propose the *correlated mobile k-hop clustered networks model*. We mainly analyze the dynamics of cluster scales and observe that the impact of correlated mobility and cluster scalability on connectivity is primarily imposed through influencing the network state transition. So in our model, we divide the network state into three categories: (1)cluster- sparse state ($\alpha+2\beta<0$) (2) cluster-dense state ($\alpha+2\beta>0$) and (3) cluster-critical state ($\alpha+2\beta=0$). We prove that the critical transmission range for cluster-sparse state is $\sqrt{\frac{\gamma logn}{k\pi n^{\alpha}}}$ when $\alpha+2\beta<0$ and for cluster-dense state is $\sqrt{\frac{logn}{k\pi n^{\alpha}}}$ when $\alpha+2\beta>\frac{1}{k}$. Our research can help to understand the nature of *correlated node movements*, *cluster scalability* (*spatial heterogeneity*) and *network state transition*, and provide insights on building and managing large-scale wireless networks.

Keywords: Connectivity, Correlated Mobility, Cluster Scalability

I. INTRODUCTION

Connectivity performance is a fundamental concern when designing and implementing wireless networks, and hence is of paramount significance. To achieve the connectivity, nodes in the networks need to reach others by adjusting their transmission power. There are numerous works exploring the *asymptotic connectivity* of wireless networks. In [1], Gupta and Kumar prove that with range $r(n)=\sqrt{\frac{\log n+c(n)}{\pi n}}$, overall connectivity can be established with probability one as $n\to\infty$, if and only if $c(n)\to\infty$. In [2], Wan *et al.* obtained the critical transmission radius for k-connectivity in an ad hoc network whose nodes are uniformly and independently placed. In [3], Xue *et al.* prove that $\Theta(\log n)$ [1] nearest neighbors are needed to achieve full connectivity in a multi-hop fashion in the networks with n randomly and independently distributed nodes.

However, these works are concentrated on *stationary non-clustered (flat) networks* where nodes are *independently* distributed and keep *stationary* and works exploring the impact of correlated mobility and cluster scalability on asymptotic connectivity are limited and their impact is not clear so far. This is partially because (1) there is no available model to characterize the networks with correlated mobility and cluster scalability; (2) correlated mobility and cluster scalability is ubiquitous in reality which makes analyzing their impact on connectivity hard to be tracked; and (3) the potential applications of correlated mobility and cluster scalability are not recognized.

Therefore, to understand the nature of correlated mobility and cluster scalability, and explore their interactions, implications and impact on asymptotic connectivity, we propose the *correlated mobile k-hop clustered networks model* in this paper to take into consideration both the *correlated mobility* and *cluster scalability*. We adopt the correlated mobility model to implement the group mobility, and suppose that there are $n^{\alpha}(0<\alpha\leq 1)$ cluster heads and $n^{\gamma}(0<\gamma\leq 1)$ clusters each of which is with the radius $R=\Theta(n^{\beta})(\beta\leq 0)$ in the whole network O which is assumed to be a unit torus. The cluster radius can scale with n, and with different values of β we can implement cluster scalability. In our analysis, we divide we divide the network state into three categories: (1)cluster- sparse state ($\alpha+2\beta<0$) (2) cluster-dense state ($\alpha+2\beta>0$) and (3) cluster-critical state ($\alpha+2\beta=0$) and derive the *critical transmission range* for each state.

[1] The following asymptotic notations are used in this paper. Given non-negative functions $f(n)>0$ and $g(n)>0$:
(1) $f(n)=o(g(n))$ means $\lim_{n\to\infty}\frac{f(n)}{g(n)}=0$.
(2) $f(n)=\omega(g(n))$ is equivalent to $g(n)=o(f(n))$.
(3) $f(n)=O\big(g(n)\big)$ means $\lim_{n\to\infty}\sup\frac{f(n)}{g(n)}<\infty$.
(4) $f(n)=\Theta\big(g(n)\big)$ means $f(n)=O(g(n)),g(n)=O(f(n))$.
(5) $f(n)=\Omega\big(g(n)\big)$ is equivalent to $g(n)=O(f(n))$.

II. Related Works

Previous works mostly put strength on studying the delay and throughput with correlated mobility or cluster scalability. Garetto *et al.* [4] implemented cluster scalability (inhomogeneity) and determined under which condition the node mobility can be exploited to increase the per-node throughput. Ciullo *et al.* [5] proposed a correlated mobility model, studied its impact on throughput-delay performance in the cluster-sparse and cluster-dense regime, and discovered that this correlation can sometimes lead to better performance than the one achievable under independent node movements.

However, works investigating the impact of correlated mobility and cluster scalability on asymptotic connectivity are extremely limited. The classical literature [1] [2] [3] [6] [7] assume nodes to be independently distributed in the network. Wang *et al.* [8] proposed the mobile k-hop clustered networks model, but the node movements are not correlated and there is no cluster scalability. La *et al.* [9] studied the impact of one-dimensional group mobility on the bidirectional connectivity in vehicular ad hoc networks. Unfortunately, there is no work investigating the impact of two-dimensional correlated mobility and cluster scalability on the connectivity.

III. Preliminaries

A. Mobile K-hop Clustered Networks

In this paper, we consider infrastructure-supported networks. Nodes in a clustered network are classified in two classes: *cluster-head nodes* and *cluster-member nodes*. Cluster-head nodes are selected to serve cluster-member nodes (clients) and their function is similar to an access point in s real network. A cluster member is connected when reaching one of the cluster heads. In our model, it is assumed that a clustered network consists of n cluster-member nodes and n^α cluster-head nodes, where α is the *cluster head exponent* and $0 < \alpha \leq 1$. For simplicity, n^α is treated as an integer.

In the mobile k-hop clustered networks model, all nodes are placed in a unit square O and O is supposed to be a unit torus to avoid boundary effects. $G(n, \alpha)$ is the initial graph in which a path connects all the n^α cluster heads and time is slotted into k time slots. Culster heads are randomly and independently distributed in O and always remain stationary. Cluster members are initially set in the same way but can move in the whole network O in the following slots according to some certain mobility pattern. During each time slot $\lambda(\lambda = 1, 2, \ldots, k)$, an edge e_{ij} would be added between a cluster-member node i and a cluster-head node j, if the Euclidean distance between them is less then $r(n, \alpha)$ in the distance-based connecting strategy. $r(n, \alpha)$ is the critical transmission range in mobile k-hop clustered networks.

For mobile k-hop clustered networks, a cluster member is connected if it can reach a cluster head within k time slots during its movement. If all cluster-member nodes are connected, we define that the whole clustered network O has full connectivity.

B. Network Deployment

In this part, we illustrate the initial network architecture deployment before cluster members begin to move. Like the mobile k-hop clustered networks model, we suppose there are n cluster-member nodes and n^α cluster-head nodes in a unit square O, where the *cluster head exponent* $0 < \alpha \leq 1$. The cluster heads are *uniformly and independently distributed* in O. Differently, cluster-member nodes are clustered into m clusters where $m = n^\gamma$ and the *cluster exponent* $0 < \gamma \leq 1$. Each cluster region is centered around a logical center (*home point*) and has a circular shape with radius R as $R = \Theta(n^\beta)$ and the *cluster radius exponent* $\beta \leq 0$. The home points are *uniformly and independently distributed* in O and the cluster-member nodes are *uniformly and independently distributed* in their belonging cluster regions. For simplicity, we assume that each cluster compromises an integer number $\varpi = \frac{n}{m} = n^{1-\gamma}$ of cluster members and we also assume n^α and n^γ to be integers.

C. Correlated Mobility

After deploying the initial network architecture, the cluster heads will remain stationary while the cluster members will move. The movement of a cluster-member node consists of two steps: (1) the movement of its home point; and (2) the relative movement of the cluster member in the cluster. We assume the *Mobile Clusters with Relatively Mobile Cluster Members Mobility Model* which is illustrated as follows:

Mobile Clusters with Relatively Mobile Cluster Members Mobility Model: After the network deployment, home points and cluster members will move. Time is slotted into k time slots and at the beginning of each time slot, each home point will *uniformly and independently* choose a position within the unit torus O and then each cluster member will *uniformly and independently* choose its location in its corresponding cluster region. In the rest of each time slot, the home points and cluster members would remain stationary. We illustrate this mobility model in Figure 1.

D. Cluster Scalability

After introducing the system model, we will present the unique characteristic of our model, ***cluster scalability***. As we can see, the cluster radius scales with n by assuming $R = \Theta(n^\beta)$ where $\beta \leq 0$. Hence, when β is small (with large absolute value) the cluster size will be small and clusters are sparsely distributed in O. While the cluster region will become relatively large as β is large (with small absolute value) which leads to the densely distributed clusters. We should note

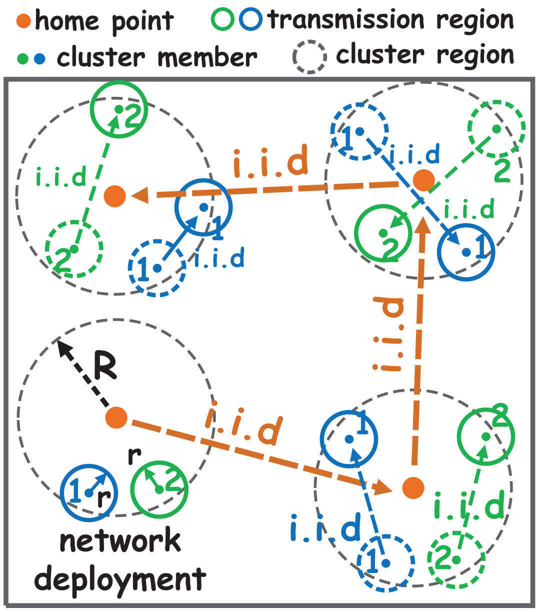

Fig. 1. Correlated Mobility Model.

that this cluster scalability also leads to node spatial heterogeneity because the node density is not the same in the whole network area.

This is the qualitative illustration of cluster scalability and we should also provide a quantitative definition. We compare the average coverage $\frac{1}{n^\alpha}$ of cluster heads with the cluster region $\pi R^2 = \Theta(n^{2\beta})$ and give the following three cases:

(C1). *Cluster-sparse state* (member-dense state).

When $\pi R^2 = o(\frac{1}{n^\alpha})$, we have $\alpha + 2\beta < 0$. The cluster size is sufficiently small compared with the average coverage of each cluster head and clusters are sparsely distributed in the whole network O. Besides, the member density of each cluster $d = \frac{\varpi}{\pi R^2} = \Theta(n^{1-2\beta-\gamma})$ is large. Thus, this is also the member-dense state and the clustering property is fairly dominant. Each cluster can be regarded as an entirety because cluster members stay so close and move so consistently.

(C2). *Cluster-dense state* (member-sparse state).

In contrast to the previous case, we have $\pi R^2 = \omega(\frac{1}{n^\alpha})$ in this case and $\alpha + 2\beta > 0$. The cluster size is relatively large, clusters are densely distributed and they might intersect with each other. The member density d is relatively small, and hence this is the member-sparse state and there is almost no substantial clustering. Every cluster-member node performs more like an independent node.

(C3). *Cluster-critical state* (member-critical state).

In this case, $\pi R^2 = \Theta(\frac{1}{n^\alpha})$, and we have $\alpha + 2\beta = 0$. It is the critical state between the cluster-sparse and cluster-dense state and also a transition phase. The cluster distribution is not so dense or so sparse, and d is medium.

The cluster scalability is shown in Figure 2. We will study its impact on the critical transmission range in the rest of this paper.

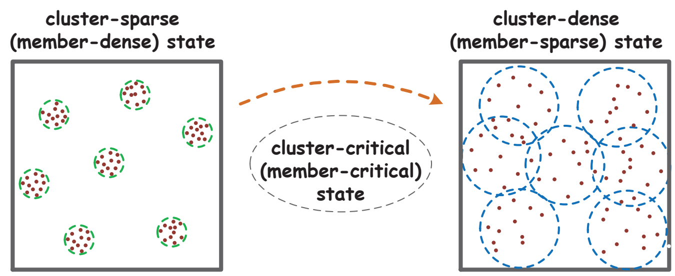

Fig. 2. Cluster Scalability.

E. Redefining Connectivity

Due to the initial network deployment and correlated mobility, the definition of connectivity in correlated mobile k-hop clustered networks is different from that in flat networks. It is similar to the that in mobile k-hop clustered networks, but still with some differences because of clustering.

We first define the *cluster-member connectivity*. A cluster member is connected if it can reach a cluster head within k slots and disconnected if it cannot reach any cluster head in k time slots, which is exactly the same to that in the mobile k-hop clustered networks. If all cluster members can be connected within k time slots, the clustered network has *full connectivity*. If we let $\mathcal{G}(n, \alpha, \beta, \gamma)$ denote the initial graph (clustered network) where a path connects all the n^α cluster heads. During each time slot $\lambda(\lambda = 1, 2, \ldots, k)$, an edge e_{ij} will be added between a cluster member i and a cluster head j into $\mathcal{G}(n, \alpha, \beta, \gamma)$ if the Euclidean distance between node i and j is less than $r(n, \alpha, \beta, \gamma)$ in the distance-based strategy where $r(n, \alpha, \beta, \gamma)$ is the *critical transmission range* in correlated mobile k-hop clustered networks from the *cluster-member connectivity prospective*. Then, $\mathcal{G}(n, \alpha, \beta, \gamma)$ has full connectivity if and only if any two cluster-member nodes can be connected by a path after k time slots.

Besides, we need to define *cluster connectivity* due to the existence of clusters. A cluster is connected if all the cluster-member nodes within it are connected (*they may connect to different cluster heads*), while a cluster is disconnected if at least one cluster member is disconnected. If all clusters are connected in k time slots, the network has full connectivity. In this definition, $\mathcal{G}(n, \alpha, \beta, \gamma)$ can be reduced to $\mathcal{G}(n, \alpha, \gamma)$ because we regard each cluster as a whole. However, it is impossible to rewrite $r(n, \alpha, \beta, \gamma)$ and give an exact transmission range to a cluster because cluster members are randomly distributed in the cluster. Therefore, we denote $r^c(n, \alpha, \beta, \gamma)$ to be the *critical transmission range* for each cluster member because we cannot omit β in this definition.

With the definition of *cluster-member connectivity* and *cluster connectivity*, we define the critical transmission range $r(n, \alpha, \beta, \gamma)$ and $r^c(n, \alpha, \beta, \gamma)$ formally.[2]

During any time slot λ, each cluster-member node would attempt to connect to the cluster-head nodes located in the circular area centered around the cluster member with the radius r which is the *transmission range* of each cluster member. Let $\mathcal{M}$ denote the event that all cluster-member nodes are connected in a given period $\Lambda(\Lambda \subseteq \{1, 2, \ldots, k\})$ and let $\mathbb{P}^\Lambda(\mathcal{M})$ be the corresponding probability. Then, the critical transmission range $r(n, \alpha, \beta, \gamma)$ is defined from the perspective of cluster-member connectivity as follows.

DEFINITION 1. *For a correlated mobile k-hop clustered network $\mathcal{G}(n, \alpha, \beta, \gamma)$ and from the perspective of cluster-member connectivity, $r(n, \alpha, \beta, \gamma)$ is the critical transmission range if*

$\lim_{n\to\infty} \mathbb{P}^\Lambda(\mathcal{M}) = 1$, if $r \geq cr(n, \alpha, \beta, \gamma)$, $c > 1$;

$\lim_{n\to\infty} \mathbb{P}^\Lambda(\mathcal{M}) < 1$, if $r \leq c'r(n, \alpha, \beta, \gamma)$, $c' < 1$.

If we consider the cluster connectivity and let $\mathcal{C}$ denote the event that all the m clusters are connected in period Λ and let $\mathbb{P}^\Lambda(\mathcal{C})$ denote the corresponding probability. The critical transmission range $r^c(n, \alpha, \beta, \gamma)$ is defined as follows.

DEFINITION 2. *For a correlated mobile k-hop clustered network $\mathcal{G}(n, \alpha, \beta, \gamma)$ and from the cluster connectivity perspective, $r^c(n, \alpha, \beta, \gamma)$ is the critical transmission range if*

$\lim_{m\to\infty} \mathbb{P}^\Lambda(\mathcal{C}) = 1$, if $r \geq cr^c(n, \alpha, \beta, \gamma)$, $c > 1$;

$\lim_{m\to\infty} \mathbb{P}^\Lambda(\mathcal{C}) < 1$, if $r \leq c'r^c(n, \alpha, \beta, \gamma)$, $c' < 1$.

Actually, we find that the cluster-member connectivity is equivalent to the cluster connectivity and we state this equivalence through the following theorem.

THEOREM 1 (CONNECTIVITY EQUIVALENCE). *Cluster connectivity and cluster-member connectivity are equivalent.*

Proof. We prove this by demonstrating: (1) if a correlated mobile k-hop clustered network $\mathcal{G}(n, \alpha, \beta, \gamma)$ has cluster-member connectivity, the corresponding $\mathcal{G}(n, \alpha, \gamma)$ must have cluster connectivity. This can be proved by the fact that during k time slots all the cluster members and clusters are connected at the same time; and (2) if $\mathcal{G}(n, \alpha, \beta, \gamma)$ is not connected from the perspective of cluster-member connectivity, the corresponding $\mathcal{G}(n, \alpha, \gamma)$ cannot have cluster connectivity. This is because there must exist at least one cluster member disconnected during k time slots, which results in some cluster disconnected. Thus, this theorem holds. □

Based on THEOREM 1, we could further find that for the same original correlated mobile k-hop clustered network, we have $r(n, \alpha, \beta, \gamma) = r^c(n, \alpha, \beta, \gamma)$ and state it as follows.

THEOREM 2 (CRITICAL TRANSMISSION RANGE EQUIVALENCE). *For an original correlated mobile k-hop clustered network $\mathcal{G}(n, \alpha, \beta, \gamma)$ and the corresponding $\mathcal{G}(n, \alpha, \gamma)$, we have $r(n, \alpha, \beta, \gamma) = r^c(n, \alpha, \beta, \gamma)$.*

Proof. We prove this theorem as follows:

(1) if $\mathcal{G}(n, \alpha, \beta, \gamma)$ is fully connected, due to DEFINITION 1, we have

$\lim_{n\to\infty} \mathbb{P}^\Lambda(\mathcal{M}) = 1$, if $r \geq cr(n, \alpha, \beta, \gamma)$, $c > 1$;

$\lim_{n\to\infty} \mathbb{P}^\Lambda(\mathcal{M}) < 1$, if $r \leq c'r(n, \alpha, \beta, \gamma)$, $c' < 1$.

Because of the CONNECTIVITY EQUIVALENCE proved in THEOREM 1 and the fact that $m \to \infty$ as $n \to \infty$, we obtain

$\lim_{m\to\infty} \mathbb{P}^\Lambda(\mathcal{C}) = 1$, if $r \geq cr^c(n, \alpha, \beta, \gamma)$, $c > 1$;

$\lim_{m\to\infty} \mathbb{P}^\Lambda(\mathcal{C}) < 1$, if $r \leq c'r^c(n, \alpha, \beta, \gamma)$, $c' < 1$.

Therefore, we have $r^c(n, \alpha, \beta, \gamma) = r(n, \alpha, \beta, \gamma)$ for $\mathcal{G}(n, \alpha, \gamma)$.

(2) if $\mathcal{G}(n, \alpha, \gamma)$ is fully connected, due to DEFINITION 1, we obtain

$\lim_{m\to\infty} \mathbb{P}^\Lambda(\mathcal{C}) = 1$, if $r \geq cr^c(n, \alpha, \beta, \gamma)$, $c > 1$;

$\lim_{m\to\infty} \mathbb{P}^\Lambda(\mathcal{C}) < 1$, if $r \leq c'r^c(n, \alpha, \beta, \gamma)$, $c' < 1$.

Because of THEOREM 1 and the observation that $n \to \infty$ as $m \to \infty$, we get

$\lim_{n\to\infty} \mathbb{P}^\Lambda(\mathcal{M}) = 1$, if $r \geq cr(n, \alpha, \beta, \gamma)$, $c > 1$;

$\lim_{n\to\infty} \mathbb{P}^\Lambda(\mathcal{M}) < 1$, if $r \leq c'r(n, \alpha, \beta, \gamma)$, $c' < 1$.

Thus, we $r(n, \alpha, \beta, \gamma) = r^c(n, \alpha, \beta, \gamma)$ for $\mathcal{G}(n, \alpha, \beta, \gamma)$.

Now, we finish the proof. This theorem lays the foundation of later analysis. □

Considering THEOREM 1 and 2, we will use r_c to denote the *critical transmission range* uniformly without leading to ambiguity. Besides, we use $\mathcal{G}(n, \alpha, \beta, \gamma, r_c)$ consistently to denote the correlated mobile k-hop clustered network with n cluster-member nodes, n^α cluster-head nodes and n^γ clusters each of which is with radius $R = \Theta(n^\beta)$ where a cluster member can connect to a cluster-head node if the Euclidean distance between them is at most r_c.

IV. MAIN RESULTS

We summarize our main results in this thesis as follows:

(1). *Cluster-sparse state* ($\alpha + 2\beta < 0$): $r_c = \sqrt{\frac{\gamma \log n}{k\pi n^\alpha}}$, where $\frac{\gamma}{k} < \alpha \leq 1, 0 < \gamma \leq 1$.

(2). *Cluster-dense state* ($\alpha + 2\beta > 0$): $r_c = \sqrt{\frac{\log n}{k\pi n^\alpha}}$, where $\frac{1}{k} < \alpha \leq 1, 0 < \gamma \leq 1$.

[2]Note that $r(n, \alpha, \beta, \gamma)$ and $r^c(n, \alpha, \beta, \gamma)$ are proposed from different prospectives and may not be equal. But in the following, we will prove that they are substantially equivalent.

V. The Critical Transmission Range r_c of the Correlated Mobile K-hop Clustered Networks for the Cluster-Sparse State

In this case, we use $\mathbb{P}_{f\_css}(n,\alpha,\beta,\gamma,r_c)$ to denote the probability that $\mathcal{G}(n,\alpha,\beta,\gamma,r_c)$ has some cluster[3] disconnected for the cluster-sparse state and our main result of this case is as follows.

Theorem 3. *In a correlated mobile k-hop clustered network $\mathcal{G}(n,\alpha,\beta,\gamma,r_c)$ for the cluster-sparse state, the critical transmission range is $r_c=\sqrt{\frac{\gamma\log n}{k\pi n^{\alpha}}}$, where $\frac{\gamma}{k}<\alpha\leq 1, \alpha+2\beta<0$* and $0<\gamma\leq 1$.

A. Necessary Condition for Theorem 3

On account of $\alpha+2\beta<0$ and the given r_c in this case, we have $\pi R^2=o(\frac{1}{n^{\alpha}})$ and $r_c=\omega(R)$. This condition is the theoretical expression of the prominent clustering property and is of great importance which serves as the basis for our analysis in this case.

We prove the necessary condition of r_c of Theorem 3 by the classical methodology introduced in [7] i.e., to demonstrate that $\mathbb{P}_{f\_css}(n,\alpha,\beta,\gamma,r_c)$ is strictly larger than zero.

We first give the following technical lemma which will be referred in the proof of the necessity of Theorem 3.

Lemma 1. *If $r=\sqrt{\frac{\gamma\log n+\xi}{k\pi n^{\alpha}}}, \alpha+2\beta<0, \frac{\gamma}{k}<\alpha\leq 1$, for any fixed $0<\theta<1$ and sufficiently large n, we have*

$$m\Big(1-\pi(r+R)^2\Big)^{kn^{\alpha}}\geq\theta e^{-\xi} \tag{1}$$

where r is the transmission range and $R=\Theta(n^{\beta})$.

Proof. Because $r=\sqrt{\frac{\gamma\log n+\xi}{k\pi n^{\alpha}}}=\Theta(r_c)$, we still have $r=\omega(R)$. We take the logarithm of the left hand side, use the power series expansion for $\log(1-x)$, and get

$$\begin{aligned}
&\log\big(\text{L.H.S. of (1)}\big)\\
=&\log m+kn^{\alpha}\log\big(1-\pi(r+R)^2\big)\\
=&\log m-kn^{\alpha}\sum_{i=1}^{\infty}\frac{\big(\pi(r+R)^2\big)^i}{i}\\
=&\log m-kn^{\alpha}\Big(\sum_{i=1}^{2}\frac{\big(\pi(r+R)^2\big)^i}{i}+\delta(n)\Big)
\end{aligned}\tag{2}$$

Here, for all sufficiently large n:

$$\begin{aligned}
\delta(n)=&\sum_{i=3}^{\infty}\frac{\big(\pi(r+R)^2\big)^i}{i}\\
\leq&\frac{1}{3}\sum_{i=3}^{\infty}\big(\pi(r+R)^2\big)^i\\
=&\frac{\big(\pi(r+R)^2\big)^3}{3\big(1-\pi(r+R)^2\big)}\leq\frac{\big(\pi(r+R)^2\big)^2}{3}
\end{aligned}\tag{3}$$

[3]Note that we study the probability of cluster disconnection in this case because we derive r_c from the cluster connectivity perspective. Therefore, we mainly investigate the situation where some cluster is disconnected in $\mathcal{G}(n,\alpha,\beta,\gamma,r_c)$.

Substituting $\delta(n)$ and $r=\sqrt{\frac{\gamma\log n+\omega}{k\pi n^{\alpha}}}$ in (2), and for all sufficient large n we get

$$\begin{aligned}
&\log\big(\text{L.H.S of (1)}\big)\\
\geq&\log m-kn^{\alpha}\Big(\sum_{i=1}^{2}\frac{\big(\pi(r+R)^2\big)^i}{i}+\frac{\big(\pi(r+R)^2\big)^2}{3}\Big)\\
&(\text{since } r=\omega(R),\ \text{i.e. },r>R)\\
\geq&\log m-kn^{\alpha}\Big(\pi(r+R)^2+\frac{5}{6}\big(\pi(2r)^2\big)^2\Big)\\
\geq&-\xi-k\pi\Theta(n^{\alpha+2\beta})-2\sqrt{k\pi}\Theta(n^{\frac{\alpha+2\beta}{2}}\sqrt{\gamma\log n+\xi})\\
&-\frac{40(\gamma\log n+\xi)^2}{3kn^{\alpha}}\\
&(\text{since } \alpha+2\beta<0,\alpha>0)\\
\geq&-\xi-\epsilon
\end{aligned}\tag{4}$$

Take the exponent of both sides and let $\theta=e^{-\epsilon}<1$, and the result follows. Therefore, we finished the proof. We should note that θ can be sufficiently near to 1. □

This technical lemma will be used in the proof of Theorem 4 which is presented as follows to bound some important terms.

Theorem 4. If $r=\sqrt{\frac{\gamma\log n+\xi(n)}{k\pi n^{\alpha}}}, \alpha+2\beta<0, \frac{\gamma}{k}<\alpha\leq 1, 0<\gamma\leq 1$, and $\lim_{n\to\infty}\xi(n)=\xi<+\infty$, we have

$$\liminf_{n\to\infty}\mathbb{P}_{f\_css}(n,\alpha,\beta,\gamma,r_c)\geq e^{-\xi}(1-e^{-\xi})$$

Proof. We first study the case where $r=\sqrt{\frac{\gamma\log n+\xi}{k\pi n^{\alpha}}}$ with a fixed ξ. Let $\mathcal{F}_j$ denote the the event that the jth cluster $\mathcal{C}_j$ is disconnected in k time slots. Then we have

$$\begin{aligned}
&\mathbb{P}_{f_css}(n,\alpha,\beta,\gamma,r_c)\\
\geq&\sum_{i=1}^{m}\Big(\mathbb{P}(\mathcal{F}_i)-\sum_{j\neq i}\mathbb{P}(\mathcal{F}_i\cap\mathcal{F}_j)\Big)\\
=&\sum_{i=1}^{m}\mathbb{P}(\mathcal{F}_i)-\sum_{i=1}^{m}\sum_{j\neq i}\Big(\mathbb{P}(\mathcal{F}_i\cap\mathcal{F}_j)\Big)
\end{aligned}\tag{5}$$

Then we evaluate the two terms on the right side of (5), respectively. For the first term, we have

$$\mathbb{P}(\mathcal{F}_j)\geq\Big(1-\pi(r+R)^2\Big)^{kn^{\alpha}} \tag{6}$$

and for the second term, we have

$$\begin{aligned}
&\mathbb{P}(\mathcal{F}_i^{\lambda}\cap\mathcal{F}_j^{\lambda})\\
\leq&4\pi(r+R)^2\big(1-\pi(r-R)^2\big)^{n^{\alpha}}+\big(1-2\pi(r-R)^2\big)^{n^{\alpha}}\\
\leq&4\pi(r+R)^2e^{-\pi n^{\alpha}(r-R)^2}+e^{-2\pi n^{\alpha}(r-R)^2}
\end{aligned}\tag{7}$$

Here we use the inequality:

$$1-x\leq e^{-x}\ \text{for}\ x\in[0,1] \tag{8}$$

Using (6) and (7) in (5), we obtain

$$
\begin{aligned}
&\mathbb{P}_{f\_css}(n,\alpha,\beta,\gamma,r_c)\\
\geq& m\big(1-\pi(r+R)^2\big)^{kn^\alpha} - m(m-1)\Big(4\pi(r+R)^2 e^{-\pi n^\alpha (r-R)^2}\\
&+ e^{-2\pi n^\alpha (r-R)^2}\Big)^k\\
\geq& \theta e^{-\xi} - m^2 e^{-2\pi k n^\alpha (r-R)^2}\left(4\pi(r+R)^2 e^{\pi n^\alpha (r-R)^2}+1\right)^k\\
=& \theta e^{-\xi} - e^{-2\xi} e^{-2k\pi\Theta(n^{\alpha+2\beta})+4\sqrt{k\pi}\Theta\left(n^{\frac{\alpha+2\beta}{2}}\sqrt{\gamma\log n+\xi}\right)}\\
&\Bigg(4\pi e^{\frac{\xi}{k}+\pi\Theta(n^{\alpha+2\beta})+2\sqrt{\frac{\pi}{k}}\Theta\left(n^{\frac{\alpha+2\beta}{2}}\sqrt{\gamma\log n+\xi}\right)}\\
&\Big(\frac{\gamma\log n+\xi}{k\pi n^{\alpha-\frac{\gamma}{k}}}+\Theta(n^{2\beta+\frac{\gamma}{k}})+2\Theta\big(\sqrt{\frac{\gamma\log n+\xi}{k\pi n^{\alpha-2\beta-\frac{2\gamma}{k}}}}\big)\Big)+1\Bigg)^k\\
&(\text{since } \alpha+2\beta<0, \alpha>\frac{\gamma}{k}, 2\beta+\frac{\gamma}{k}<0, \alpha-2\beta-\frac{2\gamma}{k}>0)\\
\geq& \theta e^{-\xi} - (1+\epsilon)e^{-2\xi}\\
&\text{for any } \epsilon>0 \text{ and for all } n>N(\epsilon,\theta,\xi).
\end{aligned}
\tag{9}
$$

Let ξ be a function $\xi(n)$ with $\lim_{n\to\infty}\xi(n)=\xi<+\infty$. Then for all $n\geq N'(\epsilon)$ and any $\epsilon>0$, $\xi(n)\leq\xi+\epsilon$. Because the disconnection probability $\mathbb{P}_{f\_css}(n,\alpha,\beta,\gamma,r_c)$ is monotonically decreasing in ξ, then we have

$$\mathbb{P}_{f\_ccs}(n,\alpha,\beta,\gamma,r_c)\geq\theta e^{-(\xi+\epsilon)}-(1+\epsilon)e^{-2(\xi+\epsilon)} \tag{10}$$

for all $n\geq\max\{N(\epsilon,\theta,\xi+\epsilon),N'(\epsilon)\}$. Taking limits and we have

$$\liminf_{n\to\infty}\mathbb{P}_{f\_ccs}(n,\alpha,\beta,\gamma,r_c)\geq\theta e^{-(\xi+\epsilon)}-(1+\epsilon)e^{-2(\xi+\epsilon)} \tag{11}$$

Since this holds for all $\epsilon>0$ and $\theta<1$, we can the result and this theorem holds. □

Consequently, considering the connectivity equivalence, we have the following corollaries to prove the necessity part of THEOREM 3 both from the cluster-member and cluster connectivity perspective.

COROLLARY 3.1. *In the cluster-sparse state of correlated mobile k-hop clustered networks, the network is to have* ***disconnected clusters*** *with positive probability bound away from zero if* $r=\sqrt{\frac{\gamma\log n+\xi(n)}{k\pi n^\alpha}}$ $(\lim_{n\to\infty}\xi(n)<+\infty)$.

COROLLARY 3.2. *In the cluster-sparse state of correlated mobile k-hop clustered networks, the network is to have* ***disconnected cluster members*** *with positive probability bound away from zero if* $r=\sqrt{\frac{\gamma\log n+\xi(n)}{k\pi n^\alpha}}$ $(\lim_{n\to\infty}\xi(n)<+\infty)$.

B. Sufficient Condition of r_c of Theorem 3

We still base our analysis on the cluster connectivity and prove the sufficiency of THEOREM 3. Assume there are at most n sessions during k time slots and let $r=cr_c(c>1)$. Therefore, due to THEOREM 1, it suffices to show that

$$\lim_{n\to\infty}\mathbb{P}(\bigcup_{j=1}^m\mathcal{F}_j)=0$$

Then we use union bound to bound $\mathbb{P}(\bigcup_{j=1}^m\mathcal{F}_j)$:

$$
\begin{aligned}
\mathbb{P}(\bigcup_{j=1}^m\mathcal{F}_j)&\leq\sum_{j=1}^m\mathbb{P}(\mathcal{F}_j)\\
&\leq\sum_{j=1}^m\Big(1-\pi(r-R)^2\Big)^{kn^\alpha}\\
(\text{due to (8)}) &\leq me^{-\pi(r-R)^2kn^\alpha}
\end{aligned}
\tag{12}
$$

Substituting $r=cr_c$ into (12), we obtain

$$
\begin{aligned}
me^{-\pi(r-R)^2kn^\alpha}&=me^{-k\pi n^\alpha\left(c\sqrt{\frac{\gamma\log n}{k\pi n^\alpha}}-R\right)^2}\\
&=\frac{m}{e^{c^2\gamma\log n}}\cdot\frac{e^{2ck\pi n^\alpha\sqrt{\frac{\gamma\log n}{k\pi n^\alpha}}R}}{e^{k\pi n^\alpha R^2}}
\end{aligned}
\tag{13}
$$

Since $c>1$, $\gamma>0$ and $\alpha+2\beta<0$, we take limits of two factors on the right hand side of Eq.(40) and get the following results.

$$\lim_{n\to\infty}\frac{m}{e^{c^2\gamma\log n}}=\lim_{n\to\infty}\frac{1}{n^{(c^2-1)\gamma}}=0$$

$$\lim_{n\to\infty}\frac{e^{2ck\pi n^\alpha\sqrt{\frac{\gamma\log n}{k\pi n^\alpha}}R}}{e^{k\pi n^\alpha R^2}}=\lim_{n\to\infty}\frac{e^{2c\sqrt{k\pi\gamma}\Theta(n^{\frac{\alpha+2\beta}{2}}\log n)}}{e^{k\pi\Theta(n^{\alpha+2\beta})}}=1$$

Then, we take limits of both sides in (13) and the result follows. Thus, we finish the proof of the sufficient condition of r_c of THEOREM 3.

VI. THE CRITICAL TRANSMISSION RANGE r_c OF THE CORRELATED MOBILE K-HOP CLUSTERED NETWORKS FOR THE CLUSTER-DENSE STATE

In this case, we consider the scenario where $\pi R^2=\omega(\frac{1}{n^\alpha})$, i.e., $\alpha+2\beta>0$. Here we assume $\alpha+2\beta>\frac{1}{k}$ because this condition is needed in Theorem 5. Let $\mathbb{P}_{f\_cds}(n,\alpha,\beta,\gamma,r_c)$ denote the probability that $\mathcal{G}(n,\alpha,\beta,\gamma,r_c)$ has some cluster member[4] disconnected for the cluster-dense state. Our main result is given as follows.

THEOREM 5. *In a correlated mobile k-hop clustered network* $\mathcal{G}(n,\alpha,\beta,\gamma,r_c)$ *for the cluster-dense state, the critical transmission range is* $r_c=\sqrt{\frac{\log n}{k\pi n^\alpha}}$, *where* $\frac{1}{k}<\alpha\leq 1,\alpha+2\beta>\frac{1}{k}$, and $0<\gamma\leq 1$.

Combining $\alpha+2\beta>0$ with the given r_c, we can further get $r_c=o(R)$ which is the foundation of later analysis.

A. Necessary Condition of r_c of Theorem 5

The methodology in this case is different from that in the cluster-sparse case. Instead of bounding $\mathbb{P}_{f\_cds}(n,\alpha,\beta,\gamma,r_c)$ from the cluster connectivity perspective, we investigate this case mainly from the perspective of cluster-member connectivity because the cluster members behave more like independent nodes now. But, we cannot simply use the solutions in previous works because the network deployment enforces

[4]Here, we consider the disconnected cluster members because we provide our analysis from cluster-member connectivity perspective.

clustering. Therefore, we propose a new solution which considers both the clustering and independence characteristic.

First, we give a technical lemma as follows.

LEMMA 2. *If* $r = \sqrt{\frac{\log n + \xi}{k\pi n^\alpha}}, \alpha + \beta > \frac{1}{k}, \frac{1}{k} < \alpha \leq 1$, *for any fixed* $\theta < 1$ *and sufficiently large* n, *we have*

$$n\big(1 - \pi r^2\big)^{kn^\alpha} \geq \theta e^{-\xi} \tag{14}$$

Proof. Employing the similar technique in the proof of LEMMA 1, for all sufficient large n, we can obtain the following results.

$$\begin{aligned}
&\log\big(\text{L.H.S. of (14)}\big) \geq \\
&\log n - kn^\alpha\big(\pi r^2 + \frac{5}{6}(\pi r^2)^2\big) \\
&(\text{since } \pi r^2 < \frac{1}{2} \text{ for all sufficient large } n) \\
=& -\xi - \frac{5(\log n + \xi)^2}{6kn^\alpha} \\
\geq& -\xi - \epsilon
\end{aligned} \tag{15}$$

Take the exponent of both sides in (15), let $\theta = e^{-\epsilon} < 1$ and the result follows. Therefore, we finished the proof. Note that θ can be sufficiently near to 1. □

Then, we have the following theorem.

THEOREM 6. *If* $r = \sqrt{\frac{\log n + \xi(n)}{k\pi n^\alpha}}, \alpha + 2\beta > 0, \frac{1}{k} < \alpha \leq 1, 0 < \gamma \leq 1$ *and* $\lim_{n\to\infty} \xi(n) = \xi < +\infty$, *we have*

$$\liminf_{n\to\infty} \mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c) \geq e^{-\xi}(1 - e^{-\xi})$$

Proof. Different from the proof of THEOREM 3, we regard the problem from the respect of cluster-member connectivity. Let $f_{j\kappa}^\lambda$ denote the event that the session initiated by the κth node in $\mathcal{C}_j$ at time slot λ is failed and $f_{j\kappa}$ denote that sessions initiated by the κth node in $\mathcal{C}_j$ during k time slots are all failed. We can get

$$\begin{aligned}
&\mathbb{P}_{f_cds}(n, \alpha, \beta, \gamma, r_c) \\
\geq& \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\Big(\mathbb{P}(f_{j\kappa}) - \sum_{\kappa'\neq\kappa}\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'}) - \sum_{i\neq j}\mathbb{P}(f_{j\kappa}\cap\mathcal{F}_i)\Big) \\
=& \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa}) - \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{\kappa'\neq\kappa}\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'}) \\
&- \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{i\neq j}\mathbb{P}(f_{j\kappa}\cap\mathcal{F}_i)
\end{aligned} \tag{16}$$

Then we evaluate the three terms on the right hand side of (16), respectively.

For the first term, we have

$$\mathbb{P}(f_{j\kappa}) \geq (1 - \pi r^2)^{kn^\alpha} \tag{17}$$

The second term is to estimate the probability that two sessions in the same cluster are failed. By considering the possible positions of two cluster members and employing the conditional probability we have

$$\begin{aligned}
&\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'}) \\
\leq&\Big(1\cdot(1 - 2\pi r^2)^{n^\alpha} + \frac{4r^2}{R^2}(1 - \pi r^2)^{n^\alpha}\Big)^k \\
&(\text{due to (8)}) \\
\leq&\Big(e^{-2\pi n^\alpha r^2} + \frac{4r^2}{R^2}e^{-\pi n^\alpha r^2}\Big)^k
\end{aligned} \tag{18}$$

For the third term, we need to compute the probability that one is a failed session in cluster $\mathcal{C}_j$ and the other one is a disconnected cluster $\mathcal{C}_i (i \neq j)$. In this situation, we should consider both the cluster connectivity and cluster-member connectivity, because: (1) from the perspective of the network deployment, the positions of cluster-member nodes in $\mathcal{C}_i$ are interrelated, so we cannot consider these nodes separately; and (2) viewing the probability that $\mathcal{C}_i$ is disconnected is a more accurate estimation than considering the cluster members. This is the main difference from the previous literature and the solution in the cluster-sparse state. Based on these intuitions, we bound the third term as follows.

$$\begin{aligned}
&\mathbb{P}(f_{j\kappa}\cap\mathcal{F}_i) \\
\leq&\Big(1\cdot(1 - 2\pi r^2)^{n^\alpha} + \pi(2r + R)^2(1 - \pi r^2)^{n^\alpha}\Big)^k \\
&(\text{due to (8)}) \\
\leq&\Big(e^{-2\pi n^\alpha r^2} + \pi(2r + R)^2 e^{-\pi n^\alpha r^2}\Big)^k
\end{aligned} \tag{19}$$

First, we study the case where $r = \sqrt{\frac{\log n + \xi}{k\pi n^\alpha}}$ with a fixed ξ. By substituting (17)-(19) and r into (16) we can bound $\mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c)$ as follows.

$$\begin{aligned}
&\mathbb{P}_{f_cds}(n, \alpha, \beta, \gamma, r_c) \\
\geq&\sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(f_{j\kappa}) - \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{\kappa'\neq\kappa}\mathbb{P}(f_{j\kappa}\cap f_{j\kappa'}) \\
&- \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\sum_{i\neq j}\mathbb{P}(f_{j\kappa}\cap\mathcal{F}_i) \\
&(\text{due to (17)-(19)}) \\
\geq& n(1 - \pi r^2)^{kn^\alpha} - n\varpi\Big(e^{-2\pi n^\alpha r^2} + \frac{4r^2}{R^2}e^{-\pi n^\alpha r^2}\Big)^k \\
&- nm\Big(e^{-2\pi n^\alpha r^2} + \pi(2r + R)^2 e^{-\pi n^\alpha r^2}\Big)^k \\
&(\text{due to LEMMA 2}) \\
\geq&\theta e^{-\xi} - n^{2-\gamma}e^{-2k\pi n^\alpha r^2}\Big(1 + \frac{4r^2}{R^2}e^{\pi n^\alpha r^2}\Big)^k \\
&- n^{1+\gamma}e^{-2k\pi n^\alpha r^2}\Big(1 + \pi(2r + R)^2 e^{\pi n^\alpha r^2}\Big)^k \\
=&\theta e^{-\xi} - n^{-\gamma}e^{-2\xi}\Big(1 + \frac{4(\log n + \xi)}{k\pi\Theta(n^{\alpha + 2\beta - \frac{1}{k}})}e^{\frac{\xi}{k}}\Big)^k - n^{\gamma - 1}e^{-2\xi}\Big(1 + \\
&\Big(\frac{4(\log n + \xi)}{kn^{\alpha - \frac{1}{k}}} + \pi\Theta(n^{2\beta + \frac{1}{k}}) + \Theta\big(\sqrt{\frac{4\pi\log n}{kn^{\alpha - 2\beta - \frac{2}{k}}}}\big)\Big)e^{\frac{\xi}{k}}\Big)^k
\end{aligned}$$

Then, we consider two cases in terms of different γ and bound $\mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c)$, respectively. Because we have $\alpha + 2\beta > \frac{1}{k}, \alpha > \frac{1}{k}, 2\beta + \frac{1}{k} < 0, \alpha - 2\beta - \frac{2}{k} > 0$, it is easy to get the following results.

Case 1: When $\gamma = 1$, we can further bound $\mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c)$ as follows.

$$\mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c) \geq \theta e^{-\xi} - \epsilon - e^{-2\xi}(1 + \epsilon) \tag{20}$$

Case 2: When $0 < \gamma < 1$, we can further bound $\mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c)$ as follows.

$$\mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c) \geq \theta e^{-\xi} - \epsilon \tag{21}$$

In all, no matter what γ is, we have (21) for any $\epsilon > 0$ and for all $n > N(\epsilon, \theta, \xi)$.

Then, we let ξ be a function $\xi(n)$ with $\lim_{n\to\infty} \xi(n) = \xi < +\infty$. Then for all $n \geq N'(\epsilon)$, any $\epsilon > 0$ and any $\xi(n) \leq \xi + \epsilon$. Considering that $\mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c)$ is monotonically decreasing in ξ, we have

$$\mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c) \geq \theta e^{-(\xi+\epsilon)} - \epsilon - (1 + \epsilon)e^{-2(\xi+\epsilon)}$$

for all $n \geq \max\{N(\epsilon, \theta, \xi + \epsilon), N'(\epsilon)\}$. Taking limits and we have

$$\liminf_{n\to\infty} \mathbb{P}_{f\_cds}(n, \alpha, \beta, \gamma, r_c) \geq \theta e^{-(\xi+\epsilon)} - \epsilon - (1+\epsilon)e^{-2(\xi+\epsilon)}$$

Since this holds for all $\epsilon > 0$ and $\theta < 1$, we can finish the proof. □

We can derive the following corollary to prove the necessity part of THEOREM 5.

COROLLARY 5.1. *In the cluster-dense state of correlated mobile k-hop clustered networks, the network is to have* ***disconnected cluster members*** *with positive probability bound away from zero if* $r = \sqrt{\frac{\log n + \xi(n)}{k\pi n^{\alpha}}}(\lim_{n\to\infty} \xi(n) < +\infty)$.

By employing THEOREM 1, we can illustrate this from the cluster connectivity perspective and obtain the following corollary.

COROLLARY 5.2. *In the cluster-dense state of correlated mobile k-hop clustered networks, the network is to have* ***disconnected clusters*** *with positive probability bound away from zero if* $r = \sqrt{\frac{\log n + \xi(n)}{k\pi n^{\alpha}}}(\lim_{n\to\infty} \xi(n) < +\infty)$.

B. Sufficient Condition of r_c of Theorem 5

The idea of this proof is to treat cluster members as non-clustered nodes because the clustering characteristic is not obvious in this case. We suppose there are at most n sessions during k time slots. Let each node have the transmission range $r = cr_c$, where $c > 1$. Then, we use the union bound and obtain the proof as follows.

$$\begin{aligned}
\mathbb{P}\Big(\bigcup_{j=1}^{m}\big(\bigcup_{\kappa=1}^{\varpi}(\bigcap_{\lambda=1}^{k} f_{j\kappa}^{\lambda})\big)\Big) &\leq \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\mathbb{P}(\bigcap_{\lambda=1}^{k} f_{j\kappa}^{\lambda}) \\
&\leq \sum_{j=1}^{m}\sum_{\kappa=1}^{\varpi}\Big((1-\pi r^2)^{n^{\alpha}}\Big)^{k} \\
\text{(due to (8))} &\leq n e^{-k\pi n^{\alpha} r^2} \\
&= \frac{1}{n^{c^2-1}}
\end{aligned}$$

Due to $c > 1$, taking limits of both sides, we can easily get the following result.

$$\lim_{n\to\infty} \mathbb{P}\Big(\bigcup_{j=1}^{m}\big(\bigcup_{\kappa=1}^{\varpi}(\bigcap_{\lambda=1}^{k} f_{j\kappa}^{\lambda})\big)\Big) = 0$$

Here, we finish the proof of the sufficient condition of r_c of THEOREM 5.

VII. CONCLUSIONS

In this paper, we have proposed the *correlated mobile k-hop clustered networks* model to explore the impact of *correlated mobility* and *cluster scalability* on the connectivity performance in large-scale wireless networks while bounding the transmission delay as $\Theta(1)$ for any finite k. The critical transmission range r_c have been investigated. We show that there are three states for the correlated mobile k-hop clustered networks, *cluster-sparse, cluster-dense* and *cluster-critical state*, and we prove the exact value of r_c for the first two states under certain conditions. Based on these results, we discover that cluster scalability can on some level control the degree of correlated mobility and their impact on connectivity is largely induced by the dynamics of cluster scales.

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