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Auction-Based Power Allocation for Multiuser Two-Way Relaying Networks with Network Coding

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***Abstract*—In this paper, we consider the power allocation problem in a network coded multiuser two-way relaying network, where multiple pairs of users exchange information with the help of a relay node. An auction-based power allocation scheme is proposed to solve the power competition among multiple user pairs. To maximize the utility of the whole pair, each two users in the pair bid as a single player, with the total pair payment shared by the pair users in proportion to the amount of power they obtained. The convergence of the proposed auction game is theoretically proved by using a standard function. Moreover, the outage behavior is systematically analyzed and a closed form of the system outage probability is derived. Finally, the performance of the proposed scheme is verified by the simulation results.**

***Index Terms*—Power allocation, two-way relaying, network coding, auction, outage**

I. Introduction

With the increasing demands on wireless ad hoc and peer-to-peer communications, two-way relaying emerges as a promising technique for relay-assisted information exchange [1]. For traditional two-way relaying system, in order to avoid interference at the relay node, four time slots are usually required for one complete information exchange between two source nodes. The seminal work of Ahlswede et al. [2] demonstrated that network coding achieves the multicast network capacity. Inspired by this innovative throughput-boosting approach, network coding has been proposed in two-way relaying to improve transmission efficiency [3].

In a network-coded (i.e., analog network coding [4]) two-way relaying system, the required time slots for one round information exchange can be reduced from four to two. In the first time slot, two users transmit simultaneously to the relay. In the second time slot, the relay broadcasts the additive message by amplify-and-forward operations.

The topics on network coding based two-way relaying, such as rate regions [5], relay selection [6], relaying protocols [1], and beamforming structure, have been studied in the literature. Xue et al. [5] investigated the achievable end-to-end rate regions by MAC-layer network coding and physcial-layer network coding. In [6], a new decode-and-forward two-way relaying protocol was proposed, with which one single best relay can be selected. Chen et al. [1] presented a jointly demodulate-and-XOR forward relaying scheme for power savings and user capacity improvement. In [3], an efficient algorithm to compute the optimal beamforming matrix was proposed for a multi-antenna relay channel. However, the critical issue of how to allocate power at the relay node for multiple competing user pairs is still untouched in the literature.

The overall performance of a cooperative relaying system largely depends on resource allocation schemes. In [7], an outage probability bound is derived, based on which the power allocation is optimized for the multinode amplify-and-forward protocol. Zhao et al. [8] considered optimal power allocation among multiple relay nodes for maximum system throughput with total and individual power constraints. The authors in [9] studied the optimal transmit power allocation problem for various cooperation protocols with multiple relay nodes. However, little attention has been focused on power allocation in two-way relaying or in cooperative communications with network coding. To fill in the blanks, in this work, we consider optimal power allocation in network-coded two-way relaying systems with game theory.

Game theory [10]- [12] is a simple and useful tool for studying interaction and competition among autonomous users in wireless networks. In [10], two auction mechanisms are designed to coordinate the relay power allocation for cooperative communications. The problem of resource sharing between two selfish nodes in cooperative relaying networks was considered in [11], and the Nash bargaining solution was used to achieve a win-win strategy. A distributed game-theoretical framework over multiuser cooperative networks was proposed in [12] to achieve optimal relay selection and power allocation. Unlike these strategies, in our auction game, the relay power is coordinated among multiple user pairs that want to exchange information, instead of allocated for individual users.

In this paper, we address the power allocation problem in a network coded multiuser two-way relaying network, where multiple pairs of users communicate with their partners via a common relay node. Our main contributions are as follows: First, we propose a relay power allocation scheme on the basis of an auction game, in which two users in each pair bid as a single player for a maximum utility of the whole pair, and the total pair payment is shared by the pair users in proportion to the amount of the power they obtained. Second, the convergence of the proposed auction game is theoretically proved by using a standard function. Last, we analyze the

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system outage behavior and derive a closed form of the system outage probability.

The rest of this paper is organized as follows: Sec. II presents the system model. In Sec. III, power auction mechanism is described. The system outage probability is analyzed in Sec. IV. Simulation results are shown in Sec. V. Finally, Sec. VI concludes the paper.

II. System Model

Consider a two-way relaying network with a set $\mathcal{N} = (1, 2, ..., N)$ of user pairs and a single relay node, where each user pair a_i-b_i, $i \in \mathcal{N}$, intends to exchange information with the help of relay r. Assume that each user has a single antenna and works in a half-duplex manner. Also, assume that there is no direct transmission between each pair of users. We employ the analog network coding and amplify-and-forward relaying protocol at the relay. To avoid inter-pair interference at the relay, we allocate non-overlapping frequency bands for different user pairs.

In the first time slot, user a_i and b_i in pair i transmit simultaneously to relay r with a fixed transmit power P. Thus, the combined signal received at relay r can be written as

$$Y_i^r = \sqrt{PG_{a_i}^r}X_{a_i} + \sqrt{PG_{b_i}^r}X_{b_i} + n_{r_i} \tag{1}$$

where $G_{a_i}^r$ and $G_{b_i}^r$ respectively are channel gains from user a_i and b_i to relay r, X_{a_i} and X_{b_i} are information symbols transmitted by a_i and b_i, respectively, and $n_{\{\cdot\}}$ is the additive white Gaussian noise with variance σ^2.

In the second time slot, relay r amplifies Y_i^r and forwards it back to both users. At this time slot, the signal received by a_i is

$$Y_r^{a_i} = \sqrt{P_{NC_i}G_r^{a_i}}X_{NC_i} + n_{a_i} \tag{2}$$

where X_{NC_i} is the normalized energy data symbol of Y_i^r, which is given by

$$X_{NC_i} = \frac{\sqrt{PG_{a_i}^r}X_{a_i} + \sqrt{PG_{b_i}^r}X_{b_i} + n_{r_i}}{\sqrt{PG_{a_i}^r + PG_{b_i}^r + \sigma^2}} \tag{3}$$

The signal $Y_r^{a_i}$ received at user a_i contains the information of both X_{a_i} and X_{b_i}, where X_{a_i} is a self-interference that can be completely removed since user a_i knows the data sent. The required signal at a_i is

$$\hat{Y}_r^{a_i} = \frac{\sqrt{P_{NC_i}G_r^{a_i}}}{\sqrt{PG_{a_i}^r + PG_{b_i}^r + \sigma^2}}(\sqrt{PG_{b_i}^r}X_{b_i} + n_{r_i}) + n_{a_i} \tag{4}$$

Then the signal-to-noise ratio (SNR) of user b_i's signal obtained at a_i is

$$\Gamma_{b_i} = \frac{P_{NC_i}G_r^{a_i}PG_{b_i}^r}{\sigma^2(P_{NC_i}G_r^{a_i} + PG_{a_i}^r + PG_{b_i}^r + \sigma^2)} \tag{5}$$

Thus, the rate user b_i can get is

$$R_{b_i} = \frac{1}{2}W\log_2(1 + \Gamma_{b_i}) \tag{6}$$

where W is the signal bandwidth.

Similarly, the received signal, the useful signal, and the SNR at user b_i in the second time slot are respectively given by

$$Y_r^{b_i} = \sqrt{P_{NC_i}G_r^{b_i}}X_{NC_i} + n_{b_i} \tag{7}$$

$$\hat{Y}_r^{b_i} = \frac{\sqrt{P_{NC_i}G_r^{b_i}}}{\sqrt{PG_{a_i}^r + PG_{b_i}^r + \sigma^2}}(\sqrt{PG_{a_i}^r}X_{a_i} + n_{r_i}) + n_{b_i} \tag{8}$$

$$\Gamma_{a_i} = \frac{P_{NC_i}G_r^{b_i}PG_{a_i}^r}{\sigma^2(P_{NC_i}G_r^{b_i} + PG_{a_i}^r + PG_{b_i}^r + \sigma^2)} \tag{9}$$

where $G_r^{a_i}$ and $G_r^{b_i}$ are the channel gains from relay r to a_i and b_i. Clearly, they are similar to $G_{a_i}^r$ and $G_{b_i}^r$, respectively.

Therefore, the rate user a_i can achieve is

$$R_{a_i} = \frac{1}{2}W\log_2(1 + \Gamma_{a_i}) \tag{10}$$

III. Power Auction Mechanism

A. Power Auction Game

Before the power auction starts, the relay issues a reserve bid $\xi > 0$ and a price $\delta > 0$ to all participating users. The user then submits its bid, denoted as f_{a_i} or f_{b_i}, to the relay. After receiving all the bids, the relay allocates transmit power for user pair i by [10]

$$P_{NC_i} = \frac{f_i}{\sum_{j\in\mathcal{N}} f_j + \xi}P_r \tag{11}$$

where P_r is the total power of the relay r, and $f_i = f_{a_i} + f_{b_i}$ is the integrated bid of pair i. Note that each user pair i is regarded as a single player which participates in the auction with an integrated bid f_i. In return, pair i pays the relay $C_i = \delta P_{NC_i}$.

To depict a pair's satisfaction with the power assigned by the relay, we define a utility function for user pair i as:

$$U_i(f_i; \mathbf{f}_{\underline{i}}) = R_i - C_i \tag{12}$$

where $R_i = R_{a_i} + R_{b_i}$ is the achievable rate for pair i, and $\mathbf{f}_{\underline{i}} = (f_i, ..., f_{i-1}, f_{i+1}, ..., f_N)$ denotes the supplementary bidding profile of pair i's bid f_i.

Correspondingly, the utility functions for each user in pair i are given by:

$$\begin{aligned} U_{a_i} &= R_{a_i} - \alpha_i \cdot C_i \\ U_{b_i} &= R_{b_i} - (1 - \alpha_i) \cdot C_i \end{aligned} \tag{13}$$

where α_i represents the proportion of user a_i's payment in the overall pair payment.

Definition 1. *The optimal bidding profile $\mathbf{f}^* = (f_1^*, ..., f_N^*)$ is the desirable outcome of a power auction game, with which the user pair can achieve the maximum utility, i.e.,*

$$U_i(f_i^*; \mathbf{f}_{\underline{i}}^*) \geq U_i(f_i; \mathbf{f}_{\underline{i}}^*),\ \forall i \in \mathcal{N}.$$

When the optimal bidding profile $\mathbf{f}^*$ emerges, the auction game reaches a Nash Equilibrium (NE).

Theorem 1. *The proposed auction game with the optimal bidding profile defined in Definition 1 is a NE.*

Proof: Following the similar analysis in [13], we differentiate the utility function in Eq. 12 with respect to f_i and yield:

$$\frac{\partial U_i(f_i;\mathbf{f}_{\underline{i}})}{\partial f_i} = \left(\frac{\partial(R_{a_i}+R_{b_i})}{\partial P_{NC_i}} - \delta\right)\frac{\sum_{j\neq i} f_j+\xi}{(\sum_{j\in\mathcal{N}} f_j+\xi)^2}P_r \tag{14}$$

where

$$\frac{\partial P_{NC_i}}{\partial f_i} = \frac{\sum_{j\neq i} f_j+\xi}{(\sum_{j\in\mathcal{N}} f_j+\xi)^2}P_r > 0$$

Further differentiate the first term in (14) with respect to f_i and have:

$$\frac{\partial\left(\frac{(\partial R_{a_i}+R_{b_i})}{\partial P_{NC_i}}-\delta\right)}{\partial f_i} < 0 \tag{15}$$

Since this term is strictly decreasing in f_i, the utility function $U_i(f_i;\mathbf{f}_{\underline{i}})$ is a strictly quasi-concave function of f_i. Thus, there exists a unique optimal bidding profile for a NE.

Clearly, the desirable power allocation can be characterized as an optimal solution of the following optimization problem:

$$\max \quad U_i(f_i;\mathbf{f}_{\underline{i}}) \tag{16}$$

In this game, each pair will strategically select its bid f_i to maximize its utility function $U_i(f_i;\mathbf{f}_{\underline{i}})$.

By taking the derivative of (13) with respect to f_i, we can get the optimal assigned power P_{NC_i}, the optimal bid f_i, and the proportion α_i. That is

$$\frac{\partial U_{a_i}}{\partial f_i} = 0 \;\; \& \;\; \frac{\partial U_{b_i}}{\partial f_i} = 0 \tag{17}$$

For the equations in (17), we can get three solutions to P_{NC_i} as follows,

$$P^1_{NC_i} = \frac{-n-2\sqrt{A}\cos\theta/3}{3m} \tag{18}$$

$$P^2_{NC_i} = \frac{-n+\sqrt{A}(\cos\theta/3+\sin\theta/3)}{3m} \tag{19}$$

$$P^3_{NC_i} = \frac{-n+\sqrt{A}(\cos\theta/3-\sin\theta/3)}{3m} \tag{20}$$

where we have $s = PG^r_{a_i} + PG^r_{b_i} + \sigma^2$, $\beta = \frac{Ws}{2\ln 2\delta}$, $m = G^r_{a_i}G^r_{b_i}$, $n = s(G^r_{a_i}+G^r_{b_i})$, $o = s^2-(G^r_{a_i}+G^r_{b_i})\beta$, $p = -2s\beta$, $A = n^2-3mo$, $B = no-9mp$, $T = \frac{2An-3mB}{2A^{3/2}}$, $\theta = \arccos T$.

Theorem 2. *The solutions to Eq. 17 may not be unique, but there exists one and only one feasible solution.*

Proof: Since

$$P^1_{NC_i}+P^2_{NC_i}+P^3_{NC_i} = -\frac{n}{m} < 0,$$

we infer that at least one solution is negative. Further, along with

$$P^1_{NC_i}P^2_{NC_i}P^3_{NC_i} = 54\beta(G^r_{a_i})^2(G^r_{b_i})^2 s > 0,$$

we get the conclusion that there is one positive and two negative solutions at any time.

1) If $P^1_{NC_i} > 0$, pair i's bid is updated by:

$$F_i(\mathbf{f}_{\underline{i}},\delta) = k^1_i(\delta)(\sum_{j\neq i} f_j+\xi) \tag{21}$$

where $k^1_i(\delta) = \frac{-n-2\sqrt{A}\cos\theta/3}{3mP_r+n+2\sqrt{A}\cos\theta/3}$.

2) If $P^2_{NC_i} > 0$, we have

$$F_i(\mathbf{f}_{\underline{i}},\delta) = k^2_i(\delta)(\sum_{j\neq i} f_j+\xi) \tag{22}$$

where $k^2_i(\delta) = \frac{-n+\sqrt{A}(\cos\theta/3+\sin\theta/3)}{3mP_r-(-n+\sqrt{A}(\cos\theta/3+\sin\theta/3))}$.

3) If $P^3_{NC_i} > 0$, we have

$$F_i(\mathbf{f}_{\underline{i}},\delta) = k^3_i(\delta)(\sum_{j\neq i} f_j+\xi) \tag{23}$$

where $k^3_i(\delta) = \frac{-n+\sqrt{A}(\cos\theta/3-\sin\theta/3)}{3mP_r-(-n+\sqrt{A}(\cos\theta/3-\sin\theta/3))}$.

B. Distributed Iterative Algorithm

According to Eq. 21-23, if we want to calculate the optimal bidding profile $\mathbf{f}^*$ in a distributed fashion, each pair needs the global information to obtain other pairs' best bids, thus may not be possible in practice. Therefore, we adopt the following distributed iterative algorithm, which allows each pair to update its own bid separately [10].

If $P^l_{NC_i} > 0, l = 1,2,3$, pair i updates its bid by

$$f_i(t) = k^l_{i,d}(\delta, t-1)f_i(t-1) \tag{27}$$

where $k^l_{i,d}, l = 1,2,3$, are computed with Eq. 24-26, respectively.

Definition 2. *[12] An iterative function $F_i(\mathbf{f}_{\underline{i}})$ is a standard function, if for all $\mathbf{f}_{\underline{i}} > 0$, the following properties are satisfied:*

- *Positivity: $F_i(\mathbf{f}_{\underline{i}}) > 0$.*
- *Monotonicity: If $\mathbf{f}_{\underline{i}} > \mathbf{f}'_{\underline{i}}$, $F_i(\mathbf{f}_{\underline{i}}) > F_i(\mathbf{f}'_{\underline{i}})$.*
- *Scalability: For all $\alpha > 1, \alpha F_i(\mathbf{f}_{\underline{i}}) > F_i(\alpha\mathbf{f}_{\underline{i}})$.*

Lemma 1. *[14] For a standard function $F_i(\mathbf{f}_{\underline{i}})$ defined in Definition 2, it converges to unique fixed point from any feasible initial value.*

Theorem 3. *The proposed bid iterative function in Eq. 21-23 is a standard function, which converges to the unique optimum.*

Proof:

Positivity: Take the case of $P^1_{NC_i} > 0, P^2_{NC_i} < 0, P^3_{NC_i} < 0$ as an example, then we have:

$$k^1_i(\delta) = \frac{-n-2\sqrt{A}\cos\theta/3}{3mP_r+n+2\sqrt{A}\cos\theta/3} = \frac{P^1_{NC_i}}{P_r-P^1_{NC_i}}$$

$P^1_{NC_i}$ is the power allocated by the relay that is always smaller than the relay's total power P_r, therefore, $k^1_i(\delta)$ is always positive. Meanwhile, other pairs' bids $\mathbf{f}_{\underline{i}}$ are also positive. As a result, the function $F_i(\mathbf{f}_{\underline{i}})$ is positive.

Monotonicity: Since $F_i(\mathbf{f}_{\underline{i}})$ is a linear increasing function of $\mathbf{f}_{\underline{i}}$, the larger $\mathbf{f}_{\underline{i}}$ is, the larger $F_i(\mathbf{f}_{\underline{i}})$ would be.

$$k^1_{i,d}(\delta,t-1)=\frac{P_r-P^1_{NC_i}(t-1)}{P^1_{NC_i}(t-1)}\frac{-n-2\sqrt{A}\cos\theta/3}{3mP_r+n+2\sqrt{A}\cos\theta/3}f_i(t-1) \tag{24}$$

$$k^2_{i,d}(\delta,t-1)=\frac{P_r-P^2_{NC_i}(t-1)}{P^2_{NC_i}(t-1)}\frac{-n+\sqrt{A}(\cos\theta/3+\sin\theta/3)}{3mP_r-(-n+\sqrt{A}(\cos\theta/3+\sin\theta/3))}f_i(t-1) \tag{25}$$

$$k^3_{i,d}(\delta,t-1)=\frac{P_r-P^3_{NC_i}(t-1)}{P^3_{NC_i}(t-1)}\frac{-n+\sqrt{A}(\cos\theta/3-\sin\theta/3)}{3mP_r-(-n+\sqrt{A}(\cos\theta/3-\sin\theta/3))}f_i(t-1) \tag{26}$$

Scalability: For all $\alpha>1$,

$$\alpha F_i(\mathbf{f}_{\underline{i}})=F_i(\alpha\mathbf{f}_{\underline{i}}+\alpha\xi)=k^1_i(\delta)(\alpha\sum_{j\neq i}f_j+\alpha\xi)$$
$$>F_i(\alpha\mathbf{f}_{\underline{i}})=k^1_i(\delta)(\alpha\sum_{j\neq i}f_j+\xi)$$

For other two cases, we can obtain the same conclusion. Therefore, $F_i(\mathbf{f}_{\underline{i}})$ is a standard function, thus would finally reach the convergence.

A complete algorithm for updating the bidding profile is shown in Algorithm 1.

Algorithm 1 BIDS IN A DISTRIBUTED FASHION

1:**Initialization**
$t=0$, initialize N pairs' bids: $f_1,f_2,\cdots,f_N$;
Defining an upper bound iterative times M;
2:**Definition**
Define the iterative function $T_1,T_2,\cdots,T_N$ for each pair, the way to fix T_i is as follows:
if $P^1_{NC_i}>0$, $T_i=k^1_{i,d}(\delta,t-1)$;
else if $P^2_{NC_i}>0$, $T_i=k^2_{i,d}(\delta,t-1)$;
else if $P^3_{NC_i}>0$, $T_i=k^3_{i,d}(\delta,t-1)$;
where $P^1_{NC_i},P^2_{NC_i},P^3_{NC_i}$ are the three solutions of the optimum allocated power for pair i.
3:**Auction progress**
Pairs: iterative bids $f_1(t)$, $f_2(t)$, $\cdots$, $f_N(t)$;
Relay: allocation power for each pair as $P_{NC_1},P_{NC_2},\cdots,P_{NC_N}$;
for($t=0;t<M;t++$)
$\{P_{NC_i}(t)=\frac{f_i(t)}{f_1(t)+f_2(t)+\cdots+f_N(t)+\xi}P_r$;
$f_i(t+1)=T_i*f_i(t);\}$
4:**Convergence**
Check whether they are in convergence or not. If convergent, game is over; else increasing M.

IV. OUTAGE PROBABILITY

In this section, we investigate the impact of the proposed resource allocation scheme on the performance of the system outage probability. Without loss of generality, we consider a system outage occurs when any of the pair users fails to recover the information from its counterpart.

In each transmission, the mutual information at user a_i and b_i are defined respectively as:

$$I_{a_i}=log_2(1+\Gamma_{b_i});\quad I_{b_i}=log_2(1+\Gamma_{a_i})$$

Definition 3. *The outage probability $P(R)$ is the probability of the mutual information I_i falling below a certain rate R, i.e., $P(R)=P(I_i<R)$.*

Suppose that the transmission from a_i to b_i is independent of its opposite process, then the outage probability of pair i can be represented as [15]

$$\begin{aligned}P^{out}_i&=1-P(I_{a_i}>R,I_{b_i}>R)\\&<1-P(I_{a_i}>R)P(I_{b_i}>R)\\&\doteq P(I_{a_i}<R)+P(I_{b_i}<R)\end{aligned}$$

in the high SNR regime.

Thus, the system outage probability P^{out} is

$$\begin{aligned}P^{out}&=1-\prod_{i=1}^{N}(1-P^{out}_i)\\&\doteq 1-\prod_{i=1}^{N}\{1-P(I_{a_i}<R)-P(I_{b_i}<R)\}\end{aligned}$$

Note that I_i is a function of channel gains $G^{\{.\}}_{\{.\}}$, and the channel gain $G^{\{.\}}_{\{.\}}$ is assumed to be an exponential distributed random variable with mean value $\frac{1}{\lambda_{G^{\{.\}}_{\{.\}}}}$. In this paper, we define $\frac{1}{\lambda_{G^{\{.\}}_{\{.\}}}}=G^{\{.\}}_{\{.\}}$

Theorem 4. *In the high SNR regime, pair i's outage probability of the proposed resource allocation algorithm is*

$$P^{out}_i=2-e^{-\lambda_u g(\beta)\rho}e^{-\lambda_v g(\beta)(\rho+1)}-e^{-\lambda_v g(\beta)\rho}e^{-\lambda_u g(\beta)(\rho+1)}$$

where $u=G^r_{a_i}$, $v=G^r_{b_i}$, $\gamma=\frac{P}{\sigma^2}$, $\rho=\frac{P}{P_{NC_i}}$, $\beta=\frac{1}{\gamma}$, $g(\beta)=(2^R-1)\beta$.

Theorem 5. *In the high SNR regime, the system outage probability is*

$$\begin{aligned}P^{out}=1-\prod_{i=1}^{N}\{&e^{-\lambda_u g(\beta)\rho}e^{-\lambda_v g(\beta)(\rho+1)}\\&+e^{-\lambda_v g(\beta)\rho}e^{-\lambda_u g(\beta)(\rho+1)}-1\}\end{aligned} \tag{28}$$

V. SIMULATION RESULTS

In this section, we present simulation results to demonstrate the performance of the proposed power allocation algorithm. The scenario is shown in Fig. 1, where three user pairs are respectively located at (-20m, 50m), (20m, 50m); (-40m, 30m), (40m, 30m) and (-30m, -40m), (30m, 40m). The Y coordinate of the relay r is fixed at $0m$, while its X coordinate varies from $-50m$ to $50m$. The channel gains is $\frac{0.097}{d^4}$, where d is the distance between two users. The transmit power of each user is $P = 0.1W$, the total power of the relay is $P_r = 1W$, and the noise variance is $\sigma^2 = 10^{-13}$.

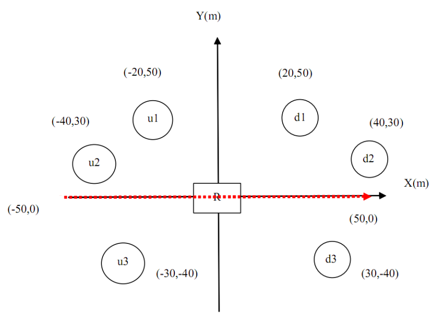

Fig. 1. Topology of a two-way relaying network

Fig. 2 plots the evolutions of user rates with the relay movement. Due to space limit, we take pair 1 users as an example, while other pairs produce similar results. It is observed that, when the relay r moves relatively closer to user a_1 than to user b_1, a_1 achieves a larger rate compared with b_1. Specifically, when the relay moves to -20m (X-axis) where the distance between a_1 and r is shortest, a_1 would reach its peak rate. On the contrary, as the relay moves closer to user b_1, b_1's rate gradually increases, while a_1's rate slowly declines. It reveals the fact that the user rate has a close relationship with the relay's location, and the more closer a user to the relay, the larger its rate would be.

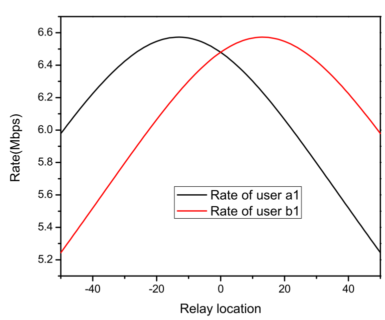

Fig. 2. User rate vs. relay location (X-axis)

Fig. 3 shows the impact of relay location on the value of α. As previously mentioned, α_i represents the proportion of user a_i's payment in the total pair payment. Clearly, the larger the α_i, the more payment a_i has to give, meanwhile, the less payment leaves for b_i. As can be seen that, when the relay is more closer to a_i, i.e., the relay is set within the range [-50m, 0m], α_i is always bigger than 0.5. That is to say, user a_i will pay more than user b_i, and vice versa. From Fig. 2 and 3, we can find that, for the pair user that is closer to the relay than its counterpart, it can achieve a relatively higher rate with more payment.

Fig. 4 displays the variations of the optimal allocated power with the relay movement. Since the performance of the left part

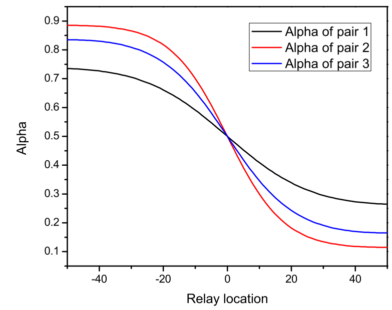

Fig. 3. Alpha vs. relay location (X-axis)

(from -50m to 0m) is symmetric to that in the right part (from 0m to 50m), we take the left part for demonstration. It is found that when the relay moves from -50m to 0m, the assigned power for each pair gradually decreases, and finally reaches the minimum at the point 0m. It implies that the preferred location for the relay is at the middle of the pair users, because the total signal attenuation of the pair is minimal at this place, the pair thus can achieve the best performance with the least power.

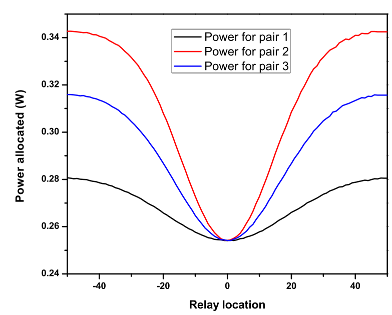

Fig. 4. Allocated power vs. relay location (X-axis)

The relationship between the user utility and the relay location is shown in Fig. 5. We can see that when the relay is set at -20m or 20m, one user in the pair can achieve its maximum utility, but the pair utility is not the largest. In this paper, we aim at maximizing the utility of the whole pair, rather than the interest of single user in the pair. Therefore, in this case, the optimal point for the NE is 0m, where the pair achieves the largest utility.

Fig. 6 displays the convergence behavior of the bids of three pairs, where the relay is located at (-10m, 0m). It is seen that the proposed scheme converges at a fast speed. After about 20 iterations, these three pairs converged to the optimum bids.

Finally, we demonstrate the performance of the system. As can be seen in Fig. 7, for a given threshold rate R, the system outage probability decreases with the increase of the SNR. Since the larger the SNR, the stronger the received signal would be. On the other hand, when we fix the SNR, the outage probability increases with the rate R. Therefore, the SNR is inversely proportional to the system outage probability, while

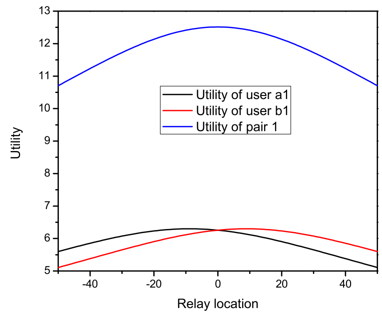

Fig. 5. Utility vs. relay location (X-axis)

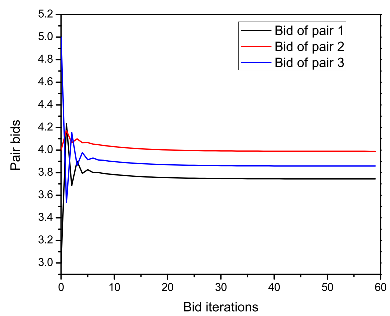

Fig. 6. Convergence of pair bids

the rate R is proportional to the system outage probability. Fig.8 shows that transmission power allocated by the relay increases with the SNR.

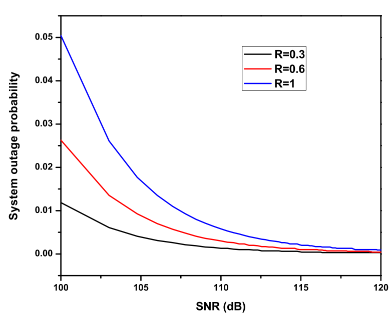

Fig. 7. Performance of system outage probability

VI. Conclusion

The relay power allocation is a critical issue in cooperative communications. In this article, we tackled the power allocation problem for network-coded multiuser two-way cooperative networks, where the relay broadcasts the combined information of two users in the pair by the amplify-and-forward protocol. A pair-based, instead of user-based power auction and allocation scheme is proposed for maximizing the utility of the whole pair. Also, the convergence performance as well as the system outage behavior are theoretically analyzed.

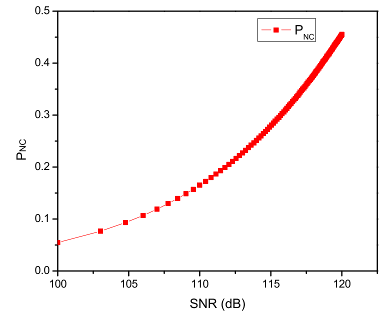

Fig. 8. The optimal transmission power

In our future work, further investigation may be focused on the scenarios of multiple relays or single relay equipped with multi-antennas.

References

[1] M. Chen and A. Yener, “Multiuser two-way relaying: detection and interference management strategies, *IEEE Trans. wireless communications*, vol. 8, no. 8, pp. 4296-4305, Aug. 2008.

[2] R. Ahlswede, N. Cai, S. Y. Li, and R. W. Yeung, “Network information flow, *IEEE Trans. Information Theory*, vol. 46, no. 4, pp. 1204-1216, Jul. 2000.

[3] R. Zhang, Y. C. Liang, C. C. Chai, and S. Cui, “Optimal Beamforming for Two-Way Multi-Antenna Relay Channel with Analogue Network Coding,” *IEEE Journal on Selected Areas in Communications*, Vol. 27, No. 5, pp. 699-712, June 2009.

[4] C. C. Chai, and C. Yuen, “On Two-way Communications for Cooperative Multiple Source Pairs Through a Multi-antenna Relay,“ submitted to *IEEE Trans. Information Theory*, 2010.

[5] F. Xue, C. H. Liu, and S. Sandhu, “MAC-layer and PHY-layer network coding for two-way relaying: achievable rate regions and opportunistic scheduling, *Proc. of Allerton Conf. on Communication, Control and Computing*, Sep. 2007.

[6] Q. Zhou, Y. Li, F. Lau, and B. Vucetic, “Decode-and-Forward Two-Way Relaying with Network Coding and Opportunistic Relay Selection,” *IEEE Trans. Communications*, vol. 58, no. 11, Nov. 2010.

[7] K. G. Seddik, A. K. Sadek, W. Su, and K. J. Liu, “Outage Analysis and Optimal Power Allocation for Multinode Relay Networks, *IEEE Signal Processing Letters*, vol. 14, no. 6, pp. 377-380, June 2007.

[8] Y. Zhao, R. Adve, and T. Lim, “IImproving Amplify-and-Forward Relay Networks: Optimal Power Allocation versus Selection, *IEEE Trans. Wireless Communications*, vol. 6, no. 8, pp. 3314-3123, Aug. 2007.

[9] R. Annavajjala, P. Cosman, and L. B. Milstein, “Statistical Channel Knowledge-Based Optimum Power Allocation for Relaying Protocols in the High SNR Regime, *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 292-305, Feb. 2007.

[10] J. Huang, Z. Han, M. Chiang, and H. V. Poor, “Auction-Based Resource Allocation for Cooperative Communications,” *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 7, pp. 1226-1237, Sep. 2008.

[11] B. Wang, Z. Han, and K. J. Liu, “Distributed Relay Selection and Power Control for Multiuser Cooperative Communication Networks Using Buyer/Seller Game,” *IEEE INFOCOM*, pp. 544-552, May 2007.

[12] B. Wang, Z. Han, and K. J. Liu, “Distributed Relay Selection and Power Control for Multiuser Cooperative Communication Networks Using Stackelberg Game,” *IEEE Trans. Mobile Computing*, vol. 8, no. 7, pp. 975-990, July 2009.

[13] J. Huang, R. Berry, and M. L. Honig, “Auction-based spectrum sharing,” *Mobile Networks and Applications*, vol. 11, no. 3, pp. 405-418, June 2006.

[14] R. D. Yates, “A framework for uplink power control in cellular radio systems,” *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 7, pp. 1341-1347, Sep. 1995.

[15] A. Zhan, C. He, and L. Jiang, “A Channel Statistic Based Power Allocation in a Butterfly Wireless Network with Network Coding,” *Proc. IEEE International Conference on Communications (ICC)*, Capetown, May 2010.