Dynamic Network Coding for Multiple Sessions Based on Flow Gain

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***Abstract*—This paper proposes a dynamic coding policy (MSNC) which allows network coding applied across multiple sessions and develops a necessary condition for MSNC by denoting the benefit gaining from coding. With MSNC, intermediate nodes can independently judge whether the coding operation is appropriate and easily decide which packet to transmit among the incoming flows. To make the network coding feasible in practic, a control mechanism for an intermediate node is presented, which illustrates the operations for the incoming packet flows. Simulation results demonstrate the advantage of MSNC in terms of both throughput and fairness.**

***Keywords*—Network coding, Throughput gain, Multiple Sessions, Control strategy**

I. Introduction

Since the seminal work of Ahlswede et al. [1], network coding has established its theoretical ability to improve network throughput by mixing packets at intermediate nodes rather than simply adopting a replicate and forward strategy. For transmission with multiple sessions originated from different sources, network coding can be classified into two types: intra and inter session coding. For the former one, coding is performed within the packets belonging to the same source. In comparison to coding with a single session scenario, network coding performing across multi-session is much more complex and deciding what linear coding operations to perform is an NP-hard problem [2]. In this work, we consider a network coding scheme that allows coding performing across different sessions, and with which the intermediate node can easily decide whether to make a combination among the incoming flows.

In order to make network coding work across different sessions, Katti et al. in [3] introduced an opportunistic approach to network coding for mesh wireless networks in advantage of the shared nature of the wireless medium. Cui et al. in [4] proposed an optimization approach for network coding based on decomposition multiple sessions into a superposition of multicast and unicast sessions. However, these studies on network coding for wireless networks have to follow an opportunistic scheduling strategy, which decides a set of nodes whose packets are XORed according to the instantaneous link conditions. Erylimaz et al. [5] proposed a routing-scheduling-coding rule to decide whether coding is to be performed according to the queues status. Unfortunately, the performance of network coding is infeasible as the limitation of the physical layer. A pairwise intersession network coding proposed in [6] eliminated unnecessary queue length information exchange among intermediate nodes. Note, however, that coding is allowed only between two flows and is found to be only modest depending on network topologies. Li et al. in [7] presented the differences between coding and pure routing and investigated the benefit of network coding for multiple unicast sessions. But it is not clear when and how the coding should be applied at the relay node. Unlike the previous work, we propose the concept of flow gain, and develop a coding policy MSNC for serving multi-session networks based on it, with which the intermediate nodes can independently judge whether the coding operation is appropriate while guaranteeing decodability at the terminals.

In this paper, we consider a network coding approach for multi-session networks and propose a dynamic coding strategy MSNC across different sources. Our main contributions are as follows. First, we develop a necessary condition for coding by denoting the benefit gaining from network coding. According to this condition, intermediate nodes can independently judge whether the coding operation is appropriate. Second, to increase the throughput and guarantee the decodability simultaneously, a dynamic strategy for multiple session network coding MSNC is proposed. The intermediate node can easily decide which packet to transmit among the incoming flows with MSNC. Further, to make MSNC feasible in practical, a control mechanism for an intermediate node is presented, which illustrates how the incoming packet flow operates.

The rest of this paper is organized as follows. Sec.II discusses the network model and the control strategy of MSNC. Sec.III describes the details above the proposed MSNC strategy and the implementation of the coding. Numerical results are presented in Sec.IV, and we conclude this article at the last section.

II. Control strategy for multiple session network coding

A. Settings and system model

Network can be modeled as a directed acyclic graph $G = (V, E)$, where V is the set of nodes and E is the set of links. $S = \{s_1, s_2, ..., s_N\}$ and $T = \{t_1, t_2, ..., t_N\}$ denote the set of

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source nodes and destination nodes, respectively. Each node $s \in S$ can communicate with other nodes and deliver its data to the corresponding sink node $t \in T$. Let (s,t) describe each pair of source node s and its sink node t. Each link $(i,j) \in E$ has a fixed capacity c_{ij}.

Let us assume that there are N independent random processes, denoted by $\chi = \{X_1, X_2, ..., X_N\}$ originating at each source node. Suppose that there exist multiple paths from a source node to the sink node. Let $J(s,t)$ denote the set of paths between source s and its sink t. For each t, let $R_s^{t,m}$ be the transmission rate from s over path m ($m \in J(s,t)$). Moreover, let $g_{ij}^{s,t,m}$ represent the information flow rate of link (i,j) allocated by s over path m. And $V_{out}(i)$ the outgoing node setf or each node $i \in V$.

B. Link capacity region

As shown in Fig.1, there are two source-sink pairs (s_1,t_1) and (s_2,t_2). Each source node s_i transmits the data to the corresponding sink t_i through multiple relay nodes. Since there is only one path from source s_i to sink node t_i shown in Fig.1, $m = 1$. Each link has a unit capacity ($c_{ij} = 1$Mbps). Clearly, flows from different sources share the capacity of wireless links. Without network coding, the link capacity region satisfies:

$$\sum_{(s,t)} \sum_{m \in J(s,t)} g_{ij}^{s,t,m} \leq c_{ij}, \qquad \forall (i,j) \in E, \tag{1}$$

Inequation 1 represents that the rate for each path is less than the min capacity of the links occupied. Take node v_2 as an example, without network coding, the capacity of link (v_2,v_6) is shared by source nodes s_1 and s_2, and sink nodes t_1 and t_2 cannot achieve the communication capacity simultaneously.

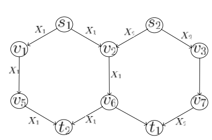

Fig. 1. The multiple session network

When network coding is allowed, flow rate can be increased. More precisely, we can apply linear combinations (e.g. XOR operation) over different flows at the intermediate nodes. In Fig.1, the intermediate node v_2 can gain the packets X_1 and X_2 from source nodes s_1 and s_2. As network coding is adopted at node v_2 which can code the packets from different sessions and then send the coded flow $X_1 \oplus X_2$ to its downstream nodes t_1 and t_2. Then, we can define the link capacity region with coding as:

$$\sum_{(s,t)} \sum_{m \in J(s,t)} g_{ij}^{s,t,m} \leq c_{ij} \qquad \forall (i,j) \in \{E - (v_2,v_6)\}, \tag{2}$$

$$\max_{(s,t)} \sum_{m \in J(s,t)} g_{v_2,v_6}^{s,t,m} \leq c_{v_2,v_6}. \tag{3}$$

Let f'_{v_2,v_6} represent the encoded flow rate at node v_2, and $f'_{v_2,v_6} = \max_{(s,t)} \sum_{m \in J(s,t)} g_{v_2,v_6}^{s,t,m}$. Clearly, the achievable

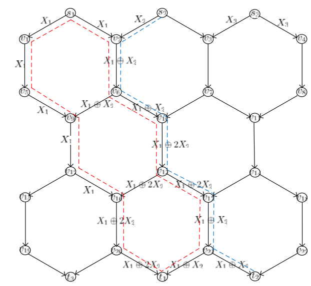

Fig. 2. The network with three pairs of source and sink node

region for (s_1,t_1) and (s_2,t_2) is increased which is in fact the communication capacity region.

Obviously, when the exogenous arrival rates at node v_2, it is likely that link (v_2,v_6) represents a bottleneck. In this case, we could alleviate the congestion at node v_2 by coding them together and sharing the limited capacity. If node t_1 and t_2 can gain remedy flow from other links, the coded packets sent by node v_2 are able to be decoded at the downstream nodes.

In Fig.2, the network topology enlarges and more pairs of source and sink node are exited. It is clear that it is not efficient for node v_2 to code the incoming packets if link (v_5,v_9) and (v_7,v_{10}) are congested themselves. For example, the original packet X_1 and X_2 can be mixed into coded packet $X_1 \oplus X_2$ at node v_2 in Fig.2. The coded packet can be decoded at the downstream node v_9 and v_{10} as long as they gain remedy packets X_1 and X_2 from other paths. If the original packet X_2 is coded with packet X_3 at intermediate node v_3, it is impossible for node v_{10} to receive the remedy packet X_2. As a result, the code packet $X_1 \oplus X_2$ is useless at node v_{10} as the original packet cannot be recovered. Network coding can be adopted only if coded packets are decodable at the downstream nodes. Therefore, the decision rule must be to exploit the network coding advantage while guaranteeing decidability at the receivers. Moreover, the coded packet can be recoded at the downstream node, such as node v_{13}.

Definition 1: For each link (m,n), denote $\rho_{mn}^{s_i}$ as the flow gain for forwarding the data originated from s_i over link (m,n), $\sigma_{mn}^{s_i,s_j}$ as the flow gain for mixing data from both s_i and s_j:

$$\rho_{m,n}^{s_i} \triangleq \sum_{k \in J(s_i,t_i)} g_{m,n}^{s_i,t_i,k} \tag{4}$$

$$\begin{aligned} \sigma_{m,n}^{s_i,s_j} \triangleq & \sum_{k \in J(s_i,t_i)} g_{m,n}^{s_i,t_i,k} - f'_{m,n} + \sum_{k \in J(s_j,t_j)} g_{m,n}^{s_j,t_j,k} \\ & - f'_{m,n} - \sum_{k \in J(s_i,t_i)} \sum_{(p,q) \in E - \{(n,q)\}} g_{p,q}^{s_i,t_i,k} \\ & - \sum_{k \in J(s_j,t_j)} \sum_{(p,q) \in E - \{(n,q)\}} g_{p,q}^{s_j,t_j,k} \end{aligned} \tag{5}$$

where $q \in V_{out}(n)$ and $r_{p,q}$ represents the remedy flow rate used for decoding at the outgoing node of n, and $r_{p,q} = \sum_{m \in J(s,t)} \sum_{(p,q) \in E - \{(n,q)\}} g_{p,q}^{s,t,m}$. The flow gain for each session and the remedy flow cost for coding constitute the flow gain $\sigma_{m,n}^{s_i,s_j}$.

Taking Fig.2 as an example, source s_i maintains its own data and sends them to the sink node. At first the source node divides the collected data into generations, and continuously produces original packet streams from a generation. Fig.2 shows that the packet flows originating from each source node through multiple paths towards the respective destination. Assume v_6 is able to observe the data its neighbor nodes v_9 and v_{10} gain. If node v_9 and v_{10} can receive the remedy flows for decoding from node s_1 and s_2 respectively, node v_2 serves the coded flow using whatever capacity it has on link (v_2, v_6). Otherwise, link (v_2, v_6) sends the uncoded flow to node v_9 and v_{10}.

And node v_2 makes the coding decision based on the flow gains: $\rho^{s_1}_{v_2,v_6}$, $\rho^{s_2}_{v_2,v_6}$ and $\sigma^{s_1,s_2}_{v_2,v_6}$. If $\sigma^{s_1,s_2}_{v_2,v_6}$ is greater than $\max(\rho^{s_1}_{v_2,v_6}, \rho^{s_2}_{v_2,v_6})$ and the remedy flow is available for node v_9 and v_{10}, then coding is performed. Node v_2 combines the two different flows packet by packet forming a single coded packet and transmits the coded packet on link (v_2, v_6). Node v_2 repeatedly forms coded packets and sends them for as much capacity is available on link (v_2, v_6).

C. Necessary condition for multiple session network coding

In this section, we introduce a necessary condition for MSNC to share the limited bandwidth among nodes. For each link $(m, n) \in E$, two sets of weights are computed, one corresponding to uncoded $\{\rho^{s_i}_{m,n}\}$ and the other corresponding to coded packets $\{\sigma^{s_i,s_j}_{(m,n)}\}$. Here, $\rho^{s_i}_{m,n}$ represents the flow gain associated with link (m, n) serving for source node s_i without coding across sessions, while $\sigma^{s_i,s_j}_{(m,n)}$ is the flow gain associated with coding packets across session i and j.

Once these weights are computed, the maximizing s_i for $\rho^{s_i}_{m,n}$, denoted by s^*_i, and the maximizing (s_i, s_j) for $\sigma^{s_i,s_j}_{(m,n)}$, denoted by (s^*_i, s^*_j), are computed. Then, we define the weights associated with the best decisions without and with MSNC as: $\rho^{*s_i}_{(m,n)} \triangleq \rho^{s^*_i}_{(m,n)}$, $\sigma^{*s_i,s_j}_{(m,n)} \triangleq \sigma^{s^*_i,s^*_j}_{(m,n)}$.

When the remedy flow can arrive at the outgoing nodes of node n from the corresponding sources, the decision for network coding is performed based on the comparison between $\rho^{*s_i}_{(m,n)}$ and $\sigma^{*s_i,s_j}_{(m,n)}$.

Theorem 1: For each bottleneck link (m, n) with $\rho^{*s_i}_{(m,n)}$ and $\sigma^{*s_i,s_j}_{(m,n)}$, multiple session network coding (MSNC) scheme exists if the following conditions hold:

- $\sigma^{*s_i,s_j}_{(m,n)} \geq \rho^{*s_i}_{(m,n)}$,
- and the remedy flow is available for the outgoing nodes of link (m, n), $\sum_{m\in J(s,t)} \sum_{(p,q)\in E-\{(n,q)\}} g^{s,t,m}_{p,q} \geq 0, \quad q \in V_{out}(n)$.

III. Implementation for Multiple Session Network Coding

We use M_{mn} to denote the message along link (m, n). As in MSNC scheme, the packet M along any link is a linear combination $M = c_1 \cdot X_1 \oplus c_2 \cdot X_2$ of the original packets X_1 and X_2, where the corresponding coding coefficients c_1 and c_2 are strictly positive integers. Depending on the correlation among the packets receiving at an intermediate node v_i, we have the following three cases:

Case 1: If all the incoming packets are uncoded and not identical, node v_i can adopt MSNC among these messages based on **Theorem 1**.

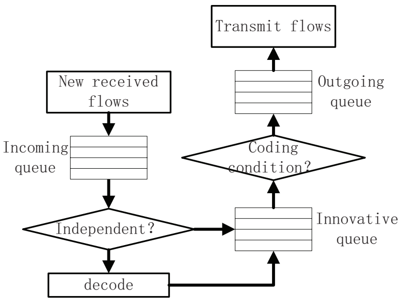

Fig. 3. The control flow at an intermediate forwarder

Case 2: If a part of the incoming messages is coded and the rest is from source node directly, the correlation among them is calculated at node v_i. And it would decode the correlation messages before recoding them. The decoding is applied at node v_i before recoding.

Case 3: If all the incoming messages are identical, MSNC is not applied at node v_i as the incomings are from the same session and coding is useless.

Definition 2: For link (m, n), packet M_{mn} is identical to the incoming messages, if all the incoming packet flows of (m, n) are identical including the case when there is only one incoming message.

Definition 3: For link (m, n), packet M_{mn} is not identical to the incoming messages, if the incoming packet flows are not identical and linearly independent.

Fig.3 illustrates the packet flow of operation at an intermediate node where MSNC is implemented. Whenever a packet is gained, it first undergoes innovative check which determines whether it is linearly independent of existing packets. If so, it will be put into an innovative packet queue. Otherwise, the node decodes the identical packet first before pushing into the queue. All outgoing packets are generated by re-encoding and transmitting to the downstream nodes.

We describe the control mechanism of MSNC in Algorithm 1. For a link (m, n), its source node m can code the incoming packets based on the Algorithm 1. To ensure the decodability of MSNC, the outgoing message must be in the span of all coded incoming messages and the coding coefficients must be sent with the coded packets to the next node. Then packet decoding at the terminals is simple. Each destination node maintains a packet pool, where each original packet whether it has received or recovered is kept. The packets are stored in a hash table indexed by the packet ID. When an encoded packet is received, the sink node goes through the IDs of the native packets one by one, and recovers the corresponding packet as possible as it can.

IV. Performance Evaluations

In this section, the simulation results are presented to evaluate the performance of the proposed strategy of MSNC. Numerical experiments are conducted over the network shown in Fig.2, in which three source nodes can transmit the data to the corresponding sink node. We present our simulation results and compare different approaches.

Algorithm 1 MNSC procedure for intermediate node

Pick packets X_i and X_j at the head of the innovative queue for different sessions;
Original Packets = X_i, X_j;
$V_{out}(m)$= { the downstream nodes of$m$};
Gain $\rho^{*s_i}_{(m,n)}$ and $\sigma^{*s_i,s_j}_{(m,n)}$;
if $\rho^{s_i}_{(m,n)} > \sigma^{*s_i,s_j}_{(m,n)}$ **then**
  MSNC is not adopted at node m;
  $f_{mn} = \sum_{(s,t)} \sum_{m \in J(s,t)} g^{s,t,m}_{v_2,v_6}$;
else if $i \in V_{out}(m)$ can gain remedy packets **then**
  MSNC is adopted;
  $f_{mn} = \max_{(s,t)} \sum_{m \in J(s,t)} g^{s,t,m}_{v_2,v_6}$;
end if

A. Achievable rate region

We consider the throughput benefits with the proposed MSNC scheme. First, we assume that each link of the network in Fig.2 with a unit capacity (1Mbps), and only consider the session 1 and 2. As link (v_2, v_6), (v_6, v_{10}), (v_{10}, v_{13}), (v_{13}, v_{17}) and (v_{17}, v_{21}) are shared by both s_1 and s_2, the max flow of session 1 and 2 cannot be gain, simultaneously. The achievable rate region with and without MSNCg are depicted in Fig.4.

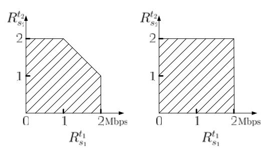

Fig. 4. The achievable rate region with and without MSNC

B. Throughput Gain for network coding

As the data communication within different sessions suffer from congestion and error-prone, the reliable bandwidth of each link is time-varying. Then we assume that each link with a random capacity and display the throughput for each session in Fig.5. To evaluate the throughput gains of MSNC, we define the throughput gain of the proposed coding scheme as: $G = \frac{\sum_i R_i^{MSNC} - \sum_i R_i}{\sum_i R_i}$, where $\sum_i R_i$ represents the total throughput of the network when network coding is not allowed, and $\sum_i R_i^{MSNC}$ the total throughput with MSNC. And MSNC can offer throughput benefits for the whole network as $G = 0.944$.

C. Fairness for MSNC

The most compelling benefits of network coding might be in term of fairness among sessions. We choose the popular max-min fairness criteria [8] to evaluate the fairness performance among the three sessions. In accordance with the rate allocation, we define our max-min fairness index as: Max-min Fairness Index $= \frac{\min R_i}{\max R_i}$, where $\min R_i$ represents the minimum rate of all sessions, and $\max R_i$ corresponds to the maximum lifetime among all the sensors.

As shown in Fig.2, there are four paths between s_1 and t_1, four paths between s_2 and t_2, and only one path between s_3 and t_3. Also, the path between s_3 and t_3 overlaps with all the other paths between (s_1, t_1) and (s_2, t_2). Therefore, without MSNC, there is no available capacity for source s_3 and t_3 even each source transmit its data along the shortest path or the max flow paths to the sink node. Since t_3 can gain the remedy packets form s_1 to recovery the coded packets, source s_3 can send the data to t_3 with the proposed coding scheme.

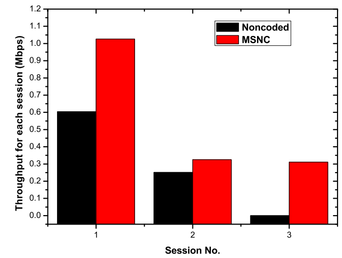

Fig. 5. The throughput for each session with and without MSNC

Obviously, t_3 cannot gain the achievable rate from s_3 whether with SPT or Max-flow algorithm and the rate allocation among three sessions is unfair. However, with MSNC, each session can gain a fair rate from the corresponding source node. Using MSNC, the fairness index is increased to 1 with a decrease in rate R_2 and a considerable increase in R_3.

V. Conclusion

In this paper, we develop MSNC for the multi-session network and propose a necessary condition for coding by denoting the flow gaining from network coding. The relay node can easily decide which flow to transmit among the incoming flows using MSNC to increase the throughput and guarantee the decodability simultaneously. Moreover, we develop a control mechanism for the intermediate node to make the network coding feasible in practic. Our results have proven that the throughput advantage of MSNC can be achieved and each session can share the limited capacity in a fairness way.

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