Joint Spectrum and Power Allocation in Coded Cooperative Cognitive Radio Networks

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***Abstract*—Cooperative cognitive radio (CR) improves quality of service (e.g. rate) of primary users by cooperative transmission from secondary uses. However, as owners of the spectrum, primary users' traffic demands are relatively easy to be satisfied. The rationale is that they would be more interested in the benefits of other format (e.g. revenue), instead of the enhanced rate. In this paper, we propose a new cooperative CR framework, where primary users assist transmissions of secondary users. In exchange for this concession, primary users receive the payments of secondary users for the spectrum and cooperative transmit power being used in cooperation. An auction-theoretic model with multiple auctioneers, multiple bidders and multiple commodities is developed for a joint spectrum and cooperative power allocation. Finally, we mathematically prove the convergence of the proposed auction, and verify the performance of the proposed scheme by numerical results.**

***Index Terms*—Cooperative cognitive radio, spectrum and power allocation, auction, Walrasian Equilibrium**

I. Introduction

With the rapid deployment of wireless services in the last decade, radio spectrum is becoming a valuable and scarce resource. On the other side, the report from the Federal Communications Commissions reveals that most of the licensed spectrum is severely underutilized [1]. As a promising technique, cognitive radio (CR) [2] is proposed to deal with the dilemma between spectrum scarcity and spectrum underutilization. CR allows unlicensed users (or secondary users) to access licensed bands under the condition that the induced interference to licensed users (or primary users) does not reach an unacceptable level.

Cooperative communications [3] enables users to relay data for each other and thus creates a virtual multiple-input-multiple-output (MIMO) system for cooperative diversity. Recently, incorporations of cooperation concept into CR networks has become a new cognitive radio paradigm. Cooperation in CR networks is mainly classified into two categories: i) cooperation among secondary users; ii) cooperation between primary users and secondary users. The first category aims to improve the performance of secondary transmission, in which a secondary user acts as a relay and assists transmissions of other secondary users [4]. Generally, the solutions for traditional cooperative communications are valid for cooperation among secondary users.

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The second category benefits both primary and secondary users in which different rights of primary users and secondary users to the spectrum are taken into account, thus is more challenging than the first category. Simeone et al. in [5] proposed a cooperation-based spectrum leasing scheme, where a primary user leases the owned spectrum to a subset of secondary users for a fraction of time in exchange for cooperation from secondary users. Zhang et al. in [6] proposed a cooperative cognitive radio network framework, in which some secondary users are selected by primary users as cooperative relays and in return, they obtain more spectrum access opportunities.

The existing works on cooperation between primary and secondary users fall into just one direction, i.e., secondary users relay the traffic for primary users, from which primary users achieve the improvement of quality of service (e.g., transmission rate or outage probability). Actually, as owners of the spectrum, primary users' traffic demands are relatively easy to be satisfied, especially for those who have a light traffic load. To these users, the enhanced rate or the cooperation from secondary users might have less attractions. Instead, they would be more interested in the benefits of other format (e.g. revenue). In this paper, we propose a new cooperation way between primary and secondary users, where primary users assist transmissions of secondary users. In return, they receive the payments of secondary users for the spectrum and cooperative power being used in cooperation.

Network coding was first proposed to achieve the capacity gain of wired networks. The broadcast nature of wireless medium enables a relay to overhear messages from multiple sources, which facilitates the application of network coding in cooperative communications [7]. In this study, we consider a network-coded cooperative CR environment, in which primary users are endowed with network coding capability. Hence, they do not forward secondary users' data directly, but perform a combination of those with its own data (see Sec. II for details).

In the proposed cooperation, secondary users require to compete at primary users for channel bands and cooperative transmit power. Auction theory [8] is a simple and powerful tool for distributed resource allocation in interactive multiuser systems. Auction-based power and channel allocation have been extensively studied in the literature [9]-[11]. The existing power or spectrum auction models are characterized by single auctioneer multiple bidders for single commodity as in [10], or multiple auctioneers multiple bidders for single commodity, for example, for power in [9] and for channel in [11]. In order to jointly address spectrum and power allocation problem, we propose a new auction structure with multiple auctioneers, multiple bidders and multiple commodities. Primary users (auctioneers) sell a portion of the channel access time and cooperative transmit power to secondary users for utility enhancement; Secondary users (bidders) purchase power as well as channels from primary users for utility maximization.

The contributions of our work are summarized as follows:

(1) A new cooperative CR framework, i.e., primary users cooperate for secondary users, is proposed, where primary users earn the revenue by selling channels and cooperative power to secondary users. While secondary users maximize the utility by finding an optimal trade-off between the expense on channel and power and the achievable rate.

(2) An auction-theoretic model with multiple auctioneers, multiple bidders and multiple commodities is developed for the proposed cooperative CR networks. Unlike traditional multi-commodity auction, the spectrum and cooperative power are not two independent commodities, but offered as a bundle. Moreover, they are two different types of commodities. The channel is indivisible and thus can be assigned either totally or nothing. While the cooperative power is divisible and therefore can be offered at any quantity. These properties are specifically considered in the proposed auction strategy.

The remainder of this paper is organized as follows: Sec. II presents the system model. Sec. III describes a joint spectrum and power auction mechanism. Numerical results are presented in Sec. IV. Finally, Sec. V concludes the paper.

II. NETWORK MODELING AND NOTATIONS

Consider a CR system consisting of a primary network composed of M primary links, and a secondary network composed of N secondary links. In the primary network, each primary user (PU) j is equipped with a primary transmitter (PT) and a primary receiver (PR), and has K_j non-overlapping narrowband channels. Assume that the channels owned by the same PU have the same carrier frequency, while those of different PUs are different so as to avoid interference of different primary transmissions. In the secondary network, each secondary user (SU) i is equipped with a secondary transmitter (ST) and a secondary receiver (SR). The PTs act as relays to assist SUs' transmissions by network coding and the amplify-and-forward relaying protocol. Also, assume that each channel of the PT can be accessed by only one ST at the same time, and the channel occupancy by the STs is maintained by each PT itself. For simplicity, we consider the scenario where the total number of channels in the system equals to the number of SUs, i.e., $N = \sum_{j=1}^{M} K_j$, such that each SU can access one channel.

The structure of our CR frame consists of an auction slot and a data transmission slot. In the auction slot, the ST, which intends to send data to its SR, selects a desired PT and joins the channel and power auction organized by that PT. The data transmission slot is for primary and secondary transmission that is further divided into three phases, as shown in Fig. 1. At each channel of PU j, in the first phase, PT j sends its data to its receiver. Meanwhile, the data are overheard by SR i who is designated to that channel by PT j; In the second phase, ST i transmits its data to its receiver, which are also overheard by PR j; In the third phase, PT j combines together its own data sent in the first phase and the data overheard in the second phase and sends the additive data out. Then both PR j and SR i can recover their desired data. Note that in Fig. 1, the solid lines indicate the intended communications, and the dotted lines represent the interference.

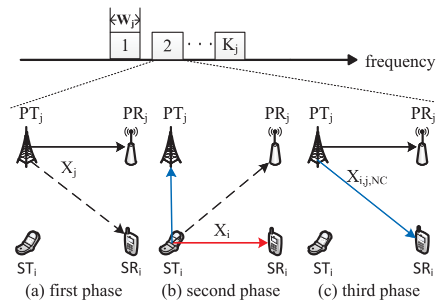

Fig. 1. Transmissions in three phases

Assume that the channels change slowly and the channel gain is stable within each frame. Also, assume that the channel state information (CSI) can be accurately measured at each receiver, and this information can be sent to other receivers through an error-free control channel.

In the first phase: At each channel of PU j, PT j transmits its signal to its destination PR j with power P_j. Assume that the total transmit power P_{U_j} of PT j is equally used at each channel, i.e., we have $P_j = P_{U_j}/K_j$. The signals received at PR j and SR i are respectively given by

$$Y_{PT_j}^{PR_j} = \sqrt{P_j} G_{PT_j}^{PR_j} X_j + n_{PR_j}, \tag{1}$$

$$Y_{PT_j}^{SR_i} = \sqrt{P_j} G_{PT_j}^{SR_i} X_j + n_{SR_i}, \tag{2}$$

where Y_A^B represents the signal received at B from A, X_j is the information symbol transmitted by PT j with $E[|X_j|^2] = 1$, and $n_{\{\cdot\}}$ is the additive white Gaussian noise (AWGN) with variance σ^2. G_A^B denotes the channel gains from A to B, which is also the channel gains from B to A. The amplitude $|G_A^B|^2$ is exponentially distributed, with rate parameter $\lambda_A^B = (d_A^B)^\alpha$, where d_A^B denotes the distance between A and B, and α is the path-loss exponent.

The signal-to-noise-ratio (SNR) of X_j at PR j in the first phase is

$$\Gamma_{PR_j}(1) = \frac{P_j |G_{PT_j}^{PR_j}|^2}{\sigma^2}. \tag{3}$$

In the second phase: ST i transmits its signal with power P_i. The signals received by SR i, PT j and PR j are respectively

$$Y_{ST_i}^{SR_i} = \sqrt{P_i} G_{ST_i}^{SR_i} X_i + n_{SR_i}, \tag{4}$$

$$Y_{ST_i}^{PT_j} = \sqrt{P_i} G_{ST_i}^{PT_j} X_i + n_{PT_j}, \tag{5}$$

$$Y_{ST_i}^{PR_j} = \sqrt{P_i} G_{ST_i}^{PR_j} X_i + n_{PR_j}, \tag{6}$$

where X_i is the signal transmitted by ST i in this phase with $E[|X_i|^2] = 1$.

Thus, the SNR of X_i at SR i in the second phase is

$$\Gamma_{SR_i}(2) = \frac{P_i|G_{ST_i}^{SR_i}|^2}{\sigma^2}. \quad (7)$$

In the third phase: PT j makes a combination of $Y_{ST_i}^{PT_j}$ and its own signal X_j, and amplifies and forwards the combined signal $X_{i,j,\text{NC}}$. Then, the signals received by PR j and SR i are

$$Y_{PT_j,\text{NC}}^{PR_j} = \sqrt{P_{i,j}} G_{PT_j}^{PR_j} X_{i,j,\text{NC}} + n_{PR_j}, \quad (8)$$

$$Y_{PT_j,\text{NC}}^{SR_i} = \sqrt{P_{i,j}} G_{PT_j}^{SR_i} X_{i,j,\text{NC}} + n_{SR_i}, \quad (9)$$

where $P_{i,j}$ is PT j's cooperative power for ST i, and $X_{i,j,\text{NC}}$ is the normalized energy data symbol defined as

$$X_{i,j,\text{NC}} = \frac{X_j + \sqrt{P_i} G_{ST_i}^{PT_j} X_i + n_{PT_j}}{\sqrt{1 + P_i|G_{ST_i}^{PT_j}|^2 + \sigma^2}}. \quad (10)$$

The signal $Y_{PT_j,\text{NC}}^{PR_j}$ received at PR j contains the information of both X_j and X_i, where X_i is the interference signal overheard in the second phase that can be completely removed. This yields

$$\hat{Y}_{PT_j,\text{NC}}^{PR_j} = \frac{\sqrt{P_{i,j}} G_{PT_j}^{PR_j}}{\sqrt{1 + P_i|G_{ST_i}^{PT_j}|^2 + \sigma^2}} \left(X_j + n_{PT_j} \right) + n_{PR_j}. \quad (11)$$

In the third phase, the SNR at PR j is then given by

$$\Gamma_{PR_j}(3) = \frac{P_{i,j}|G_{PT_j}^{PR_j}|^2}{\sigma^2(1 + P_i|G_{ST_i}^{PT_j}|^2 + P_{i,j}|G_{PT_j}^{PR_j}|^2 + \sigma^2)}. \quad (12)$$

Correspondingly, the achievable rate from PT j to PR j when relaying for ST i is

$$R_{j,i} = \frac{W_j}{3} \log_2(1 + \Gamma_{PR_j}(1) + \Gamma_{PR_j}(3)), \quad (13)$$

where W_j is the bandwidth of the channel.

Therefore, the achievable rate from PT j to PR j, when relaying for K_j STs over all the channels, is

$$R_{j,\text{C}} = \sum_{i=1}^{K_j} \frac{W_j}{3} \log_2(1 + \Gamma_{PR_j}(1) + \Gamma_{PR_j}(3)). \quad (14)$$

Similarly, the useful signal, the SNR at SR i, and the achievable rate from ST i to SR i are respectively given by

$$\hat{Y}_{PT_j,\text{NC}}^{SR_i} = \frac{\sqrt{P_{i,j}} G_{PT_j}^{SR_i}}{\sqrt{1 + P_i|G_{ST_i}^{PT_j}|^2 + \sigma^2}} \Big(\sqrt{P_i} G_{ST_i}^{PT_j} X_i + n_{PT_j} \Big) + n_{SR_i}, \quad (15)$$

$$\Gamma_{SR_i}(3) = \frac{P_{i,j}|G_{PT_j}^{SR_i}|^2 P_i |G_{ST_i}^{PT_j}|^2}{\sigma^2(1 + P_i|G_{ST_i}^{PT_j}|^2 + P_{i,j}|G_{PT_j}^{SR_i}|^2 + \sigma^2)}, \quad (16)$$

$$R_{i,j} = \frac{W_j}{3} \log_2(1 + \Gamma_{SR_i}(2) + \Gamma_{SR_i}(3)). \quad (17)$$

III. JOINT SPECTRUM AND POWER AUCTION

A. PU's Utility Function

Each PU $j \in \{1, ..., M\}$, sells two heterogeneous commodities (channels and cooperative power) among N SUs. The supply of PU j can be denoted by a vector $\mathbf{S_j} = (K_j, P_{U_j})$, which consists of the number of the licensed channels and the available cooperative power PU j has. Let λ_j^1 and λ_j^2 be the prices of a channel and a power unit PU j asks for. The price vector of PU j then is denoted by $\lambda_\mathbf{j} = (\lambda_j^1, \lambda_j^2)$. It is noted that the channels owned by the same PU are assumed to be identical, i.e., they have the same bandwidth, carrier frequency, modulating scheme, etc., thus they sell at the same price.

The utility of PU j is defined as the summation of its achievable rate and the payoff it receives in channel and power auction. That is

$$U_{j,\text{C}}(\mathbf{S_j}, \lambda_\mathbf{j}) = g \sum_{i=1}^{K_j} R_{j,i} + \lambda_j^1 K_j + \lambda_j^2 P_{U_j}. \quad (18)$$

where g is a positive constant providing conversion of units.

B. SU's Utility Function

When purchasing the channel and cooperative power, each SU i wishes to maximize its transmission rate with minimum cost. Formally, we formulate SU i's purchase strategy as its bids to be submitted to all the PUs:

$$\mathbf{Q_i} = (\mathbf{Q_{i,1}}, ..., \mathbf{Q_{i,j}}, ..., \mathbf{Q_{i,M}})^T, \quad (19)$$

where $\mathbf{Q_{i,j}} = (C_{i,j}, P_{i,j})$ is a resource demand vector. $P_{i,j}$ represents the required cooperative power of SU i from PU j. $C_{i,j} \in \{0, 1\}$ specifies that whether SU i is willing to buy a channel from PU j. If it is, $C_{i,j} = 1$; Otherwise $C_{i,j} = 0$. It is worth mentioning that the channel and cooperative power are two different types of commodities, of which the channel is indivisible and the cooperative power is divisible. Therefore, the channel is available in a supply of one and thus can be assigned either totally or nothing; The cooperative power can be offered at any quantity of $P_{i,j}$, subject to the constraint that $0 \le \sum_{i=1}^N P_{i,j} \le P_{U_j}$.

Furthermore, the proposed cooperation architecture requires the SU to buy the channel and power from the same PU. It implies that the channel and power should be offered as a bundle. For each SU, it can only purchase the entire bundle or nothings. When SU i does not buy the channel from PU j, i.e., $C_{i,j} = 0$, it will not receive any power from that PU, i.e., we have $P_{i,j} = 0$. Similarly, if PU j does not assign any power to SU i, i.e., $P_{i,j} = 0$, it is not allowed to sell the channel to that SU, i.e., we have $C_{i,j} = 0$.

To depict a SU's satisfaction with the received channel and power from PU j, we define a utility function of SU i to PU j as:

$$U_{i,j}(\mathbf{Q_{i,j}}, \lambda_\mathbf{j}) = g R_{i,j}(P_{i,j}) - \lambda_j^1 C_{i,j} - \lambda_j^2 P_{i,j}. \quad (20)$$

Then the utility function of SU i is defined as:

$$U_i = \max_{j \in \{1, ..., M\}} \quad U_{i,j}(\mathbf{Q_{i,j}}, \lambda_\mathbf{j}). \quad (21)$$

If SU i decides to purchase channel and power from PU j, i.e., $C_{i,j}=1$, and $C_{i,k}=0$ for $k\neq j$, then the optimal cooperative power demand $P^*_{i,j}$ of SU i can be achieved by solving the following utility maximization problem:

$$\begin{aligned}\max_{P_{i,j}} \quad & U_{i,j}(\mathbf{Q_{i,j}},\lambda_{\mathbf{j}}) = gR_{i,j}(P_{i,j}) - \lambda^1_j - \lambda^2_j P_{i,j} \\ \text{s.t.} \quad & 0 \le P_{i,j} \le P_{U_j}.\end{aligned} \tag{22}$$

C. Ascending Clock Auction Mechanism

We model a multi-auctioneer, multi-bidder and multi-commodity auction game to efficiently allocate the channels and cooperative power of M PUs among N SUs. Each PU j, i.e. the auctioneer, iteratively announces the prices of its two commodities. Each SU i, i.e. the bidder, responds to each PU j by submitting its demand $\mathbf{Q_{i,j}}$, which reports the quantities of the channel and the power it wishes to purchase at these prices. PU j then calculates the *cumulative clinch* by ascending clock auction algorithm [12]. Thereafter, PU j adjusts the prices according to the relationship between the total demand and the total supply. This process repeats until the prices converge at which the total demand is no more than the total supply.

1) PU selection

The PU selection occurs on each SU at each auction clock, by which the SU determines from which PU it buys a channel and how much cooperative power it requests from that PU. For example, at the auction clock τ, each PU j announces its current prices in a form of $\lambda_{\mathbf{j}}(\tau) = (\lambda^1_j(\tau), \lambda^2_j(\tau))$ to all the SUs. Based on these prices, SU i selects the desired PU. To do that, SU i sets $C_{i,j}=1, \forall j \in \{1,...,M\}$, and separately solves M utility maximization problems defined in (22). Then it finds out the desired PU j that incurs the maximum utility, and places its bids to that PU as $C_{i,j}(\lambda^1_j(\tau)) = 1$, and $P_{i,j}(\lambda^2_j(\tau)) = P^*_{i,j}(\lambda^2_j(\tau))$. For any other PU $k\neq j$, it sets the bids to $C_{i,k}(\lambda^1_k(\tau)) = 0$ and $P_{i,k}(\lambda^2_k(\tau)) = 0$.

2) Resource crediting

At each auction clock τ, PU j collects N SUs' bids, and computes the total required channels and power of these SUs. Let $C^{\text{tal}}_j(\lambda^1_j(\tau)) = \sum_{i=1}^N C_{i,j}(\lambda^1_j(\tau))$ and $P^{\text{tal}}_j(\lambda^2_j(\tau)) = \sum_{i=1}^N P_{i,j}(\lambda^2_j(\tau))$ represent the total channel and power demand at PU j at clock τ, respectively. Further, let $E^1_j(\lambda^1_j(\tau)) = C^{\text{tal}}_j(\lambda^1_j(\tau)) - K_j$ and $E^2_j(\lambda^2_j(\tau)) = P^{\text{tal}}_j(\lambda^2_j(\tau)) - P_{U_j}$ represent the excess channel and power demand at PU j, respectively. Then PU j adjusts its price vector according to the excess demand.

Case 1: $E^1_j(\lambda^1_j(\tau)) > 0$ and $E^2_j(\lambda^2_j(\tau)) > 0$. It tells that the total demand for the power and the channel both exceed the supply. Due to the indivisibility of the channel, none of the channels would be credited to any SU. For bundling sale, the power would not be credited to any SU, either. So we have

$$\hat{C_{i,j}}(\lambda^1_j(\tau)) = 0,\ \hat{P_{i,j}}(\lambda^2_j(\tau)) = 0,\ \forall i \in \{1,...,N\}, \tag{23}$$

where $\hat{C_{i,j}}(\lambda^1_j(\tau))$ and $\hat{P_{i,j}}(\lambda^2_j(\tau))$ are the cumulative clinch, which are the amounts of the channel and power that are credited to SU i at the price $\lambda_{\mathbf{j}}(\tau)$. Thereafter, PU j updates its price vector with $\lambda^1_j(\tau+1) = \lambda^1_j(\tau) + \mu^1_j$, and $\lambda^2_j(\tau+1) = \lambda^2_j(\tau) + \mu^2_j$, where $\mu^1_j > 0$ and $\mu^2_j > 0$ are step sizes, and announces this new price vector to all SUs.

Case 2: $E^1_j(\lambda^1_j(\tau)) > 0$ and $E^2_j(\lambda^2_j(\tau)) \le 0$. In this case, there are more than K_j SUs competing for PU j's channels, whose total power demand is less than PU j's supply. Similar to Case 1, neither the channel nor the power would be credited to any SU. Therefore, the cumulative clinch to the SUs are also calculated by (23). Finally, the price of the channel is updated by $\lambda^1_j(\tau+1) = \lambda^1_j(\tau) + \mu^1_j$. While the price of per unit power remains unchanged as $\lambda^2_j(\tau+1) = \lambda^2_j(\tau)$ for the sake that the total power demand does not exceed the supply.

Case 3: $E^1_j(\lambda^1_j(\tau)) \le 0$ and $E^2_j(\lambda^2_j(\tau)) > 0$. As the supply of the channels is sufficient, the channels can be credited to all the SUs who bid for them. Moreover, the power can be credited to each SU in terms of their opponents' demands. Thus, for each SU i with $C_{i,j}(\lambda^1_j(\tau)) = 1$, we have

$$\begin{aligned}&\hat{C_{i,j}}(\lambda^1_j(\tau)) = 1,\\ &\hat{P_{i,j}}(\lambda^2_j(\tau)) = \max\Big(0, P_{U_j} - \sum_{k=1,k\neq i}^{N} P_{k,j}(\lambda^2_j(\tau))\Big).\end{aligned} \tag{24}$$

For each SU i with $C_{i,j}(\lambda^1_j(\tau)) = 0$, we have

$$\hat{C_{i,j}}(\lambda^1_j(\tau)) = 0, \quad \hat{P_{i,j}}(\lambda^2_j(\tau)) = 0. \tag{25}$$

Thereafter, PU j updates the price vector with $\lambda^1_j(\tau+1) = \lambda^1_j(\tau)$, and $\lambda^2_j(\tau+1) = \lambda^2_j(\tau) + \mu^2_j$.

Case 4: $E^1_j(\lambda^1_j(\tau)) \le 0$ and $E^2_j(\lambda^2_j(\tau)) \le 0$. It shows that the supply of both channel and power is sufficient for all the competitors. Therefore, each competitor would be credited according to its demand. Namely

$$\hat{C_{i,j}}(\lambda^1_j(\tau)) = C_{i,j}(\lambda^1_j(\tau)),\ \hat{P_{i,j}}(\lambda^2_j(\tau)) = P_{i,j}(\lambda^2_j(\tau)). \tag{26}$$

Then two prices are kept unchanged with $\lambda^1_j(\tau+1) = \lambda^1_j(\tau)$, and $\lambda^2_j(\tau+1) = \lambda^2_j(\tau)$.

Additionally, the demand $\mathbf{Q_{i,j}}(\lambda_{\mathbf{j}}(\tau))$ of SU i from PU j is a function of PU j's announced price $\lambda_{\mathbf{j}}(\tau)$. If PU j's price is too high at τ, SU i which chose PU j at $\tau-1$ might give up it and choose another PU at τ, then all the channel and power clinched to SU i before at PU j become unclinched. Correspondingly, all the credits SU i received before should be cleared. So, in the above four cases, for SU i with $C_{i,j}(\lambda^1_j(\tau)) = 0$ and $C_{i,j}(\lambda^1_j(\tau-1)) = 1$, we have

$$\hat{C_{i,j}}(\lambda^1_j(\tau')) = 0, \hat{P_{i,j}}(\lambda^2_j(\tau')) = 0, \forall \tau' \in \{0,...,\tau-1\}. \tag{27}$$

3) Payment calculation

Assuming that the supply meets the total demand for each PU at clock $\tau = T$, i.e., $E^1_j(\lambda^1_j(\tau)) = 0$ and $E^2_j(\lambda^2_j(\tau)) \le 0, \forall j \in \{1,...,M\}$, then the auction converges to an equilibrium price vector $\lambda_{\mathbf{j}}{}^* = \lambda_{\mathbf{j}}(T)$. Consider that the supply P_{U_j} might not be fully covered at price $\lambda_{\mathbf{j}}{}^*$, i.e. $E^2_j(\lambda^2_j(\tau)) < 0$. For each SU i with $\hat{P_{i,j}}(\lambda^2_j(T)) \neq 0$, its cumulative clinch is re-calculated by [12]:

$$\begin{aligned}&\hat{P_{i,j}}(\lambda^2_j(T)) = P_{i,j}(\lambda^2_j(T)) + \\ &\frac{P_{i,j}(\lambda^2_j(T-1)) - P_{i,j}(\lambda^2_j(T))}{\sum\limits_{i=1}^N P_{i,j}(\lambda^2_j(T-1)) - \sum\limits_{i=1}^N P_{i,j}(\lambda^2_j(T))}\left[P_{U_j} - \sum_{i=1}^N P_{i,j}(\lambda^2_j(T))\right]\end{aligned} \tag{28}$$

Finally, the quantities of the channel and power that are assigned to the SU is given by

$$C^*_{i,j} = \hat{C_{i,j}}(\lambda^1_j(T)),\ P^*_{i,j} = \hat{P_{i,j}}(\lambda^2_j(T)). \tag{29}$$

Correspondingly, the payment for the channel from SU i to PU j is

$$\begin{aligned} V^1_{i,j} =& \lambda^1_j(0)\hat{C_{i,j}}(\lambda^1_j(0)) \\ &+ \sum_{\tau=1}^{T} \lambda^1_j(\tau)\Big(\hat{C_{i,j}}(\lambda^1_j(\tau)) - \hat{C_{i,j}}(\lambda^1_j(\tau-1))\Big), \end{aligned} \tag{30}$$

and the payment for the power from SU i to PU j is

$$\begin{aligned} V^2_{i,j} =& \lambda^2_j(0)\hat{P_{i,j}}(\lambda^2_j(0)) \\ &+ \sum_{\tau=1}^{T} \lambda^2_j(\tau)\Big(\hat{P_{i,j}}(\lambda^2_j(\tau)) - \hat{P_{i,j}}(\lambda^2_j(\tau-1))\Big). \end{aligned} \tag{31}$$

D. Existence of Walrasian Equilibrium

First, we specify a generic economic model: M auctioneers wish to allocate units of each of K heterogeneous commodities among N bidders. For each auctioneer j, its available supply is $\mathbf{S_j} = (S^1_j, ..., S^K_j)$, its announced price vector is $\lambda_\mathbf{j} = (\lambda^1_j, ..., \lambda^K_j)$, and its allocation to bidder i is $\mathbf{A_{i,j}} = (A^1_{i,j}, ..., A^K_{i,j})$. For each bidder i, its demand from auctioneer j at price $\lambda_\mathbf{j}$ is $\mathbf{Q_{i,j}}(\lambda_\mathbf{j}) = (Q^1_{i,j}(\lambda^1_j), ..., Q^K_{i,j}(\lambda^K_j))$, its payment to auctioneer j is $V_{i,j}$, and it has a function $F_{i,j}(\mathbf{Q_{i,j}})$ with respect to $\mathbf{Q_{i,j}}$.

Definition 1. *A Walrasian Equilibrium is a $M \times K$ price vector* $\lambda^* = \begin{pmatrix} \lambda^{1*}_1 & \lambda^{2*}_1 & \dots & \lambda^{K*}_1 \\ \vdots & \vdots & & \vdots \\ \lambda^{1*}_M & \lambda^{2*}_M & \dots & \lambda^{K*}_M \end{pmatrix}$ *and a $N \times M \times K$ allocation vector* $\mathbf{A}^* = \begin{pmatrix} \mathbf{A}^*_{\mathbf{1,1}} & \mathbf{A}^*_{\mathbf{1,2}} & \dots & \mathbf{A}^*_{\mathbf{1,M}} \\ \vdots & \vdots & & \vdots \\ \mathbf{A}^*_{\mathbf{N,1}} & \mathbf{A}^*_{\mathbf{N,2}} & \dots & \mathbf{A}^*_{\mathbf{N,M}} \end{pmatrix}$*, such that* $\mathbf{Q}(\lambda^*) = \begin{pmatrix} \mathbf{Q_{1,1}}(\lambda^*) & \mathbf{Q_{1,2}}(\lambda^*) & \dots & \mathbf{Q_{1,M}}(\lambda^*) \\ \vdots & \vdots & & \vdots \\ \mathbf{Q_{N,1}}(\lambda^*) & \mathbf{Q_{N,2}}(\lambda^*) & \dots & \mathbf{Q_{N,M}}(\lambda^*) \end{pmatrix} = \mathbf{A}^*$*, and* $S^k_j = \sum_{i=1}^{N} A^{k*}_{i,j}, \forall j \in \{1, ..., M\}, \forall k \in \{1, ..., K\}$.

Lemma 1. *[12] An auction has a Walrasian Equilibrium if it satisfies :*

(1) Pure private values: Bidder i's value, i.e. $F_{i,j}(\mathbf{Q_{i,j}})$, for the demand vector $\mathbf{Q_{i,j}}$ does not change when bidder i learns other bidders' information.

(2) Quasilinearity: Bidder i's utility from receiving the demand vector $\mathbf{Q_{i,j}}$ in return for the payment $V_{i,j}$ is given by $F_{i,j}(\mathbf{Q_{i,j}}) - V_{i,j}$.

(3) Monotonicity: The function $F_{i,j}(\mathbf{Q_{i,j}})$ is increasing, i.e., if $\mathbf{Q'_{i,j}} > \mathbf{Q_{i,j}}$, then $F_{i,j}(\mathbf{Q'_{i,j}}) > F_{i,j}(\mathbf{Q_{i,j}})$.

(4) Concavity: The function $F_{i,j}(\mathbf{Q_{i,j}})$ is concave.

Theorem 1. *The proposed multi-auctioneer multi-bidder and multi-commodity auction has a Walrasian Equilibrium.*

It is easy to find out that the proposed auction model satisfies four conditions in Lemma 1.

IV. SIMULATION RESULTS

We consider a scenario as shown in Fig. 2, where there are two PUs and six SUs in the network. PU 1 has 2 channels and PU 2 has 4 channels. The channel gains are $(\frac{0.097}{d^\alpha})^{\frac{1}{2}}$, where d is the distance between two nodes, and the path-loss exponent is $\alpha = 4$. The transmit power of each SU is $0.01W$, the transmit power of PU 1 and PU 2 are respectively $2W$ and $1W$, and the noise variance is $\sigma^2 = 10^{-13}$.

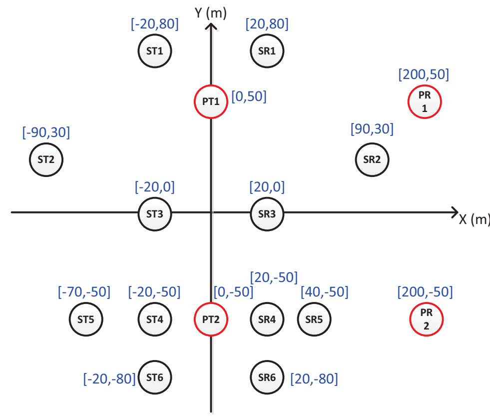

Fig. 2. A six-SU two-PU simulation network

Fig. 3 shows the convergence performance of the proposed resource allocation algorithm, where the step sizes are set to $\mu^1_1 = \mu^1_2 = 0.2$, and $\mu^2_1 = \mu^2_2 = 0.5$. It is observed that four prices converge at different speeds, and the auction of the entire system converges after 71 iterations. Compared the convergent power prices (i.e. the optimal values) of two PUs, we find that the optimal power price of PU 2 is larger than that of PU 1. It indicates that the power competition at PU 2 is more stronger than that at PU 1, thus leading to more auction clocks and a higher convergent power price. Also, it is noticed that the channel price of PU 2 keeps unchanged throughout the entire auction process. The reason is that the number of SUs that choose PU 2 is always no more than the number of PU 2's available channels. Therefore, the channel price of PU 2 keeps at the initial price during the whole auction.

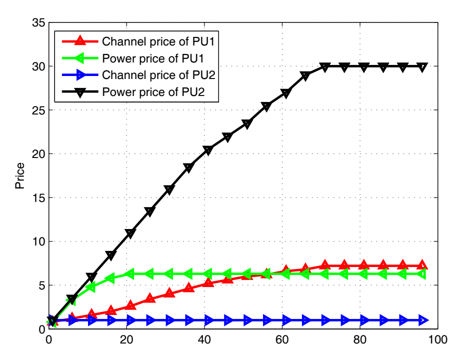

Fig. 3. Convergence of PUs' prices

Fig. 4 displays the evolution of the PU selection at SU 3, where "1" (Y-axis) represents SU 3 selects PU 1, and "2" represents it chooses PU 2. It is found that SU 3's selection varies between PU 1 and PU 2, and is finally stable at PU 2. According to the proposed algorithm, the SU at each auction clock, chooses the PU that currently incur the maximum utility. Due to different excess resource demand, two PUs might conduct different price adjustments at each clock. As a result, the PU selected by SU 3 at clock τ might not bring it the maximum utility at $\tau+1$, then SU 3 leaves that PU and selects another one at $\tau+1$. For instance, SU 3 selects PU 2 at $\tau = 9$ and changes to PU 1 at $\tau = 10$.

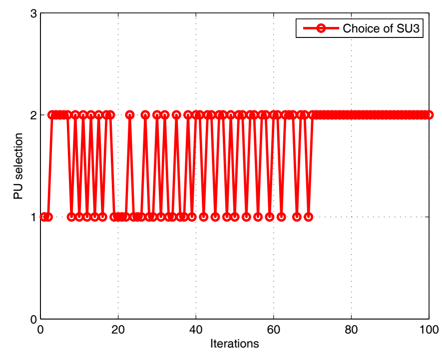

Fig. 4. The PU selection at SU 3

We now adjust the transmit power supply of each PU within a range of $[1W, 7W]$. It is seen in Fig. 5, that the optimal power price of each PU decreases with the increase of its power supply. Since the less power available at a PU, the smaller possibility that this supply can meet the total power demand of the SUs, the higher optimal power price would be, and vice versa. Fig. 6 display the utility achieved at each SU, where the simulation results show that SU 1 and SU 2 finally choose PU 1 and SU 3$\sim$6 choose PU 2. Clearly, the more power available at the PU, the more power is assigned to each SU, the lower optimal power price at the PU. As a result, the utility of each SU increases with the power supply of the PU.

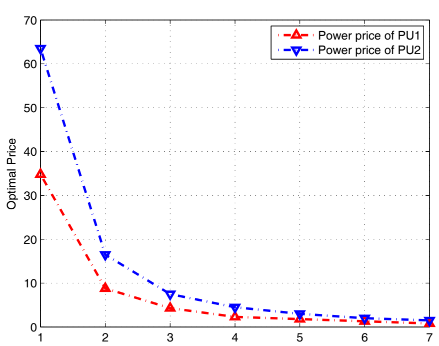

Fig. 5. Optimal power price of each PU

V. Conclusion

In this article, we tackled the joint power and spectrum allocation problem under a new cooperative cognitive radio framework, where primary users assist secondary transmissions and earn the revenue from selling the spectrum and cooperative power to secondary users. The trade between primary users and secondary users is modeled as an auction with multiple bundling commodities. The auction algorithm and its convergence performance are investigated. Future work can be extended to the cases in which the spectrum can be sold either separately or together with the cooperative power. In this way, secondary users can choose cooperative transmission (buy the spectrum and power both) or direct transmission (only buy the spectrum) for maximizing the utility.

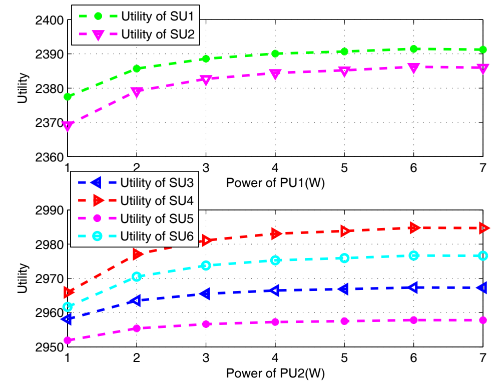

Fig. 6. The utility achieved at SUs

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